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Generating Skolem Difference Mean Graphs

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ABSTRACT

A graph *G (V, E)* with *p* vertices and *q* edges is said to have skolem difference mean labeling if it is possible to label the vertices $x \in V$ with distinct elements *f(x)* from *{1,2,….,p+q}* in such a way that the edge *e = uv* is labeledwith $\frac{|f(u)-f(v)|}{2}$ if $|f(u)-f(v)|$ is even and $\frac{|f(u)-f(v)|+1}{2}$ if $|f(u)-f(v)|$ is odd and the resulting edges get distinct labels from *{1,2,…,q}*. A graph that admits skolem difference mean labeling is called a Skolem difference mean graph A graph *G = (V, E)* with *p* vertices and *q* edges is said to have Near skolem difference mean labeling if it is possible to label the vertices $x \in V$ with distinct elements *f(x)* from *{1,2,….,p+q-1,p+q+2}* in such a way that each edge *e = uv*, is labeled as *f*(e)* = $\frac{|f(u)-f(v)|}{2}$ if $|f(u)-f(v)|$ is even and *f*(e)* =$\frac{|f(u)-f(v)|+1}{2}$ if $|f(u)-f(v)|$ is odd. The resulting labels of the edges are distinct and from *{1,2,…,q}*. A graph that admits a Near skolem difference mean labeling is called a Near Skolem difference mean graph. In this paper, the authors generate skolem difference mean graphs from near skolem difference mean graphs.

***Keywords*:** Skolem difference mean graphs, Near skolem difference mean graphs, Duplication of graph elements

1. INTRODUCTION

The graphs represented in this paper are finite, undirected and simple one. The vertex set and the edge set of a graph G are denoted by V(G) and E(G) respectively. Terms and notations

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not defined here are used in the sense of Harary [2]. For number theoretic terminologies, [1] is referred.

A graph labeling is an assignment of integers to the vertices or edges or both vertices and edges subject to certain conditions. Rosa [13] introduced β-valuation of a graph in the year 1966 and Golomb [4] called it as graceful labeling. These are several types of graph labeling and a detailed survey is found in [7]. The notion of skolem difference mean labeling was due to Murugan and Subramanian. They studied skolem difference mean graphs of H-graphs in [10]. Various Skolem Difference Mean Graphs were studied in [3, 5, 6, 9, 11, 12, 14, 17].

The notion of near skolem difference mean labeling was due to Shebagadevi and Nagarajan [15]. They studied near skolem difference mean labeling of some special types of trees and duplicating the components of the path in [15, 16]. In this paper some skolem difference mean graphs are generated from near skolem difference mean graphs. The following definitions are necessary for the present study.

Labeled graphs are becoming an increasing useful family of mathematical models for a broad range of applications like designing go-ray crystallographic analysis, formulating a communication network addressing system, determining optimal circuit layouts, problems in additive number theory etc. Applications of Graph Labeling in Communication Networks were studied in [8].

Definition 1.1: The H-graph of a path P_n is the graph obtained from two copies of P_n with vertices $v_1, v_2, \ldots, v_n$ and $u_1, u_2, \ldots, u_n$ by joining the vertices $v_{\frac{n+1}{2}}$ and $u_{\frac{n+1}{2}}$ if n is odd and the vertices $v_{\frac{n}{2}+1}$ and $u_{\frac{n}{2}}$ if n is even.

Definition 1.2: The Banana tree denoted by $Bt(n_1, n_2, \ldots, n_m)$ (m times n) is a graph obtained by connecting a vertex v_0 to one leaf of each of m number of stars.

Definition 1.3: A lily graph I_n, $n \geq 2$ can be constructed by two-star graphs $2K_{1,n}$, $n \geq 2$ joining two path graphs $2P_n$, $n \geq 2$ with sharing a common vertex, that is $I_n = 2K_{1,n} + 2P_n$.

Definition 1.4: The coconut tree graph $T(n,m)$ is obtained by identifying the central vertex of $K_{1,n}$ with a pendant vertex of the path P_m.

Definition 1.5: Let v be a vertex of a graph G. Then the duplication of v is a graph $G(v)$ obtained from G by adding a new vertex v' with $N(v') = N(v)$.

Definition 1.6: Let $e = uv$ be an edge of G. Then duplication of an edge $e = uv$ is a graph $G(uv)$ obtained from G by adding a new edge $u'v'$ such that $N(u') = N(u) \cup \{v'\} - \{v\}$ and $N(v') = N(v) \cup \{u'\} - \{u\}$.

Definition 1.7: Duplication of a vertex v_k by a new edge $e' = u'v'$ in a graph G produces a new graph G' such that $N(v') = \{v_k, u'\}$ and $N(u') = \{v_k, v'\}$.

Definition 1.8: Duplication of an edge $e = uv$ by a new vertex v' in a graph G produces a new graph G' such that $N(v') = \{u,v\}$.

Definition 1.9 [10]: A graph *G (V, E)* with *p* vertices and *q* edges is said to have skolem difference mean labeling if it is possible to label the vertices $x \in V$ with distinct elements *f(x)* from *{1,2,….,p+q}* in such a way that the edge *e = uv* is labeled with $\frac{|f(u)-f(v)|}{2}$ if $|f(u)-f(v)|$ is even and $\frac{|f(u)-f(v)|+1}{2}$ if $|f(u)-f(v)|$ is odd and the resulting edges get distinct labels from *{1,2,…,q}*. A graph that admits skolem difference mean labeling is called a Skolem difference mean graph.

Definition 1.10 [15]: A graph *G = (V, E)* with *p* vertices and *q* edges is said to have Near skolem difference mean labeling if it is possible to label the vertices $x \in V$ with distinct elements *f(x)* from *{1,2,….,p+q-1,p+q+2}* in such a way that each edge *e = uv*, is labeled as $f^*(e) = \frac{|f(u)-f(v)|}{2}$ if $|f(u)-f(v)|$ is even and $f^*(e) = \frac{|f(u)-f(v)|+1}{2}$ if $|f(u)-f(v)|$ is odd. The resulting labels of the edges are distinct and from *{1,2,…,q}*. A graph that admits a Near skolem difference mean labeling is called a Near Skolem difference mean graph.

2. RESULTS

Theorem 2.1: If the *H*-graph *G* is near skolem difference mean, then it is skolem difference mean.

Proof: *Let* $V(G) = \{v_i, u_i / 1 \le i \le n\}$ and
$E(G) = \{v_i v_{i+1}, u_i u_{i+1} / 1 \le i \le n-1\} \cup \{v_{\frac{n+1}{2}} u_{\frac{n+1}{2}}$ (when *n* is odd) and $v_{\frac{n}{2}+1} u_{\frac{n}{2}}$(when *n* is even)$\}$

Then $|V(G)| = 2n$ and $|E(G)| = 2n-1$.

Let $f: V(G) \rightarrow \{1,2,3,…,p+q-1,p+q+2\}$ be a near skolem difference mean labeling of *G* as defined in [15].

Let $g: V(G) \rightarrow f$ be defined as follows.

Case i: when *n* is odd

$g(v_{2i+1}) = f(v_{2i+1}) - 2;\ 0 \le i \le \frac{n-1}{2}$
$g(v_2) = f(v_2) - 2;$
$g(v_{2i}) = f(v_{2i}) - 2;\ 2 \le i \le \frac{n-1}{2}$
$g(u_{2i+1}) = f(u_{2i+1}) - 2;\ 0 \le i \le \frac{n-1}{2}$
$g(u_{2i}) = f(u_{2i}) - 2;\ 1 \le i \le \frac{n-1}{2}$

Case ii: when *n* is even.

$g(v_{2i+1}) = f(v_{2i+1}) - 2;\ 0 \le i \le \frac{n-2}{2}$
$g(v_2) = f(v_2) - 2;$
$g(v_{2i}) = f(v_{2i}) - 2;\ 2 \le i \le \frac{n}{2}$
$g(u_{2i+1}) = f(u_{2i+1}) - 2;\ 0 \le i \le \frac{n-2}{2}$

$g(u_{2i}) = f(u_{2i}) - 2;\ 1 \le i \le \frac{n}{2}$

Let g^* be the induced edge labelling of g.

Then $g^*(v_i v_{i+1}) = f^*(v_i v_{i+1});\ 1 \le i \le n-1$
$g^*(v_{\frac{n+1}{2}} u_{\frac{n+1}{2}}) = f^*(v_{\frac{n+1}{2}} u_{\frac{n+1}{2}})$; when n is odd.
$g^*(v_{\frac{n}{2}+1} u_{\frac{n}{2}}) = f^*(v_{\frac{n}{2}+1} u_{\frac{n}{2}})$; when n is even.
$g^*(u_i u_{i+1}) = f^*(u_i u_{i+1});\ 1 \le i \le n-1$.

In both cases, the induced edge labels are all distinct and are $g^*(E(G)) = \{1,2,\ldots,2n-1\}$.

Hence the H-graph G is a skolem difference mean graph.

Theorem 2.2: If the banana tree $Bt\ (n,n,\ldots,n)$, (m times n) is a near skolem difference mean graph, then it is skolem difference mean graph.

Proof: Let the graph be denoted by G.

Let $V(G) = \{v_0, v_j, u_{ij} / 1 \le i \le n \text{ and } 1 \le j \le m\}$ and $E(G) = \{v_0 u_{1j}, v_j u_{ij} / 1 \le i \le n, 1 \le j \le m\}$.

Then $|V(G)| = mn+m+1$ and $|E(G)| = mn+m$.

Let $f: V(G) \rightarrow \{1,2,\ldots,2mn+2m,2mn+2m+3\}$ be a near skolem difference mean labelling of G as defined in [15].

Let $g: V(G) \rightarrow f$ be defined as follows.
$g(v_0) = f(v_0) - 2;$
$g(v_j) = f(v_j) - 2;\ 1 \le j \le m$
$g(u_{1j}) = f(u_{1j}) - 2;\ 1 \le j \le m$
$g(u_{ij}) = f(u_{ij}) - 2;\ 2 \le i \le n-1 \text{ and } 1 \le j \le m$
$g(u_{nj}) = f(u_{nj}) - 2;\ 1 \le j \le m-1$
$g(u_{nm}) = f(u_{nm}) - 2;$

Let g^* be the induced edge labelling of g.

Then $g^*(v_0 u_{1j}) = f^*(v_0 u_{1j});\ 1 \le j \le m$
$g^*(v_j u_{ij}) = f^*(v_j u_{ij});\ 1 \le i \le n-1 \text{ and } 1 \le j \le m$.
$g^*(v_j u_{nj}) = f^*(v_j u_{nj});\ 1 \le j \le m$.

Thus the induced edge labels are all distinct and are $g^*(E(G)) = \{1,2,\ldots,mn+m\}$.

Hence the banana tree $Bt\ (n,n,\ldots,n)$ is skolem difference mean.

Theorem 2.3: If the lily graph $I_n\ (n \ge 2)$ admits near skolem difference mean, then it is skolem difference mean.

Proof: Let G be the lily graph I_n with $n \ge 2$.
Let $V(G) = \{u_i, v_j / 1 \le i \le 2n, 1 \le j \le 2n-1\}$ and $E(G) = \{v_n u_i, v_j v_{j+1} / 1 \le i \le 2n, 1 \le j \le 2n-2\}$.
Then $|V(G)| = 4n-1$ and $|E(G)| = 4n-2$.

Let $f: V(G) \rightarrow \{1,2,\ldots,8n\text{-}4,8n\text{-}1\}$ be a near skolem difference mean labelling of G as defined in [15].

Let $g: V(G) \rightarrow f$ be defined as follows.

Case i: when n is odd.
$g(v_1) = f(v_1) - 2;$
$g(v_{2i+1}) = f(v_{2i+1}) - 2; 1 \leq i \leq n\text{-}1.$
$g(v_{2i}) = f(v_{2i}) - 2; 1 \leq i \leq n\text{-}1.$
$g(u_i) = f(u_i) - 2; 1 \leq i < 2n.$

Case ii: when n is even.
$g(v_{2i+1}) = f(v_{2i+1}) - 2; 0 \leq i \leq n\text{-}1.$
$g(v_2) = f(v_2) - 2;$
$g(v_{2i}) = f(v_{2i}) - 2: 1 \leq i \leq n\text{-}1.$
$g(u_i) = f(u_i) - 2; 1 \leq i \leq 2n.$

Let g^* be the induced edge labeling of g.

Then $g^*(v_jv_{j+1}) = f^*(v_jv_{j+1}); 1 \leq i \leq 2n\text{-}2.$
$g^*(v_nu_i) = f^*(v_nu_i); 1 \leq i \leq 2n.$

Thus the induced edge labels are all distinct and are $g^*(E(G)) = \{1,2,\ldots,4n\text{-}2\}$.

Hence the lily graph admits skolem difference mean labeling for $n \geq 2$.

Theorem 2.4: If the coconut tree $T(n, m)$ is near skolem difference mean for every $n, m \geq 1$, then it is skolem difference mean.

Proof: Let G be the coconut tree $T(n, m)$.

Let $V(G) = \{v_i, u_j / 1 \leq i \leq n, 1 \leq j \leq m\}$ and $E(G) = \{v_iv_{i+1}, v_1u_j / 1 \leq i \leq n\text{-}1, 1 \leq j \leq m\}$.

Then $|V(G)| = n+m$ and $|E(G)| = n+m\text{-}1$.

Let $f: V(G) \rightarrow \{1,2,\ldots,2n+2m\text{-}2,2n\text{-}2m+1\}$ be a near skolem difference mean labelling of Gas defined in [15].

Let $g: V(G) \rightarrow f$ be defined as follows.

Case i: when n is odd.
$g(v_{2i+1}) = f(v_{2i+1}) - 2; 0 \leq i \leq \frac{n-1}{2}.$
$g(v_2) = f(v_2) - 2;$
$g(v_{2i}) = f(v_{2i}) - 2; 1 \leq i \leq \frac{n-1}{2}.$
$g(u_i) = f(u_i) - 2; 1 \leq i \leq m.$

Case ii: when n is even.
$g(v_{2i+1}) = f(v_{2i+1}) - 2; 0 \leq i \leq \frac{n-2}{2}.$
$g(v_2) = f(v_2) - 2;$

$g\,(v_{2i}) = f\,(v_{2i}) - 2;\ 1 \le i \le \frac{n}{2}$.
$g\,(u_i) = f\,(u_i) - 2;\ 1 \le i \le m$.

In both cases let g^* be the induced edge labelling of g.

Then $g^*\,(v_i v_{i+1}) = f^*\,(v_i v_{i+1});\ 1 \le i \le n-1$.
$g^*\,(v_1 u_j) = f^*\,(v_1 u_j);\ 1 \le j \le m$.

The induced edge labelling are all distinct and are $g^*(E\,(G)) = \{1,2,\ldots,n+m-1\}$.

Hence the coconut tree is a skolem difference mean for every $n, m \ge 1$.

Theorem 2.5: The graph obtained by duplicating an arbitrary vertex of P_n is near skolem difference mean, then it is skolem difference mean.

Proof: Let P_n be the path.

Let v be the new vertex which is adjacent to both v_{i-1} and v_{i+1}, thus forming a new graph G.

The graph G is obtained by duplicating an arbitraty vertex v_i, $1 \le i \le n$ of the given path.

Let $V\,(G) = \{v, v_i \,/\, 1 \le i \le n\}$ and
$E\,(G) = \{vv_{i+1}, v_i v_{i+1} \,/\, 1 \le i \le n-1\}$ if v_i is a pendant vertex
$= \{vv_{i-1}, vv_{i+1}, v_i v_{i+1} \,/\, 1 \le i \le n-1\}$ if v_i is a not pendant vertex

Then $|V(G)| = n+1$ and $|E(G)| = \begin{cases} n \text{ if } v_i \text{ is a pendant vertex} \\ n+1 \text{ if } v_i \text{ is not a pendant vertex} \end{cases}$

Case i: when v_i is a pendant vertex.

Let $f\!: V\,(G) \to \{1, 2, \ldots, 2n, 2n+3\}$ to be near skolem difference mean labeling of G as defined in [16].

Let $g\!: V\,(G) \to f$ be defined as follows.

$g\,(v_{2i+1}) = f\,(v_{2i+1}) + 1\ ;\begin{matrix} 0 \le i \le \frac{n-1}{2}, if\ n\ is\ odd. \\ 0 \le i \le \frac{n-2}{2}, if\ n\ is\ even. \end{matrix}$

$g\,(v_{2i}) = f\,(v_{2i}) + 1\ ;\begin{matrix} 1 \le i \le \frac{n-1}{2}, if\ n\ is\ odd. \\ 1 \le i \le \frac{n}{2}, if\ n\ is\ even. \end{matrix}$

$g\,(v) = f\,(v) + 1$; if v is the duplicating vertex of v_1 or v_n.

Let g^* be the induced edge labelling of g. Then
$g^*(v_i v_{i+1}) = f^*(v_i v_{i+1});\ 1 \le i \le n-1$.
$g^*(vv_2) = f^*(vv_2)$;
$g^*(vv_{n-1}) = f^*(vv_{n-1})$;

The induced edge labels are distinct and are $\{1,2,\ldots,n\}$.

Case ii: When v_i is not a pendant vertex.

Let $f\!: V\,(G) \to \{1,2,\ldots,2n+1,2n+4\}$ be a near skolem difference mean labeling of G as defined in [16].

Let $g\!: V\,(G) \to f$ be definedas follows

$g\,(v_{2i+1}) = f\,(v_{2i+1}) - 1\ ;\begin{matrix} 0 \le i \le \frac{n-1}{2},\ \ if\ n\ is\ odd. \\ 0 \le i \le \frac{n-2}{2},\ \ if\ n\ is\ even. \end{matrix}$

$g\,(v_2) = f\,(v_2) - 2;$

$g\,(v_{2i}) = f\,(v_{2i})\ ;\begin{matrix} 2 \le i \le \frac{n-1}{2},\ \ if\ n\ is\ odd. \\ 2 \le i \le \frac{n}{2},\ \ if\ n\ is\ even. \end{matrix}$

$g\,(v) = f\,(v) - 1;\ \begin{matrix} i\ is\ even \\ i\ is\ odd \end{matrix}$

Let g^* be the induced edge labeling of g. Then
$g^*(v_iv_{i+1}) = f^*(v_iv_{i+1});\ 1 \le i \le n-1.$
$g^*(vv_{i-1}) = f^*(vv_{i-1});$
$g^*(vv_{i+1}) = f^*(vv_{i+1});$

The induced edge labels are distinct and are $\{1,2,\ldots,n+1\}$.

Hence the theorem.

Theorem 2.6: The graph obtained by duplicating an arbitrary edge of P_n is near skolem difference mean, then it is skolem difference mean.

Proof: Let P_n be the path.

Let $e = uv$ be the duplicated edge of v_kv_{k+1}.

Let G be the resulting graph.

Let $V(G) = \{u,v,\ v_i\ /\ 1 \le i \le n\}$ and
$E(G) = \{uv,\ v_iv_{i+1}\ /\ 1 \le i \le n-1\} \cup \{vv_3$ or vv_{n-2} when uv is the duplication of v_1v_2 or $v_{n-1}v_n$ respectively$\}$
$= \{uv, uv_{k-1},\ vv_{k-2},\ v_iv_{i+1}\ /\ 1 \le i \le n-1\}$ when uv is the duplication of an edge v_kv_{k+1} which is not a pendent edge.

Case i: when $uv$is the duplication of a pendant edge $v_1v_2$or $v_{n-1}v_n$.

Then $|V(G)| = n+2$ and $|E\ (G)| = n+1$.

Let $f: V(G) \rightarrow \{1,2,\ldots,2n+2,2n+5\}$ be a near skolem difference mean labeling of G as defined in [16].

Let g: $V\,(G) \rightarrow f$ be defined as follows

$g\,(v_{2i+1}) = f\,(v_{2i+1}) - 1\ ;\begin{matrix} 0 \le i \le \frac{n-1}{2},\ \ if\ n\ is\ odd. \\ 0 \le i \le \frac{n-2}{2},\ \ if\ n\ is\ even. \end{matrix}$

$g\,(v_2) = f\,(v_2)\ - 2;$

$g\,(v_{2i}) = f\,(v_{2i})\ ;\begin{matrix} 2 \le i \le \frac{n-1}{2},\ \ if\ n\ is\ odd. \\ 2 \le i \le \frac{n}{2},\ \ if\ n\ is\ even. \end{matrix}$

$g\,(u) = \begin{cases} f(u)+8; & when\ v_1v_2\ is\ the\ pendant\ edge. \\ f\ (u)+6; & when\ v_{n-1}v_n\ is\ the\ pendant\ edge. \end{cases}$

$g\,(v) = \begin{cases} f(v)+5; & when\ v_1v_2\ is\ the\ pendant\ edge. \\ f\ (v)+6; & when\ v_{n-1}v_n\ is\ the\ pendant\ edge. \end{cases}$

Let g^* be the induced edge labelling of g. Then
$g^*(v_iv_{i+1}) = f^*(v_iv_{i+1});\ 1 \le i \le n-1.$
$g^*(uv) = f^*(uv);$

$g^*(vv_3) = f^*(vv_3)$; when v_1v_2 is the pendant edge.
$g^*(vv_{n-2}) = f^*(vv_{n-2})$; when $v_{n-1}v_n$ is the pendant edge.

The edge labels are all distinct and are $\{1,2,\ldots,n+1\}$

Case ii: when uv is the duplication of some non-pendant edge v_kv_{k+1}.

Then $|V(G)| = n+2$ and $|E(G)| = n+2$.

Let $f: V(G) \to \{1,2,\ldots,2n+3,2n+6\}$ be a near skolem difference mean labeling of G as defined in [16].

Let$g: V(G) \to f$ be defined as follows

Subcase i: when k is an even number.
$g(v_1) = f(v_1) - 2;$

$g(v_{2i+1}) = f(v_{2i+1}); \begin{matrix} 1 \le i \le \frac{n-1}{2}, & n \text{ is odd} \\ 1 \le i \le \frac{n-2}{2}, & n \text{ is even} \end{matrix}$

$g(v_{2i}) = \begin{cases} f(v_{2i})-1; & 1 \le i \le \frac{k}{2} \\ f(v_{2i})-1; & \begin{matrix} \frac{k}{2} \le i \le \frac{n-1}{2}, & n \text{ odd} \\ \frac{k}{2} \le i \le \frac{n}{2}, & n \text{ even} \end{matrix} \end{cases}$

$g(u) = f(u) - 1;$
$g(v) = f(v) - 1;$

Subcase ii: when k is an odd number

$g(v_{2i+1}) = \begin{cases} f(v_{2i+1})-1; & 0 \le i \le \frac{k-1}{2} \\ f(v_{2i+1})-1; & \begin{matrix} \frac{k-1}{2} \le i \le \frac{n-1}{2}, & n \text{ odd} \\ \frac{k-1}{2} \le i \le \frac{n}{2}, & n \text{ even} \end{matrix} \end{cases}$

$g(v_2) = f(v_2) - 2;$

$g(v_{2i}) = f(v_{2i}); \begin{matrix} 2 \le i \le \frac{n-1}{2}, & n \text{ odd} \\ 2 \le i \le \frac{n}{2}, & n \text{ even} \end{matrix}$

$g(u) = f(u) - 2;$
$g(v) = f(v) - 2;$

Let g^* be the induced edge labelling of g. Then
$g^*(v_iv_{i+1}) = f^*(v_iv_{i+1}); 1 \le i \le k-2$
$g^*(uv_{k-1}) = f^*(uv_{k-1}); k-1 \le i \le n-1$
$g^*(vv_{k+2}) = f^*(vv_{k+2});$
$g^*(uv) = f^*(uv);$

The edge labels are all distinct and are $\{1,2,\ldots,n+2\}$.

Hence the theorem.

Theorem 2.7: The graph obtained by duplicating a vertex by an edge in $P_n$is near skolem difference mean, then it is skolem difference mean.

Proof: Let $P_n$be the path.

Let $e = uv$ be the duplicated edge of the vertex v_k.

Let G be the resulting graph.

Let $V(G) = \{u, v, v_i \,/\, 1 \le i \le n\}$ and $E(G) = \{uv, uv_k, vv_k, v_iv_{i+1} \,/\, 1 \le i \le n-1\}$.

Then $|V(G)| = n+2$ and $|E(G)| = n+2$.

Let f: $V(G) \to \{1,2,\ldots,2n+3,2n+6\}$ be a near skolem difference mean labeling as defined in [16].

Let$g$: $V(G) \to f$ be defined as follows

Case i: let k be an odd number.

$g(v_1) = f(v_1) - 2;$

$g(v_{2i+1}) = \begin{cases} f(v_{2i+1}); & 1 \le i \le \frac{n-1}{2}, \text{ if } n \text{ is odd} \\ f(v_{2i+1})-1; & 1 \le i \le \frac{n-2}{2}, \text{ if } n \text{ is even} \end{cases}$

$g(v_{2i}) = f(v_{2i}) - 1\,; \begin{matrix} 1 \le i \le \frac{n-1}{2}, \text{ if } n \text{ is odd} \\ 1 \le i \le \frac{n}{2}, \text{ if } n \text{ is even} \end{matrix}$

$g(u) = \begin{cases} f(u)-2; & \text{if } g(v_k)=g(v_1) \\ f(u)-2; & \text{if } g(v_k)\ne g(v_1) \end{cases}$

$g(v) = \begin{cases} f(v)-2; & \text{if } g(v_k)=g(v_1) \\ f(v)-2; & \text{if } g(v_k)\ne g(v_1) \end{cases}$

Case ii: let $k$be an even number.

$g(v_{2i+1}) = f(v_{2i+1}) - 1; \begin{matrix} 0 \le i \le \frac{n-1}{2}, \text{ if } n \text{ is odd} \\ 0 \le i \le \frac{n-2}{2}, \text{ if } n \text{ is even} \end{matrix}$

$g(v_2) = f(v_2) - 2;$

$g(v_{2i}) = \begin{cases} f(v_{2i}); & 2 \le i \le \frac{n-1}{2}, \text{ if } n \text{ is odd} \\ f(v_{2i})-1; & 2 \le i \le \frac{n}{2}, \text{ if } n \text{ is even} \end{cases}$

$g(u) = \begin{cases} f(u)-2; & \text{if } g(v_k)=g(v_2) \\ f(u)-2; & \text{if } g(v_k)\ne g(v_2) \end{cases}$

$g(v) = \begin{cases} f(v)-2; & \text{if } g(v_k)=g(v_2) \\ f(v)-2; & \text{if } g(v_k)\ne g(v_2) \end{cases}$

Let g^* be the induced edge labelling of g. Then

$g^*(uv) = f^*(uv);$

$g^*(uv_k) = f^*(uv_k);$

$g^*(vv_k) = f^*(vv_k);$

$g^*(v_iv_{i+1}) = f^*(v_iv_{i+1});\ 1 \le i \le n-1.$

Thus the induced edge labels are all distinct and are $\{1,2,\ldots,n+2\}$.Hence the theorem.

Theorem 2.8: The graph obtained by duplicating an edge by a vertex in P_n is near skolem difference mean, then it is skolem difference mean.

Proof: Let $P_n$be the path.

Let v be the duplicated vertex of the edge v_kv_{k+1}.

Let G be the resulting graph.

Let $V(G) = \{v, v_i \,/\, 1 \le i \le n\}$ and $E(G) = \{vv_k, vv_{k+1}, v_iv_{i+1} \,/\, 1 \le i \le n-1\}$.

Then $|V(G)| = n+1$ and $|E(G)| = n+1$.

Let $f: V(G) \rightarrow \{1,2,....,2n+1,2n+4\}$ be a near skolem difference mean labeling as defined in [16].

Let $g: V(G) \rightarrow f$ be defined as follows

Case i: Let k be an odd number.

Subcase i: n is odd.

$g(v_1) = f(v_1) - 2;$

$g(v_{2i+1}) = f(v_{2i+1}) - 1;\ 1 \leq i \leq \frac{n-1}{2}$

$g(v_{2i}) = \begin{cases} f(v_{2i}); & 1 \leq i \leq \frac{k+1}{2} \\ f(v_{2i})-2; & \frac{k+1}{2} \leq i \leq \frac{n-1}{2} \end{cases}$

$g(v) = f(v) - 1;$

Subcase ii: n is even

$g(v_1) = f(v_1) - 2;$

$g(v_{2i+1}) = f(v_{2i+1}),\ 1 \leq i \leq \frac{n-2}{2}$

$g(v_{2i}) = \begin{cases} f(v_{2i})-1; & 1 \leq i \leq \frac{k+1}{2} \\ f(v_{2i})-1; & \frac{k+1}{2} \leq i \leq \frac{n}{2} \end{cases}$

$g(v) = f(v);$

Case ii: Let k be an even number.

Subcase i: n is odd.

$g(v_{2i+1}) = \begin{cases} f(v_{2i+1})-1; & 0 \leq i < \frac{k}{2} \\ f(v_{2i+1})-1; & \frac{k}{2} \leq i \leq \frac{n-1}{2} \end{cases}$

$g(v_2) = f(v_2) - 2;$

$g(v_{2i}) = f(v_{2i});\ 2 \leq i \leq \frac{n-1}{2}$

$g(v) = f(v) - 1;$

Subcase ii: n is even

$g(v_{2i+1}) = \begin{cases} f(v_{2i+1})-1; & 0 \leq i < \frac{k}{2} \\ f(v_{2i+1})-2; & \frac{k}{2} \leq i \leq \frac{n-2}{2} \end{cases}$

$g(v_2) = f(v_2) - 2;$

$g(v_{2i}) = f(v_{2i}) - 1;\ 2 \leq i \leq \frac{n}{2}$

$g(v) = f(v) - 2;$

Let g^* be the induced edge labelling of g in both the subcases.

Then

$g^*(vv_k) = f^*(vv_k)$

$g^*(vv_{k+1}) = f^*(vv_{k+1})$

$g^*(v_iv_{i+1}) = \begin{cases} f^*(v_iv_{i+1}); & 1 \leq i \leq k-1 \\ f^*(v_iv_{i+1}); & k \leq i \leq n-1 \end{cases}$

Thus the edge labels are all distinct and are $\{1,2,\ldots,n+1\}$.

Hence the theorem.

3. CONCLUSIONS

In this paper, the authors generated skolem difference mean labeling of H-graphs, banana tree, lily graph, coconut tree, duplicating an arbitrary vertex of a path, duplicating an arbitrary edge of a path, duplicating an arbitrary vertex by edge of a path and duplicating an arbitrary edge by a vertex of a path from near skolem difference mean labeling of their corresponding graphs. An attempt may be done for some other graphs in various related definitions of skolem difference mean graphs.

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