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Application of Game Theory in Maintaining the Academic Standard in the Nigerian Universities

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ABSTRACT

In this paper, we have developed a game theory model to maintain the academic performance of students in the Nigerian Universities. We found that the value of the game was $V = 1.9453\times10^{12}$. The optimal strategies for player (A), the lecturers were: $y_1 = 0.1754$; $y_2 = 0.1780$; $y_3 = 0.1386$; y4 = 0.1506; y5 = 0.05453; $y_6 = 0.09209$; $y_7 = 0.18185$ and $y_8 = 0.02895$. The optimal strategies for player (B), the students were: $x_7 = 0.54444$ and $x_8 = 0.45556$. Player (A) have eight strategies to play the game and their opponents, player (B), have eight strategies but can only play two out of these strategies to minimize losses, these are strategy x_7 and x_8 respectively to survive the academic realities. Player (A), who were the owner of the game maximize their gain by using the Maximin criterion while Player (B), who were the defenders minimize their losses by using Minimax criterion. The strategies by the lecturer include all the techniques adopted to eliminate examination malpractices, maintain quality and quantity of education and to keep the standard of education highest.

Keywords**:** Game theory, Linear programming, Payoff matrix, Strategies, Value of the game

1. INTRODUCTION

Game can be defined as an activity between two or more persons (lecturers and students) involving moves by person (player) according to a set of rules at the end of which each player

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receives some benefits or satisfaction or suffers loss. The quests for better performance by the students and equal measure by the lecturers to protect the integrity of the process such that the grade made by the students are a genuine one is an example of a game problem. At all times, the students fashion out strategies to maneuver their ways; while the lecturers cross match the strategies by defending the integrity of the grading process. Academic performance of students involves two major players; the lecturers trying to defend the integrity of the process and the students whose interest are to graduate with good classes at all cost. This is a two-person zero-sum game where each is trying to bring the best strategies to outweigh the other and a loss for one is a gain to the other.

Students and lecturers are in constant competitions on who gets what in their academic performance. The lecturers try to protect the integrity of their institutions by maintaining a high level of standard and reputation for their institutions while the students try at all cost to make good grade points. Both the lecturers and the students employ different strategies to achieve this. The strategies among other things include; devotion of attention and more time for studies; punctuality and attendance to lectures; participating in tests and continuous assessment; cheating during examination etc., these are on the part of students. On the part of lecturers; they lecture students; give tests and continuous assessment; examine the students and grade them properly; forward the students name for disciplinary actions in the case of examination malpractices and forgery. None of the two players wanted to be cheated by the other. Each of them has their strategies that they could combine to wine their opponents. This scenario is a game theory problem and we want to determine those strategies the student and the lecturers will apply to match each other so that the system will be in equilibrium.

Used game theory to model the interaction of attackers and defenders during a distributed denial of service attack (DDoS) [1]. In a DDoS attack scenario, the players are competing for greater share of resource of the network. An attack occurs when the attackers flood the link in such a way that legitimate users are denied access to a target resource on the network. They modeled the problem using the method of linear programming and obtained the best strategy for the defender to reducing rogue packets that seek to flood the bandwidth of the network. Game theory provides a much needed methodology for constructing games, testing rules, and deciding upon the interpretation of the observed play. It deals with processes in which the individual decision unit has only partial control over the strategic factors affecting its environment. The maxmin strategy appears to be the rational strategy to employ if one assumes that the opponent is not going to commit blunders. A strategy in the sense of game theory is a completely specified plan of action which covers all contingencies based upon the information state of the players throughout the game [2].

The theory of games has been most fully developed for the two-person situation, the conflict of two opposing individuals or groups. Almost all battle decisions involve two opposing military forces. A military commander may approach decision with either of two philosophies. He may select his course of action on the basis of his estimate of what his enemy is able to do to oppose him. Or, he may make his selection on the basis of his estimate of what his enemy is going to do. For a mixed strategy they must assign a probability of choice to each available course of action.

Thus a pure strategy is a special case of a mixed strategy, with the probability of zero assigned to the play of all strategies except one [3]. Game theory is a mathematical construct used in making decisions. It assumes two interdependent decision makers in a situation where the outcome for one participant's choice of action is dependent on the other's choice.

The appropriate course of action for either actor is what is known as a "minimax" strategy. Minimax strategy is that which minimizes the maximum loss. This minimax criterion can be used with only one actor, where he or she is confronted with two or more choices of action given variable states of nature which differentially influence those choices [4].

Game is played against nature. Making choices has been necessary for man to evolve and survive. Faced with various options, man has to make choices on the basis of his or her beliefs about the likely outcomes in terms of safety, shelter, food, procreation and so on. In some deep sense it seems likely that human actors would try to choose actions that will give the best expected result: we would expect them to act rationally and make rational choices. As the state of nature is uncertain, the outcomes are considered against a number of discrete states of nature [5]. [6] Observed that Matrix notation and matrix operations provide a model for analyzing certain situations involving conflict. Probability theory can be used to determine the best strategies for the players. [7] Observed that Game theory is usually difficult to test in the field because predictions typically depend sensitively on features that are not controlled or observed. They conduct one such test using both laboratory and field data from the Swedish lowest unique positive integer (LUPI) game. In this game, players pick positive integers and whoever chooses the lowest unique number wins. Equilibrium predictions are derived assuming Poisson distributed population uncertainty. The field and lab data show similar patterns and despite various deviations from equilibrium, there is a surprising degree of convergence toward equilibrium. Game theory, with its emphasis on strategic choice, makes a significant programmatic promise: to contribute to the development of a theory of action. Thus, scholars broadly agree that the theory of action proposed by game theory has at its core a fairly simple structure, consisting of three building blocks. They agree that game theory is driven, first, by an understanding of the process of choice making based on the expected utility model of decision making. Second, game theory is seen as generating predictions by linking the analysis of choice making to the concept of equilibrium. Third, game theory is seen as treating the rules of the game, the strategies, the way in which these choices are sequenced, the preferences of actors, and the information actors possess when they make their choices as exogenous factors that are taken as given and assumed to remain constant [8].

2. METHODOLOGY

2. 1. Method Data Collection

Secondary data was collected from the first semester comprehensive result of the second year students of Science Laboratory Technology (SLT), Federal University of Technology Owerri using systematic sampling. The students' performance/ grades were presented in a matrix form with rows and columns, see Table 2. Each of these entries forms the payoff matrix. The payoff matrix forms m x n person-zero-sum game.

2. 2. Nature of the Problem

This is a two-person zero-sum game. The players are the lecturers and the students. The lecturers try everything possible to defend the grade points to maintain the culture of excellence for which FUTO is known for while the students on the other hand try as much as possible to beat the strategies adopted by the lecturers. This is a typical two-person zero-sum game with multiple strategies on both lecturers and students, where the lecturers (defender) compete with the attacker (the students). The defender is the owner of the game and he tries as much as possible to protect the standard of education from attacker, whose interest is to attack the system and make good grade points at all cost, thereby causing a fall in the standard of education. The defender plays to maximize his payoff while the attacker chooses the best strategy to minimize his loses. Each player brings their best strategies to outweigh the other.

2. 3. Method of Analysis

Game theory bears a strong relationship with linear programming, since every finite two-person zero-sum game can be expressed as a linear program and conversely every linear program can be represented as a game. Linear programming is the most general method of solving any two-person zero-sum game. If there is no saddle point, no dominance and the method of matrices also fails, then linear programming offers the best method of solution. [6] Observed that every two person, zero-sum game is equivalent to a linear-programming problem. This finding allowed for the easy calculation of the optimal strategies for any m matrix game using the Simplex method. The Simplex method is a computational procedure for solving linear-programming problems. We shall represent the defender by (A) and the attacker by (B). We shall solve the resulting payoff matrix with mixed strategies games. We shall use MATLAB (package) to solve the modeled problem and determine the best strategies for both players (A) and (B). Finally, we shall determine the value of the game V.

Assumptions

The following assumptions are made for the development of the game model:

1. No saddle point exists in the payoff matrix.
2. No dominance exists in both rows and columns of the payoff matrix.
3. The payoff matrix is at least (3x3)
4. Both horizontal and vertical oddments of the payoff matrix are not equal.
5. Since the above conditions hold, then we develop a linear programming model for solving the games as follows:

Table 1. Payoff Matrix for Player A and B.

		Payoff Matrix				
		B				
		Y1	Y2	Y3	. . .	Yn
	X1	a11	a12	a13	. . .	a1n
A	X2	a21	a22	a23		a2n
	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
	Xm	am,1	. . .	. . .	. . .	Amn

The payoff matrix is presented in Table 1.
Let the value of the game (to A) be V and B is trying to minimize V.
Then

Against A_1: $a_{11}y_1 + a_{12}y_2 + \ldots + a_{1m}y_m \leq V$ …………………….. (1)

Against A_2: $a_{21}y_1 + a_{22}y_2 + \ldots + a_{2m}y_m \leq V$ ……………………… (2)

… … ….

Against A_n: $a_{n1}y_n + a_{n2}y_n + \ldots + a_{nm}y_{mn} \leq V$ ……………………… (3)

$y_1 + y_2 + \ldots + y_m = 1$ (sum of probabilities must be equal to 1) ……. (4)

where $y_1, y_2, \ldots, y_m \geq 0$

From equations (1) and (4), we get the following relations by dividing by V:

$$a_{11}\frac{y_1}{V} + a_{12}\frac{y_2}{V} + a_{13}\frac{y_3}{V} + \ldots + a_{1n}\frac{y_n}{V} \leq 1$$

$$a_{21}\frac{y_2}{V} + a_{22}\frac{y_2}{V} + a_{23}\frac{y_3}{V} + \ldots + a_{2n}\frac{y_n}{V} \leq 1$$

.

.

.

$$a_{m1}\frac{y_m}{V} + a_{m2}\frac{y_m}{V} + a_{m3}\frac{y_m}{V} + \ldots + a_{mn}\frac{y_m}{V} \leq 1$$

$$\frac{y_1}{V} + \frac{y_2}{V} + \frac{y_3}{V} + \ldots + \frac{y_m}{V} = 1$$

Let $\frac{y_j}{V} = Y_j$; $j = 1, 2, \cdots n$, we have

$$a_{11}Y_1 + a_{12}Y_2 + a_{13}Y_3 + \ldots + a_{1n}Y_n \leq 1$$

$$a_{21}Y_1 + a_{22}Y_2 + a_{23}Y_3 + \ldots + a_{2n}Y_n \leq 1$$

.

. (5)

.

$$a_{m1}Y_1 + a_{m2}Y_2 + a_{m3}Y_3 + \ldots + a_{mn}Y_m \leq 1$$

$$Y_1 + Y_2 + Y_3 + \ldots + Y_n = \frac{1}{V} \qquad (6)$$

$$Y_1, Y_2, Y_3, \ldots, Y_n \geq 0$$

Since B is trying to minimize V, he must maximize $\frac{1}{V}$.

Thus the problem is to maximize the objective function (equation (6)) subject to constraint equations (5) which can be solved using Simplex method of linear programming with the following steps:

Step 1:

Introduce slack variables, and the above relations can be written as

$$a_{11}Y_1 + a_{12}Y_2 + a_{13}Y_3 + \ldots + a_{1n}Y_n + S_1 = 1$$

$$a_{21}Y_1 + a_{22}Y_2 + a_{23}Y_3 + \ldots + a_{2n}Y_n + S_2 = 1$$

.

.

.

$$a_{m1}Y_1 + a_{m2}Y_2 + a_{m3}Y_3 + \ldots + a_{mn}Y_m + S_m = 1$$

and we want to

maximize $\frac{1}{V} = Y_1 + Y_2 + Y_3 + \ldots + Y_n + 0S_1 + 0S_2 + \cdots + 0S_n$

where $Y_1, Y_2, Y_3, \ldots, Y_n, S_1, S_2, S_3, \cdots, S_n \geq 0$.

Step 2:

Then present the above in a simplex tableau and do the needed iterations. S_i's are the basic variables that form the starting solutions. At the final tableau, the non-basic variables, $Y_i's$ enters the basis. Their corresponding values are given as:

$$Y_1 = \frac{y^*{}_1}{N}, Y_2 = \frac{y^*{}_2}{N}, \cdots, Y_n = \frac{y^*{}_n}{N} \quad \text{and}$$

$$\frac{1}{V} = Y_1 + Y_2 + Y_3 + \ldots + Y_n = \frac{y^*{}_1}{N} + \frac{y^*{}_2}{N} + \cdots + Y_n = \frac{K}{p}$$

where $\sum_{i=1}^{n} \frac{y_i^*}{N_i} = \frac{K}{P}$

Value of the game $V = \frac{p}{K}$; But

$$\frac{y_j}{V} = Y_j$$

$$y_1 = Y_1 * V = \frac{y_1^*}{N} \times \frac{p}{K} = \frac{py_1^*}{NK}$$

$$y_2 = Y_2 * V = \frac{y_2^*}{N} \times \frac{p}{K} = \frac{py_2^*}{NK}$$

$$\vdots$$

$$y_n = Y_n * V = \frac{y_n^*}{N} \times \frac{p}{K} = \frac{py_n^*}{NK}$$

A’s best strategies are

$$X_1 = \frac{x^*{}_1}{N}, X_2 = \frac{x^*{}_2}{N}, \cdots, X_n = \frac{x^*{}_n}{N}$$

$$x_1 = X_1 * V = \frac{x_1^*}{N} \times \frac{p}{K} = \frac{px_1^*}{NK}$$

$$x_2 = X_2 * V = \frac{x_2^*}{N} \times \frac{p}{K} = \frac{px_2^*}{NK}$$

$$\vdots$$

$$x_n = X_n * V = \frac{x_n^*}{N} \times \frac{p}{K} = \frac{px_n^*}{NK}$$

The best strategies for player A (the defender) are $(\frac{px_1^*}{NK}, \frac{px_2^*}{NK}, \ldots, \frac{px_n^*}{NK})$

The best strategies for player B (the attacker) are $(\frac{py_1^*}{NK}, \frac{py_2^*}{NK}, \ldots, \frac{py_n^*}{NK})$

The value of the game, $V = \frac{p}{K}$

2. 4. Analysis

Table 2. Numerical grades of students in eight courses

	STUDENTS : PLAYER (B)								
LECTURERS : PLAYER (A)	Strategies	1	2	3	4	5	6	7	8
	1	60	55	35	40	40	50	60	53
	2	50	52	60	36	70	60	50	62
	3	71	72	50	51	40	61	70	54
	4	52	50	40	45	41	43	60	70
	5	63	62	65	76	40	41	62	65
	6	65	60	45	46	35	60	70	55
	7	53	50	51	45	40	50	60	50
	8	70	70	62	50	40	70	70	51
	9	74	60	60	46	38	51	71	46
	10	75	60	70	46	40	60	72	40
	11	71	53	40	45	35	61	60	65

From equation (5) and augmenting the slack variables we have the following 11 constraint equations:

$$60Y_1 + 55Y_2 + 35Y_3 + 40Y_4 + 40Y_5 + 50Y_6 + 60Y_7 + 53Y_8 + S_1 = 1$$

$$50Y_1 + 52Y_2 + 60Y_3 + 36Y_4 + 70Y_5 + 60Y_6 + 50Y_7 + 62Y_8 + S_2 = 1$$

$$71Y_1 + 72Y_2 + 50Y_3 + 51Y_4 + 40Y_5 + 61Y_6 + 70Y_7 + 54Y_8 + S_3 = 1$$

$$52Y_1 + 50Y_2 + 40Y_3 + 45Y_4 + 41Y_5 + 43Y_6 + 60Y_7 + 70Y_8 + S_4 = 1$$

$$63Y_1 + 62Y_2 + 65Y_3 + 76Y_4 + 40Y_5 + 41Y_6 + 62Y_7 + 65Y_8 + S_5 = 1$$

$$65Y_1 + 60Y_2 + 45Y_3 + 46Y_4 + 35Y_5 + 60Y_6 + 70Y_7 + 55Y_8 + S_6 = 1$$

$$53Y_1 + 50Y_2 + 51Y_3 + 45Y_4 + 40Y_5 + 50Y_6 + 60Y_7 + 50Y_8 + S_7 = 1$$

$$70Y_1 + 70Y_2 + 62Y_3 + 50Y_4 + 40Y_5 + 70Y_6 + 70Y_7 + 51Y_8 + S_8 = 1$$

$$74Y_1 + 60Y_2 + 60Y_3 + 46Y_4 + 38Y_5 + 51Y_6 + 71Y_7 + 46Y_8 + S_9 = 1$$

$$75Y_1 + 60Y_2 + 70Y_3 + 46Y_4 + 40Y_5 + 60Y_6 + 72Y_7 + 40Y_8 + S_{10} = 1$$

$$71Y_1 + 53Y_2 + 40Y_3 + 45Y_4 + 35Y_5 + 61Y_6 + 60Y_7 + 65Y_8 + S_{11} = 1$$

From equation (6) we have:

$$\text{Maximize } \frac{1}{V} = Y_1 + Y_2 + Y_3 + \ldots + Y_8 + 0S_1 + 0S_2 + \cdots + 0S_{11}$$

Hence, the game problem becomes:

$$\text{Maximize } \frac{1}{V} = Y_1 + Y_2 + Y_3 + \ldots + Y_8 + 0S_1 + 0S_2 + \cdots + 0S_{11}$$

Subject to the 11 constraint equations

Solving the problem using MATLAB package, we have

```
>> f = [1; 1; 1; 1; 1; 1; 1; 1];
>> A= [60,55,35,40,40,50,60,53; 50,52,60,36,70,60,50,62; . . ; 71,53,40,45,35,61,60,65];
>> b = [1; 1; 1; 1; 1; 1; 1; 1; 1; 1; 1];
>> lb = Zeros (8, 1);
>> [y, fval, exitflag, output, lambda] = linprog(f, A, b,[ ],[ ], lb)
```

Y = 1.0e-013 * [Y_1 = 0.9018; Y_2 = 0.9150; Y_3 = 0.7127; Y_4 = 0.7739; Y_5 = 0.2803;
Y_6 = 0.4734; Y_7 = 0.9348; Y_8 = 0.1488]

fval = 5.1406e-013

Hence, Value of the game is:

$V = 1 / \text{fval} = 1.9453\times10^{12}$

Hence, the strategies for lecturers are:

$y_1 = V.\ Y_1 = 0.1754$; $y_2 = 0.1780$; $y_3 = 0.1386$; $y_4 = 0.1506$; $y_5 = 0.05453$; $y_6 = 0.09209$;
$y_7 = 0.18185$; $y_8 = 0.02895$

and the sum of these strategies (probabilities) equals one; that is

$y_1 + y_2 + . . . + y_8 = 1.0000$

Also the strategies for students are:

$x_1 = x_2 = \ldots = x_6 = 0.0000$; $x_7 = 0.54444$; $x_8 = 0.45556$

and the sum of these strategies (probabilities) equals one; that is

$x_1 + x_2 + \ldots + x_6 + x_7 + x_8 = 1.0000$

3. RESULT AND DISCURSION

The value of the game was V = 1.9453×10^{12}. The strategies by the lecturer were: y_1 = 0.1754; y_2 = 0.1780; y_3 = 0.1386; y_4 = 0.1506; y_5 = 0.05453; y_6 = 0.09209; y_7 = 0.18185 and y_8 = 0.02895. The lecturer who is the owner of the game has eight strategies to play the game and he can choose any of them to outweigh his opponent, who in this case is the student. These strategies are the probability or the proportion of time the lecturer plays each of the strategy. Hence, the sum of the strategy gives one, that is, $y_1 + y_2 + \ldots + y_8 = 1.0000$. On the other hand, the opponent, who is the student has eight strategies but can only play two out of these strategies to minimize losses, these are strategy 7 and 8. Again the sum of these strategies give one; that is, $x_7 + x_8 = 1.0000$. Player (A), the lecturer plays along the column of the payoff matrix and maximizes his gain (maintaining educational standard) by using the Maximin criterion. This is a decision principle that allows the player, especially the owner of the game to maximize his payoff. Also, Player (B), the student who is a defender in this case tries to minimize his losses by using Minimax criterion. This is a decision principle that allows the player, especially the defender of the game to minimize his payoff. The strategies used by the lecturer include all the techniques adopted to eliminate examination malpractices, maintain quality and quantity of education and to keep the standard of education to the highest point. On the part of the student, the strategies include all the skills adopted to beat the lecturer's barrier to making good grade and graduate with good point. There are good strategies like hard working, devotion to study, attendance to lectures and punctuality to classes, etc., while the bad strategies include all forms and means to cheat in examination and maneuvering of all sorts in other to make good grades.

4. CONCLUSION

In this paper, we have modeled the academic performance of students in the Federal University of Technology, Owerri using game theory. We found the value of the game to be V = 1.9453×10^{12}. The lecturers have eight strategies and they played all of them to maintain the standard of education. These strategies are the respective probabilities or the proportion of the time each of the strategies were played, and the sum of the probabilities was one. The students also have eight strategies and out of these, they played only two strategies to minimize losses; that is, to be able to survive academic realities. The game was a two- person-zero-sum game where a loss to one is a gain to the other and it was a fair game.

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