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On Detection and Correction of 2-Repeated Solid Burst Errors

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ABSTRACT

In modern age, coding theory has found widespread theoretical and practical applications in the areas ranging from communication systems to digital data transmission to modern medical science, to space communication. In different types of channels, nature of errors is also different. There are various channels in which errors occur in the form of bursts. In some particular channels, within a burst, all the digits are corrupted. Such type of errors is called 'solid burst errors'. In this paper we introduce '2-repeated solid burst error' and obtain results regarding the codes detecting and correcting such errors.

***Keywords*:** 2-repeated solid burst errors, parity-check matrix, solid burst errors, standard array and syndrome

1. INTRODUCTION

In many communication systems, errors do not occur randomly but are clustered together. This led to the study of burst-error-correcting and detecting codes. This was introduced by Fire (1959) and Regier (1960) and later nicely treated by Peterson and Weldon (1972). The advantages with burst-error-correcting and detecting codes are their easy implementation and efficient functioning. Stone (1961) and Bridwell and Wolf (1970) considered multiple burst errors. Chien and Tang (1965) considered a different type of burst known as CT burst.

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In some particular channels, viz. semiconductors memory data (2003) and supercomputer storage system (1984) it is seen that within a burst all the positions are corrupeted, i.e., in a burst of length *b*, all the b components are non-zero. Such types of burst errors, in literature, are known as 'Solid Burst Errors.'

Schillinger (1964) studied codes for correcting solid burst errors. After him Shiva and Cheng (1969) gave a simple decoding scheme for correcting multiple-solid burst errors of length *b* in binary code. Bossen (1970) Sharma and Dass (1977), Etzion (1992) and Argyrides et al. (2010) also studied solid burst errors. A systematic study for obtaining bounds on number of parity-check digits for linear codes over $GF(q)$ detecting and correcting solid burst errors of length *b* or less, was made by Das (2012).

It is well known that the nature of a burst depends upon the behavior of the channel in use. The uncertain behavior of communication channel gives rise to the concept of repeated burst errors. This concept was introduced by Berardi, Dass and Verma (2009). They defined 2-repeated burst and obtained results for their detection and correction.

In this paper, we introduce '2-repeated solid-burst' and obtain results on the number of parity check digits for linear codes detecting and correcting such type of errors.

While considering 2-repeated solid burst errors there arises following three cases:
When both the bursts in a 2-repeated solid burst are disjoint.
When both the bursts in a 2-repeated solid burst partially overlap.
When both the bursts in a 2-repeated solid burst fully overlap.

The results in this paper are obtained for case (a) only.

The paper has been organized as follows: In section 2, basic definitions related to our study are stated with some examples. In section 3, a lower bound and an upper bound on the number of parity check digits of linear codes detecting any 2-repeated solid burst of length b or less are given. In section 4, we obtain lower and upper bounds on number of parity check digits of linear code correcting 2-repeated solid burst of length *b* or less.

In what follows a linear code is considered as a subspace of the space of all n-tuples over $GF(q)$. The distance between two vectors is considered in the Hamming sense.

2. PRELIMINERIES

Definition 1: A *solid burst of length* b is a vector with non-zero entries in some b consecutive positions and zeros elsewhere.

Example 1: (001111000) is a solid burst of length 4 over $GF(2)$.

A 2-repeated burst of length b is defined as follows:

Definition 2: A *2-repeated burst of length* b is a vector of length n whose only non-zero components are confined to two distinct sets of b consecutive components the first and the last component of each set being non zero.

Example 2: (001110100010001) is a 2-repeated burst of length 5 over $GF(2)$.

Definition 3: A *2-repeated solid burst of length* b is a vector with non-zero entries in some b consecutive positions of two distinct sets and zeros elsewhere.

Example 3: (0011000110) is a 2-repeated solid burst of length 2 over $GF(2)$.

3. CODES DETECTING 2-REPEATED SOLID BURST ERRORS

Theorem 1: An (n,k) linear code over $GF(q)$ that detects any 2-repeated solid bursts of length b or less in the space of all n-tuples must have at least $1+2b$ parity-check digits.

Proof: The result is proved by considering that no detectable error vector can be a codeword. Let V be an (n,k) linear code over $GF(q)$. Let us consider a set X of all those vectors in which some fixed non-zero components are confined to two distinct sets of b consecutive nonzero components.

We claim that no two vectors of the set X can belong to the same set coset of standard array; else a codeword shall be expressible as the sum or difference of two error vectors.

Assuming on the contrary that there is a pair x_1, x_2 of X belonging to the same coset of standard array, there difference $x_1 - x_2$ must be a code vector. But $x_1 - x_2$ is a vector all of whose nonzero components are confined to two distinct sets of b consecutive nonzero components, i.e., $x_1 - x_2$ is a 2-repeated solid burst of b or less. This is a contradiction. Thus all vectors in X must belong to distinct cosets of the standard array. The number of such vectors over $GF(q)$, including the vector of all zeros is clearly $1+(q-1)^{2b}$.

Also the total number of cosets in an (n,k) linear code equals q^{n-k}.

So we must have

$$q^{n-k} \geq 1+(q-1)^{2b}$$
$$n-k \geq \log_q(1+(q-1)^{2b})$$
$$n-k \geq \log_q(1+q^{2b})$$

Taking integer value

$$n-k \geq 1+2b$$

This proves the result.

Theorem 2: There exists an (n,k) linear code over $GF(q)$ that has no 2-repeated solid burst of length b or less as a code word provided that

$$n-k > \log_q\left[(q-1)^{2(b-1)}\left[1+\frac{(n-3b+1)(n-3b+2)}{2}(q-1)^{2b}\right]\right].$$

Proof: The existence of such a code will be shown by constructing an appropriate $(n,k)\times n$ parity-check matrix H. The requisite parity-check matrix H shall be constructed as follows:

Choose any nonzero (n,k)-tuple as the first column h_1 of H. Suppose that we have selected the first $j-1$ columns $h_1, h_2, .. h_{j-1}$ of H suitably. We lay down the condition of the immediately preceding $b-1$ or fewer columns of H together with any H or fewer consecutive columns from amongst the first $j-b$ columns $h_1, h_2, .. h_{j-b}$.

In other words

$$h_j \neq (a_1 h_{j-b} + a_2 h_{j-b+2} + .. a_{b-1} h_{j-1}) + (b_1 h_i + b_2 h_{i+1} + ... + b_b h_{i+b-1})$$

where $a_i + b_i \in GF(q)$ and $i+b-1 \leq j-b$.

This condition ensures that no 2-repeated solid burst of length b or less will be a code word. There are $(q-1)^{2(b-1)}$ choices for the a_i. The coefficient $bi's$ form a 2-repeated solid burst of length b or less in vector of length $n-b$, then there are

$$\frac{(n-3b+1)(n-3b+1)}{2}(q-1)^{2b}+1$$

Choices for these coefficients, including the case in which they are all zeros. Thus the total number of coefficients is

$$(q-1)^{2(b-1)}[\frac{(n-3b+1)(n-3b+1)}{2}(q-1)^{2b}+1].$$

At worst, all these linear combinations must yield a distinct sum.

Therefore, a column h_j can be added provided that

$$q^{n-k} > (q-1)^{2(b-1)}[\frac{(n-3b+1)(n-3b+1)}{2}(q-1)^{2b}+1]$$

or

$$n-k > \log_q[(q-1)^{2(b-1)}[\frac{(n-3b+1)(n-3b+1)}{2}(q-1)^{2b}+1]].$$

4. CODES CORRECTING 2-REPEATED SOLID BURST ERRORS

Theorem 3: An (n,k) linear code over $GF(q)$ that corrects all 2-repeated solid bursts of length b or less must have at least

$$\log_q[(q-1)^{2b}[\frac{(n-2b+1)(n-2b+1)}{2}(q-1)^{2b}+1]]$$

parity-check digits.

Proof: The proof is based on counting the number of correctable error vectors and comparing it with the available number of cosets.

Let us consider a vector of length n having 2-repeated solid bursts of length b or less. Its only nonzero components are confined to two distinct sets of b consecutive nonzero entries. To make 2-repeated solid bursts of length b or less, the first burst can start from ith position, where i vary from 1 to $n-2b+1$. The second burst can then start from a position after the first one ends.

Let us first consider the vector having 2-repeated solid bursts, in which the first burst, starts from the first position, their number is clearly $(q-1)^b$, then the second burst will have $n-2b+1$ starting positions and their number will also be $(q-1)^b$. Thus, the total number of 2-repeated solid bursts in which first burst starts from first position is given by

$$(q-1)^{2b}(n-2b+1).$$

Next, we consider vector with 2-repeated solid bursts, in which first burst starts from second position, the starting positions of second having reduced by 1, their number shall be

$$(q-1)^{2b}(n-2b).$$

A little consideration will show that the process of constructing 2-repeated solid bursts will end when the second burst has just one starting position, the number then being $(q-1)^{2b}.1$. Summing these all the total number of n-vectors having 2-repeated solid bursts of length b or less will be

$$(q-1)^{2b}\sum_{i=1}^{n-2b+1} i=\frac{(n-2b+1)(n-2b+2)}{2}(q-1)^{2b}.$$

For correction, all these vectors must belong to different cosets. The total numbers of cosets available are q^{n-k}. Therefore, we must have at least

$$q^{n-k}\geq 1+\frac{(n-2b+1)(n-2b+2)}{2}(q-1)^{2b}$$

or

$$n-k\geq \log_q\left[1+\frac{(n-2b+1)(n-2b+2)}{2}(q-1)^{2b}\right].$$

This proves the theorem.

Theorem 4: There shall always exist an (n,k) linear code over $GF(q)$ that corrects all 2-repeated solid bursts of length b or less $n>4b$ provided that

$$q^{n-k} > 1 + \sum_{i=1,l=1}^{b} \frac{(n-l-2i+1)(n-l-2i+2)}{2}(q-1)^{2i+l-1}.$$

Proof: The existence of such a code will be shown by constructing an appropriate $(n-k)\times n$ parity-check matrix H.

Any nonzero (n,k) –tuple is chosen as the first column h_1 of H. Subsequent columns are added to H such that after selecting first $j-1$ columns $h_1, h_2, .. h_{j-1}$, the jth column hj is added provided that hj should not be a linear combination of $b-1$ or less columns from immediately preceding $b-1$ or less columns of H together with any three distinct sets of b or less columns amongst the first $j-1$ columns.

In other words,

$$h_j = (a_1 h_i + a_2 h_{i+1} + \ldots + a_{b-1} h_{i+b-2}) + (b_1 h_{i_1} + b_2 h_{i_1+1} + \ldots + b_b h_{i_1+b-1}) + (c_1 h_{i_2} + c_2 h_{i_2+1} + \ldots + c_b h_{i_2+b-1}) + (d_1 h_{i_3} + d_2 h_{i_3+1} + \ldots + d_b h_{i_3+b-1})$$

where $a_i, b_i, c_i, d_i \in GF(q)$ and the h_i are any $j-1$ or less columns amongst the $h_{j-b+1}, h_{j-b+2}, .. h_{j-1}$ and the $h_{i_1}, h_{i_2}, h_{i_3}$ are any b or less columns chosen from three sets of b or less consecutive columns amongst the $j-1$ columns.

The condition ensures that there would not be a codeword which is expressible as a sum or difference of two vectors each of which is a 2-repeated solid burst of length b or less.

The number of choices of these coefficients can be calculated as follows:

If a_i is chosen to be a 2-repeated solid burst of length $b-1$ or less in a $(j-b)$-tuple corresponding to the coefficients b_i, c_i, d_i is given by

$$\sum_{i=1}^{b} \frac{(j-b-2i+1)(j-b-2i+2)}{2}(q-1)^{2i}.$$

If a_i is chosen to be a 2-repeated solid burst of length $b-2$ or less then the number of 2-repeated solid bursts of length b or less in a $(j-b+1)$ tuple, corresponding to the coefficients b_i, c_i, d_i is given by

$$\sum_{i=1}^{b} \frac{(j-b+1-2i+1)(j-b+1-2i+2)}{2}(q-1)^{2i}.$$

Continuing the process, if a_i is chosen to be a 2-repeated solid burst of length 0 then the number of 2-repeated solid bursts of length b or less in a $(j-1)$ tuple, corresponding to the coefficients b_i, c_i, d_i is given by

$$\sum_{i=1}^{b}\frac{(j-1-2i+1)(j-1-2i+2)}{2}(q-1)^{2i}.$$

Therefore, the total number of possible coefficients a_i, b_i, c_i, d_i is

$$(q-1)^{2(b-1)}\sum_{i=1}^{b}\frac{(j-b-2i+1)(j-b-2i+2)}{2}(q-1)^{2i}+$$
$$(q-1)^{2(b-2)}\sum_{i=1}^{b}\frac{(j-b+1-2i+1)(j-b+1-2i+2)}{2}(q-1)^{2i}+\dots$$
$$+(q-1)^{2}\sum_{i=1}^{b}\frac{(j-2-2i+1)(j-2-2i+2)}{2}(q-1)^{2i}+\sum_{i=1}^{b}\frac{(j-1-2i+1)(j-1-2i+2)}{2}(q-1)^{2i}$$

which can be written as

$$\sum_{i=1,l=1}^{b}\frac{(n-l-2i+1)(n-l-2i+2)}{2}(q-1)^{2i+l-1}.$$

Thus the column h_j can be added provided that

$$q^{n-k}>\sum_{i=1,l=1}^{b}\frac{(n-l-2i+1)(n-l-2i+2)}{2}(q-1)^{2i+l-1}.$$

This proves the result.

Example 1: Consider a $(9,3)$ binary code with the 6×9 matrix H which has been constructed by the synthesis procedure given in the proof of theorem 4 by taking $b=3, n=9$.

$$H=\begin{bmatrix}1&0&0&0&0&0&1&0&0\\0&1&0&0&0&0&0&1&0\\0&0&1&0&0&0&0&0&1\\0&0&0&1&0&0&1&0&0\\0&0&0&0&1&0&0&1&0\\0&0&0&0&0&1&0&0&1\end{bmatrix}$$

The null space of the matrix can be used to correct all 2-repeated solid bursts of length 3 or less. It may be verified from following table that the syndromes of all 2-repeated solid bursts of length 3 or less are distinct and non-zero.

Table 1.

Error Patterns	Syndromes
2-repeated solid bursts of length 1	
110000000	110000
101000000	101000
100100000	100100
100100000	100010
100010000	100001
100001000	000100
100000100	110010
100000010	101001
100000001	011000
011000000	010100
010100000	010010
010010000	010001
010001000	110100
010000100	000010
010000010	011001
010000001	001100
001100000	001010
001010000	001001
001001000	101100
001000100	011010
001000010	000001
001000001	000110
000110000	000011
000101000	000101
000100100	100000
000100010	010010
000100001	001101
000011000	000011
000010100	100110
000010010	010000
000010001	001011
000001100	100101
000001010	010011
000001001	001000

000000110	110110
000000101	101101
000000011	011011
2-repeated solid bursts of length 2	
111100000	111100
110110000	110110
110011000	110011
110001100	010101
110000110	000110
110000011	101011
011110000	011110
011011000	011010
011001100	111101
011000110	101110
011000011	000011
001111000	001111
001101100	101001
001100110	111010
001100011	010111
000111100	100011
000110110	110000
000110011	011101
000011110	110101
000011011	011000
000001111	111110
2-repeated solid bursts of length 3	
111111000	111111
111011100	011111
111001110	001111
111000111	000111
011111100	111011
011101110	101011
011100111	100011
001111110	111001
001110111	110001
000111111	111000

5. CONCLUSIONS

In this paper we obtained result for detection and correction of 2-repeated solid burst with respect to Hamming weight. Thses results can be extended in more general and practical way by choosing other types of weights viz. Eucledian weight, Lee weight and more specifically Sharma-Kaushik weight. Practical implementation of these results while generating codes may be very helpful in channeles where this particular type of bursts occurs frequently.

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