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Propagation Characteristics of Bragg Fiber: Comparative Analysis using Two Approaches

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ABSTRACT

The propagation characteristics in air core Bragg fiber has been analyzed in this paper by using the two different approaches. Initially, we have computed the exact solution of Helmholtz equation and later the asymptotic limits have been used to calculate the propagation characteristics of TE modes in air core Bragg fiber. In last, we have compared the both results after mathematical computation and find excellent agreement between both approaches.

***Keywords*:** Bragg Fiber, Asymptotic Approximation, Bragg reflection, propagation loss

1. INTRODUCTION

The Bragg fiber was first proposed by P. Yeh, A. Yariv, and E. Marom et al. in 1978 [1]. It has many advantages as compared to conventional optical fiber. Some example is like high aperture, lower propagation loss, supporting for single mode propagation etc. In case of light transmission through convention optical fiber, the main concept is total internal reflection in core while guiding of the light in Bragg fiber is different (Bragg reflection principle).

In a Bragg fiber with high and low refraction index cladding pairs, light can be confined mostly within the center air core, which can be lead to lower propagation loss and reduce the

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threshold for nonlinear effects. Based on Bragg reflection theorem, theoretically, the numbers of cladding layers need to be achieve to infinite [1].

For reducing the number of the cladding layers of Bragg fibers, and improving the propagation characteristics, an optimum design software based on MATLAB has been set up to analyze the structures for Bragg fiber in this paper. This simulation optimum design for the structures of Bragg fiber has an important meaning for the research of light propagation characteristics, and it is useful for the manufacture of Bragg fiber.

2. FUNDAMENTAL EQUATIONS OF HYBRID MODE IN BRAGG FIBER

The Bragg fiber has a cylindrically symmetric microstructure in which the hollow core is surrounded by a multi-layered cladding (see Fig. 1). The cladding consists of an alternating dielectric medium having high and low refractive index n_1 and n_2 respectively. Also l_1 and l_2 is thickness of alternating cladding layers. The core radius is r_0, and its refractive index is n_c. Further we are using a cylindrical coordinate system (r, Φ, z) for analysis of Bragg fiber.

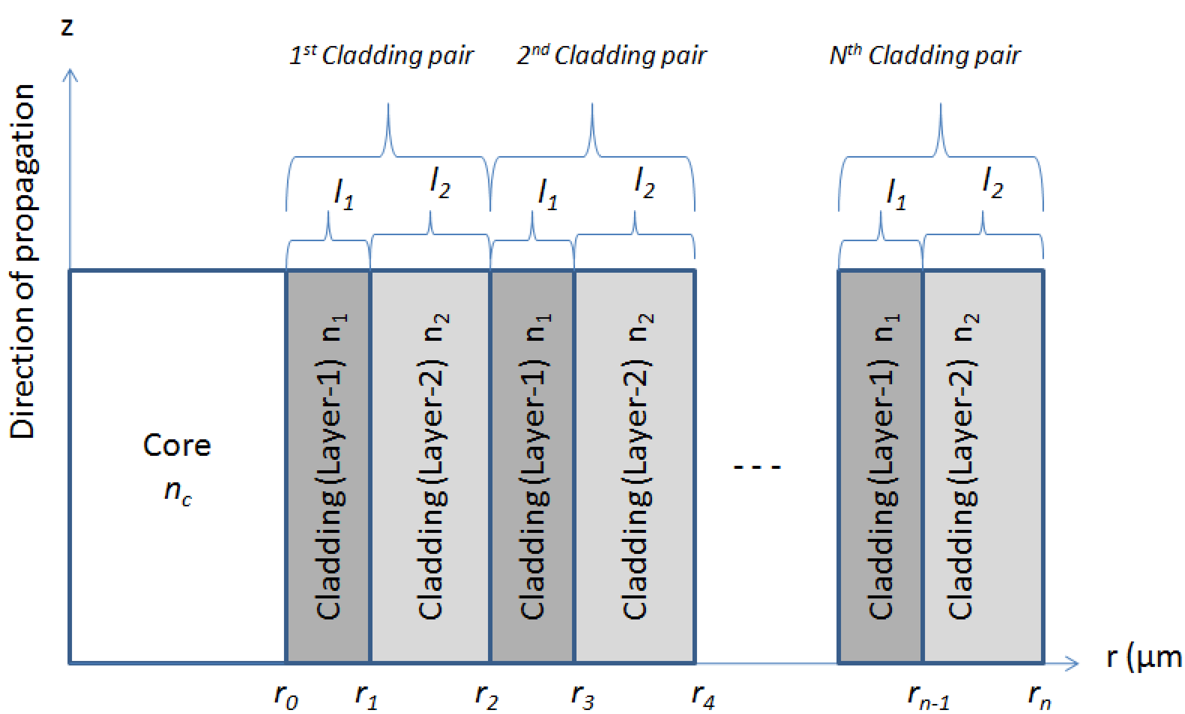

Fig. 1. Schematic of an air core Bragg fiber.

If we take the z- axis (see Fig. 1) as the direction of wave propagation, due to the translational symmetry, every field component can be represented in the following form [1]:

$$E_{(r,\Phi,z,t)} = E(r,\Phi)\, e^{i(\beta z-\omega t)} \tag{1}$$

$$H_{(r,\Phi,z,t)} = H(r,\Phi)\, e^{i(\beta z-\omega t)} \quad (2)$$

where *ω is the angular frequency and β is the propagation constant.*

According to waveguide theory we know that the transverse field components can be expressed in terms of E_z *and* H_z

$$E_r = \frac{i\beta}{\omega^2 \mu_i \epsilon_i - \beta^2}\left(\frac{\partial E_z}{\partial r} + \frac{\omega \mu_i}{\beta}\frac{\partial H_z}{r\partial \Phi}\right) \quad (3)$$

$$E_\Phi = \frac{i\beta}{\omega^2 \mu_i \epsilon_i - \beta^2}\left(\frac{\partial E_z}{r\partial \Phi} - \frac{\omega \mu_i}{\beta}\frac{\partial H_z}{\partial r}\right) \quad (4)$$

3 (a, b, c, d)

$$H_r = \frac{i\beta}{\omega^2 \mu_i \epsilon_i - \beta^2}\left(\frac{\partial H_z}{\partial r} - \frac{\omega \epsilon_i}{\beta}\frac{\partial E_z}{r\partial \Phi}\right) \quad (5)$$

$$H_\Phi = \frac{i\beta}{\omega^2 \mu_i \epsilon_i - \beta^2}\left(\frac{\partial H_z}{r\partial \Phi} + \frac{\omega \epsilon_i}{\beta}\frac{\partial E_z}{\partial r}\right) \quad (6)$$

where $k_i = (\omega^2 \mu_i \epsilon_i - \beta^2)^{1/2}$ and *i = c, 1, 2* stand for core, and periodic cladding layer 1 and 2 respectively.

Due to the cylindrical symmetry of air-core Bragg fibers, we can take the azimuthal dependence of the field components as *cos (lΦ)* or *sin (lΦ)*. For each *l*, the general solutions of E_z and H_z are the superposition of either $J_1(x)$ and $Y_1(x)$ or $I_l(x)$ and $K_l(x)$ where $x = (kr)$ [3]. In this section, we assume the solution in core layer and the electromagnetic field at a radius (r) can be written in the following matrix form [1, 3]:

$$\begin{bmatrix} E_z \\ \frac{1}{i\beta} H_\Phi \\ H_z \\ \frac{-1}{i\beta} E_\Phi \end{bmatrix} = M(k_c, r) \begin{bmatrix} A_1 \\ B_1 \\ C_1 \\ D_1 \end{bmatrix} \quad (7)$$

where A_1, B_1, C_1, and D_1 are constant amplitude coefficient in core and $M(k_i, r)$ are defined in equation (8).

$$M(k_i,r) = \begin{bmatrix} J_l(k_i r) & Y_l(k_i r) & 0 & 0 \\ \frac{\omega \epsilon_i}{\beta k_i} J'_l(k_i r) & \frac{\omega \epsilon_i}{\beta k_i} Y'_l(k_i r) & \frac{l}{k_i^2 r} J_l(k_i r) & \frac{l}{k_i^2 r} Y_l(k_i r) \\ 0 & 0 & J_l(k_i r) & Y_l(k_i r) \\ \frac{l}{k_i^2 r} J_l(k_i r) & \frac{l}{k_i^2 r} Y_l(k_i r) & \frac{\omega \mu_i}{\beta k_i} J'_l(k_i r) & \frac{\omega \mu_i}{\beta k_i} Y'_l(k_i r) \end{bmatrix} \quad (8)$$

It is important to note that the above equations (7) and (8) are the exact solution of Maxwell equation under $\beta \leq (\omega^2 \mu_i \in_i)$ without using any asymptotic approximation. Also the constant amplitude coefficient B_1 and D_1 at 1st layer (core region) are zero because the Neumann functions tends to infinity if argument tends to zero $Y_1(k_i r) \rightarrow \infty \; if \; (k_i r) \rightarrow 0$.

3. ASYMPTOTIC EXPRESSION IN CLADDING

In this section, we take the same cylindrical structure of air-core Bragg fiber (see Fig. 1) and found the different approach to observe the propagation constant of TE mode in Bragg fiber. In guided modes if we use the asymptotic expressions for the Bessel functions (Bessel function taken as exact solution of Maxwell equations) at $k_i r \rightarrow large\ (\infty)$ to describe the cladding fields, then radial dependence of E_z and H_z can be approximated as $e^{(ikr)}/\sqrt{r}$ and $e^{(-ikr)}/\sqrt{r}$ and becomes in following form[2]:

$$E_z = \frac{a_n e^{ik_1(r-r_n)} + b_n e^{-ik_1(r-r_n)}}{\sqrt{k_1 r}} \; ; r_n \leq r \leq r_{n+1} \tag{9}$$

$$E'_z = \frac{a'_n e^{ik_2(r-r'_n)} + b'_n e^{-ik_2(r-r'_n)}}{\sqrt{k_2 r}} \; ; \; r'_n \leq r \leq r'_{n+1} \tag{10}$$

$$H_z = \frac{c_n e^{ik_1(r-r_n)} + d_n e^{-ik_1(r-r_n)}}{\sqrt{k_1 r}} \; ; r_n \leq r \leq r_{n+1} \tag{11}$$

$$H'_z = \frac{c'_n e^{ik_2(r-r'_n)} + d'_n e^{-ik_2(r-r'_n)}}{\sqrt{k_2 r}} \; ; \; r'_n \leq r \leq r'_{n+1} \tag{12}$$

where a_n, b_n, c_n, d_n are the field amplitude coefficients of nth cladding layer (type -1) and $a'_n, b'_n c'_n, d'_n$ are field amplitude coefficient of nth cladding layer (type -2) and their values we find out later.

All other field component can be found by using the derivative of $E_z \; and \; H_z$ from equation (9-12) and put into equation (4) and (6). These field components are shown below in matrix form [3]:

$$\begin{bmatrix} E_z \\ H_\Phi \\ H_z \\ E_\Phi \end{bmatrix} = \begin{bmatrix} \left(\frac{a_n e^{ik_1(r-r_n)} + b_n e^{-ik_1(r-r_n)}}{\sqrt{k_1 r}} \right) \\ \frac{\omega n_1^2 \epsilon_0}{k_1} \left(\frac{a_n e^{ik_1(r-r_n)} - b_n e^{-ik_1(r-r_n)}}{\sqrt{k_1 r}} \right) \\ \left(\frac{c_n e^{ik_1(r-r_n)} + d_n e^{-ik_1(r-r_n)}}{\sqrt{k_1 r}} \right) \\ \frac{\omega \mu_i}{k_1} \left(\frac{c_n e^{ik_1(r-r_n)} - d_n e^{-ik_1(r-r_n)}}{\sqrt{k_1 r}} \right) \end{bmatrix} \tag{13}$$

and

$$\begin{bmatrix} E'_z \\ H'_\Phi \\ H'_z \\ E'_\Phi \end{bmatrix} = \begin{bmatrix} \left(\frac{a'_n e^{ik_2(r-r'_n)} + b'_n e^{-ik_2(r-r'_n)}}{\sqrt{k_2 r}}\right) \\ \frac{\omega n_2^2 \epsilon_0}{k_2}\left(\frac{a'_n e^{ik_2(r-r'_n)} - b'_n e^{-ik_2(r-r'_n)}}{\sqrt{k_2 r}}\right) \\ \left(\frac{c'_n e^{ik_2(r-r'_n)} + d'_n e^{-ik_2(r-r'_n)}}{\sqrt{k_2 r}}\right) \\ \frac{\omega \mu_0}{k_2}\left(\frac{c'_n e^{ik_2(r-r'_n)} - d'_n e^{-ik_2(r-r'_n)}}{\sqrt{k_2 r}}\right) \end{bmatrix} \tag{14}$$

To obtain the value of all amplitude coefficients, we are applying the boundary condition of all field components to continue at interface between any two adjacent dielectric layers and we found the following relations

$$\binom{a_{n+1}}{b_{n+1}} = \begin{bmatrix} A_{TM} & B_{TM} \\ B_{TM^*} & A_{TM^*} \end{bmatrix} \binom{a_n}{b_n} \tag{15}$$

$$\binom{c_{n+1}}{d_{n+1}} = \begin{bmatrix} A_{TE} & B_{TE} \\ B_{TE^*} & A_{TE^*} \end{bmatrix} \binom{c_n}{d_n} \tag{16}$$

where $A_{TE},\ B_{TE},\ A_{TM}\ and\ B_{TM}\ are\ defined\ as:$

$$A_{TE} = e^{ik_1 l_1}\left[i\ \frac{{k_1}^2 + {k_2}^2}{2k_1 k_2}\sin(k_2 l_2) + \cos(k_2 l_2)\right] \tag{17}$$

$$B_{TE} = \ ie^{-ik_1 l_1}\frac{{k_1}^2 - {k_2}^2}{2k_1 k_2}\sin(k_2 l_2) \tag{18}$$

$$A_{TM} = e^{ik_1 l_1}\left[i\ \frac{{n_2}^4 {k_1}^2 + {n_1}^4 {k_2}^2}{2{n_1}^2 {n_2}^2 k_1 k_2}\sin(k_2 l_2) + \cos(k_2 l_2)\right] \tag{19}$$

$$B_{TM} = \ ie^{-ik_1 l_1}\frac{{n_2}^4 {k_1}^2 - {n_2}^4 {k_2}^2}{2{n_1}^2 {n_2}^2 k_1 k_2}\sin(k_2 l_2) \tag{20}$$

It is important to note that all four parameters ($A_{TE}, B_{TE}, A_{TM}\ and\ B_{TM}$) are the same for all cladding layers and also after applying the Bloch theorem on the cladding fields we say that:

$$\binom{a_{n+1}}{b_{n+1}} = \lambda_{TM}\binom{a_n}{b_n} \tag{21}$$

$$\binom{c_{n+1}}{d_{n+1}} = \lambda_{TE}\binom{c_n}{d_n} \quad 17(b) \tag{22}$$

where

$$\lambda_{TM} = Re(A_{TM}) \pm \sqrt{\{[[Re(A_{TM}]^2 - 1]\}}$$

$$\lambda_{TE} = Re(A_{TE}) \pm \sqrt{\{[[Re(A_{TE}]^2 - 1]\}} \quad (23)$$

From above solutions, it is described in [Bloch theorem and Application][5] that the cylindrical Bloch theorem is guaranteed except for the vicinity of the origin. In our case, it is clear that this restriction does not prevent us from the application to the Bragg fiber. According to Bloch theorem the Bloch wave can propagate in Bragg stack only if $Re(A_{TM}) < 1$ [Analysis and Design of Bragg Fibers Using a Novel Confinement Loss Diagram Approach][5]. If $Re(A_{TM}) \geq 1$ than Bloch wave cannot propagate in our case. For the sake of easy understanding and to achieve the optimum confinement in Bragg fiber, we consider the quarter-wave layers such that $k_1 l_1 = k_2 l_2 = {}^{\pi}/_{2}$

4. SOLUTION FOR PROPAGATION CONSTANTS (β)

The propagation constant in Bragg fiber are founded by matching the value of electric and magnetic fields (exact solution in core region and asymptotic limit solution in cladding region) at interface between core and cladding layer *(let r=r₀)*, which gives us[2]

$$M(k_i, r)\begin{bmatrix} A_i \\ B_i \\ C_i \\ D_i \end{bmatrix} = \begin{bmatrix} \left(\frac{a_n e^{ik_1(r-r_n)} + b_n e^{-ik_1(r-r_n)}}{\sqrt{k_1 r}}\right) \\ \frac{\omega n_1^2 \epsilon_0}{k_1}\left(\frac{a_n e^{ik_1(r-r_n)} - b_n e^{-ik_1(r-r_n)}}{\sqrt{k_1 r}}\right) \\ \left(\frac{c_n e^{ik_1(r-r_n)} + d_n e^{-ik_1(r-r_n)}}{\sqrt{k_1 r}}\right) \\ \frac{\omega \mu_i}{k_1}\left(\frac{c_n e^{ik_1(r-r_n)} - d_n e^{-ik_1(r-r_n)}}{\sqrt{k_1 r}}\right) \end{bmatrix} \quad (24)$$

$$\frac{\omega^2}{c^2} n_c^2 \left[\frac{J_l'(k_c r_0)}{J_l(k_c r_0)} + i\,\frac{k_c n_1^2}{k_1 n_c^2}\left(\frac{\lambda_{TM} - A_{TM} - B_{TM}}{\lambda_{TM} - A_{TM} + B_{TM}}\right)\right] \times \left[\frac{J_l'(k_c r_0)}{J_l(k_c r_0)} + i\,\frac{k_c}{k_1}\left(\frac{\lambda_{TE} - A_{TE} - B_{TE}}{\lambda_{TE} - A_{TE} + B_{TE}}\right)\right] = \frac{\beta^2 l^2}{k_c^2 r_0^2} \quad (25)$$

Equation (25) is general solution which governs for guided mode in Bragg fibers. This mode can be classified called transverse electric (TE), transverse magnetic (TM), or mixture of (TE and TM) both dependence on integer value of(l). Further, if $l = 0$ than either TE mode or TM mode occurs, and $l \neq 0$ express only the mixture of TE and TM mode.

In this section, we have analyzed the accuracy of asymptotic approximation as compared to exact solution and also the validation of structure in Bragg fiber. For simplification purpose, only TE mode has been computed after considering the Bragg fiber (see Fig. 1).

All parameters are considered as$n_c = 1\,(air\ core), r_0 = 1\ \mu m,\ n_1 = 3, l_1 = 0.130\ \mu m, n_2 = 1.5, l_2 = 0.265\ \mu m$. Also, $l_1\ and\ l_2$ are selected above due to being the cladding as quarter wave stack for λ = 1.55 μm. For the analysis of only TE mode, we are taking the second terms of LHS in equation (25) and putting the value of $l = 0$ in RHS of same equation.

So, the equation (25) for TE mode is given by

$$\frac{J_l'(k_c r_0)}{J_l(k_c r_0)} + i\frac{k_c}{k_1}\left(\frac{\lambda_{TE} - A_{TE} - B_{TE}}{\lambda_{TE} - A_{TE} + B_{TE}}\right) = 0 \qquad (26)$$

A similar procedure can be applied in exact solution to obtain the propagation constant and after simplification for analysis of only TE mode it is given by [1]

$$\frac{1}{2}\left(\frac{\pi k_1 r}{2}\right)^2\left(1 - \frac{k_c^2}{k_1^2}\right) k_c [J_0'^{\,2}(k_1 r) + Y_0'^{\,2}(k_1 r)] J_0(k_c r) J_c'(k_c r) = 0 \qquad (27)$$

From these two equations, we have calculated and simulated (effective index plotted in Fig. 2) the propagation constant (β) within selected spectrum range λ between 1.36 μm to 1.61 μm. The results in both approaches are excellent with difference approximatelly zero using both methods.

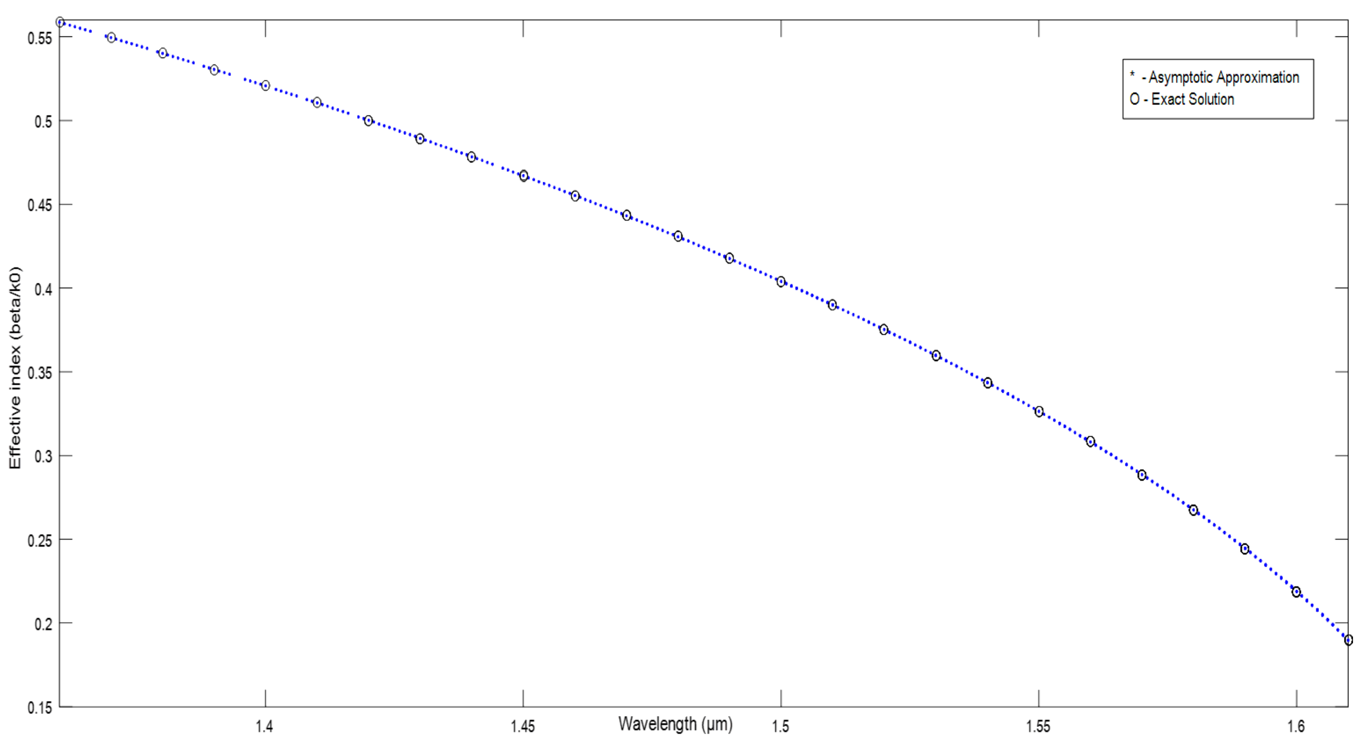

Fig. 2. Propagation Constants

Table 1. Propagation Constants

Core Radius	Wavelength (μm)	Propagation Constant	
		Exact	Asymptotic
1 μm	1.36 μm	0.5587	0.5587
	1.40 μm	0.5207	0.5207
	1.44 μm	0.4784	0.4783
	1.48 μm	0.4306	0.4306
	1.52 μm	0.3752	0.3752
	1.56 μm	0.3081	0.3081
	1.60 μm	0.2189	0.2189

5. CONCLUSIONS

The agreement between the both approach is excellent. But, we have seen that the asymptotic approximations are valid only for a core radius of at least several wavelengths.

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