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Evaluation of Forecasts Performance of ARIMA-GARCH-type Models in the Light of Outliers

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ABSTRACT

The carry-over effect of biased estimates of ARIMA-GARCH-type models parameters on forecasting accuracy is investigated in the presence of outliers by exploring the daily returns of share price series of three major banks in Nigerian. The banks considered are Diamond, United bank for Africa and Union. The data were collected from the Nigerian Stock Exchange and spanned from January 3, 2006 to December 30, 2016, comprises 2713 observations and were divided into two portions. The first portion which ranges from January 3, 2006 to November 24, 2016, comprises 2690 observations was used for model formulation and the second portion which ranges from November 25, 2016 to December 30, 2016, consisting of 23 observations was used for out-of-sample forecasting performance evaluation. The parametric bootstrap technique was used in computing the forecasts while Mean Squared Error (MSE), Root Mean Squared Error (RMSE), Mean Absolute Error (MAE) and Mean Error (ME) were the methods of forecast evaluation considered. The findings of this study showed that in the presence of outliers, the forecasts were found to be biased as indicated by ME and the accuracy reduced as shown by MSE, RMSE and MAE. However, adjusting for the outliers, only marginal improvement on the forecasts was observed, reason being that all the outliers were treated as innovations and they occurred before the forecasts origin.

***Keywords*:** ARIMA Model, Forecast, GARCH Model, Heteroscedasticity, Outlier, Volatility

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1. INTRODUCTION

In recent times, it is common to assess forecasting performance of a model using its predictive distributions. This predictive distribution incorporates parameter uncertainty in the forecasts [1]. In relation to forecasting GARCH-type processes, prior studies have shown that the estimates of model parameters are often biased in the presence of outliers and that accounting for outliers could improve the forecasting accuracy [2-5]. On the other hand, the works of [5]; [6] and [7] claimed that innovation outliers especially when occurring before the forecast origin have little or no effect on the forecasts.

Thus, the aim of this study is to trace the carry-over effects of biased estimates on the forecasting accuracy by improving on the work of [8] who modeled heroscedasticity in the presence of outliers in the share price returns of Nigerian banks.

For a robust test of forecast performance evaluation, out-of-sample forecasting approach is often applied. Out-of-sample forecast provides an excellent opportunity to look at what is called out-sample behaviour of time series data. That is, a time series will provide forecasts of new future observations which can be checked against what is actually observed [1, 9, 10]. Again, the out-of-sample forecast is accomplished when the data used for constructing the model are different from that used in forecasting evaluation. That is, the data is divided into two portions. The first portion is for model construction and the second is used for evaluating the forecasting performance with possibility of forecasting new future observations which can be checked against what is observed [1, 9-11].

2. MATERIALS AND METHODS

2. 1. Return

The return series R_t can be obtained given that P_t is the price of a unit share at time, t and P_{t-1} is the share price at time t−1.

$$R_t = \nabla lnP_t = (1-B)lnP_t = ln\,P_t - ln\,P_{t-1} \tag{1}$$

The R_t in equation (1) is regarded as a transformed series of the share price, P_t meant to attain stationarity, that is, both mean and variance of the series are stable [12]. The letter B is the backshift operator.

2. 2. Parametric bootstrap

The parametric bootstrap is used in computing nonlinear forecasts given the fact that the model used in forecasting has been rigorously checked and is judged to be adequate for the series under study [1]. Let T be the forecast origin and k be the forecast horizon (k > 0). That is, we are at time index T and interested in forecasting R_{T+k}. The parametric bootstrap considered compute realizations $R_{T+1},\ldots,R_{T+k}$ sequentially by drawing a new innovation from the specific innovational distribution of the model, and computing R_{T+i} using the model, data, and previous forecasts $R_{T+1},\ldots, R_{T+i-1}$. This results in a realization for R_{T+k}. The procedure is repeated M times to obtain M realizations of R_{T+k} denoted by $\left\{R_{T+k}^{(j)}\right\}_{j=1}^{M}$. The point forecast of R_{T+k} is then the sample average of $R_{T+k}^{(j)}$.

Consequently, Forecasts of the ARCH model are obtained recursively. Let T be the starting date for forecasting, that is forecast origin. Let F_T be the information set available at time T.

Then, the 1-step ahead forecast for conditional variance, $\sigma^2_{\mathrm{T}+1}$is

$$\sigma^2_{\mathrm{T}}(1) = \hat{\omega} + \hat{\alpha}_1 \hat{a}^2_{\mathrm{T}} + \ldots + \hat{\alpha}_p \hat{a}^2_{\mathrm{T}+1-p} \tag{2}$$

where $\hat{a}_{\mathrm{T}}$ is the estimated residual. For the 2-step ahead forecast $\sigma^2_{\mathrm{T}+2}$, we need a forecast of $a^2_{\mathrm{T}+1}$. It is given by $\sigma^2_{\mathrm{T}}(1)$.

We therefore obtain

$$\sigma^2_{\mathrm{T}}(2) = \hat{\omega} + \hat{\alpha}_1 \sigma^2_{\mathrm{T}}(1) + \hat{\alpha}_2 \hat{a}^2_{\mathrm{T}} + \ldots + \hat{\alpha}_p \hat{a}^2_{\mathrm{T}+2-p}.$$

The k-step ahead forecast for $\sigma^2_{\mathrm{T}+k}$ is

$$\sigma^2_{\mathrm{T}}(k) = \hat{\omega} + \hat{\alpha}_1 \sigma^2_{\mathrm{T}}(k-1) + \cdots + \hat{\alpha}_p \sigma^2_{\mathrm{T}}(k-p). \tag{3}$$

with $\sigma^2_{\mathrm{T}}(k-i) = \hat{a}^2_{\mathrm{T}+k-i} \; if \; k-i \leq 0.$

Forecasts of the GARCH model are obtained recursively in a similar way as that of the ARCH model. Then, the 1-step ahead forecast for $\sigma^2_{\mathrm{T}+1}$ is

$$\sigma^2_{\mathrm{T}}(1) = \hat{\omega} + \hat{\alpha}_1 \hat{a}^2_{\mathrm{T}} + \hat{\beta}_1 \hat{\sigma}^2_{\mathrm{T}} \tag{4}$$

since $a^2_{\mathrm{T}} = \sigma^2_{\mathrm{T}} e^2_{\mathrm{T}}$, the GARCH (1,1) model can be rewritten as

$$\sigma^2_{\mathrm{T}} = \omega + \alpha_1 a^2_{\mathrm{T}-1} + \beta_1 \sigma^2_{\mathrm{T}-1} = \omega + (\alpha_1 + \beta_1)\sigma^2_{\mathrm{T}-1} + \alpha_1 \sigma^2_{\mathrm{T}-1}(e^2_{\mathrm{T}-1} - 1),$$

so that, at time $\mathrm{T}+2$, we have

$$\sigma^2_{\mathrm{T}+2} = \omega + (\alpha_1 + \beta_1)\sigma^2_{\mathrm{T}+1} + \alpha_1 \sigma^2_{\mathrm{T}+1}(e^2_{\mathrm{T}+1} - 1),$$

with $E[(e^2_{\mathrm{T}+1} - 1)/F_{\mathrm{T}}] = 0$, we deduce the following 2-step ahead forecast for $\sigma^2_{\mathrm{T}+2}$:

$$\sigma^2_{\mathrm{T}}(2) = \hat{\omega} + \left(\hat{\alpha}_1 + \hat{\beta}_1\right)\sigma^2_{\mathrm{T}}(1).$$

Generally speaking, the k-step ahead forecast for $\sigma^2_{\mathrm{T}+k}$ is

$$\sigma^2_{\mathrm{T}}(k) = \hat{\omega} + \left(\hat{\alpha}_1 + \hat{\beta}_1\right)\sigma^2_{\mathrm{T}}(k-1), \; k > 1. \tag{5}$$

One of the beauties of GARCH is that volatility forecasts for any horizon can be constructed from the estimated model. The estimated GARCH model is used to get forecasts of instantaneous forward volatilities, that is, the forecast for $\sigma^2_{\mathrm{T}+k}$ made at time T and for every k step ahead.

For EGARCH model, assuming that the model parameters are known and the observations are standard Gaussian, for EGARCH (1,1) model, we have

$$ln\sigma_T^2 = (1-\alpha_1)\omega + \alpha_1 ln\sigma_{T-1}^2 + g(\epsilon_{T-1}),$$

$$g(\epsilon_{T-1}) = \theta\epsilon_{T-1} + \gamma(|\epsilon_{T-1}| - \sqrt{2/\pi}). \quad (6)$$

Taking exponentials, the model becomes

$$\sigma_T^2 = \sigma_{T-1}^{2\alpha_1} \exp[(1-\alpha_1)\omega] \exp[g(\epsilon_{T-1})],$$

$$g(\epsilon_{T-1}) = \theta\epsilon_{T-1} + \gamma(|\epsilon_{T-1}| - \sqrt{2/\pi}) \quad (7)$$

For the 1-step ahead forecast, σ_{T+1}^2 we have

$$\sigma_T^2(1) = \sigma_T^{2\alpha_1} \exp[(1-\alpha_1)\omega] \exp[g(\epsilon_T)] \quad (8)$$

The 2-step-ahead forecast of σ_{T+2}^2 is given by

$$\sigma_T^2(2) = \hat{\sigma}_T^{2\alpha_1}(1) \exp[(1-\alpha_1)\omega] E_T\{\exp[g(\epsilon_T)]\}$$

where E_T denotes a conditional expectation taken at the time origin T with

$$E\{\exp[g(\epsilon_T)]\} = \exp(-\gamma\sqrt{2/\pi})[e^{(\theta+\gamma)^2/2}\Phi(\theta+\gamma) + e^{(\theta-\gamma)^2/2}\Phi(\gamma-\theta)]$$

where $\Phi(x)$ is the cumulative density function of the standard normal distribution (see [1] for more details).

Hence,

$$\hat{\sigma}_T^2(2) = \hat{\sigma}_T^{2\alpha_1}(1)\exp[(1-\alpha_1)\omega - \gamma\sqrt{2/\pi})]$$

$$\times\{\exp[(\theta+\gamma)^2/2]\Phi(\theta+\gamma) + \exp[(\theta-\gamma)^2/2]\ \Phi(\gamma-\theta)\}.$$

Generally, the k-step -ahead forecast can be obtained as

$$\hat{\sigma}_T^2(k) = \hat{\sigma}_T^{2\alpha_1}(k-1)\exp[(1-\alpha_1)\omega - \gamma\sqrt{2/\pi})]$$

$$\times\{\exp[(\theta+\gamma)^2/2]\Phi(\theta+\gamma) + \exp[(\theta-\gamma)^2/2]\ \Phi(\gamma-\theta)\}. \quad (9)$$

2. 3. Model Evaluation Criteria

The methods of forecast evaluation based on forecast error include Mean Squared Error (MSE), Root Mean Squared Error (RMSE) and Mean Absolute Error (MAE). These criteria measure forecast accuracy. The forecast bias is measured by Mean Error (ME).

The measures are computed as follows:

$$\text{MSE} = \frac{1}{n}\sum_{i=1}^{n} \text{e}_i^2 \quad (10)$$

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} e_i^2} \tag{11}$$

$$\text{MAE} = \frac{1}{n}\sum_{i=1}^{n}|e_i| \tag{12}$$

$$\text{ME} = \frac{1}{n}\sum_{i=1}^{n}(e_i) \tag{13}$$

where e_i is the forecast error and n is the number of forecast error. Also, it should be noted that in this work, the forecasts of the returns are used as proxies for the volatilities as they are not directly observable.

2. 4. Outliers in Time Series

Generally, a time series might contain several, say k outliers of different types and we have the following general outlier model:

$$Y_t = \sum_{j=1}^{k} \tau_j V_j(\text{B}) I_t^{(T)} + X_t, \tag{14}$$

where $X_t = (\theta(B)) / (\varphi(B))\, a_t$, $V_j(\text{B}) = 1$ for an AO, and $V_j(\text{B}) = \frac{\theta(B)}{\varphi(B)}$ for an IO at t = T_j, $V_j(\text{B}) = (1 - B)^{-1}$ for a LS, $V_j(\text{B}) = (1 - \delta B)^{-1}$ for an TC, and τ is the size of outlier.

For more details on the types of outliers and estimation of the outliers effects (see [5, 11, 13-15]).

Moreover, in financial time series, the residual series, a_t is assumed to be uncorrelated with its own past, so additive, innovative, temporary change and level shift outliers coincide, and where both the mean and variance equations evolves together, we have for example GARCH(1,1) model:

$$R_t - \mu_t = \tilde{a}_t + \tau I_t^{(T)}. \tag{15}$$

$$\tilde{a}_t = \sigma_t e_t. \tag{16}$$

$$\sigma_t^2 = \omega + \alpha_1 \tilde{a}_{t-1}^2 + \beta_1 \sigma_{t-1}^2. \tag{17}$$

where $\tilde{a}_t$ is the outliers contaminated residuals.

One approach for correcting the series for outliers is using standard criteria and then estimates the conditional variance. This approach involves detecting and correcting of outliers before estimating the conditional variance [16-18].

3. RESULTS AND DISCUSSION

3. 1. Plot Analysis

The upward and downward movements of the share prices in Figures 1-3 indicate that the series are not stationary.

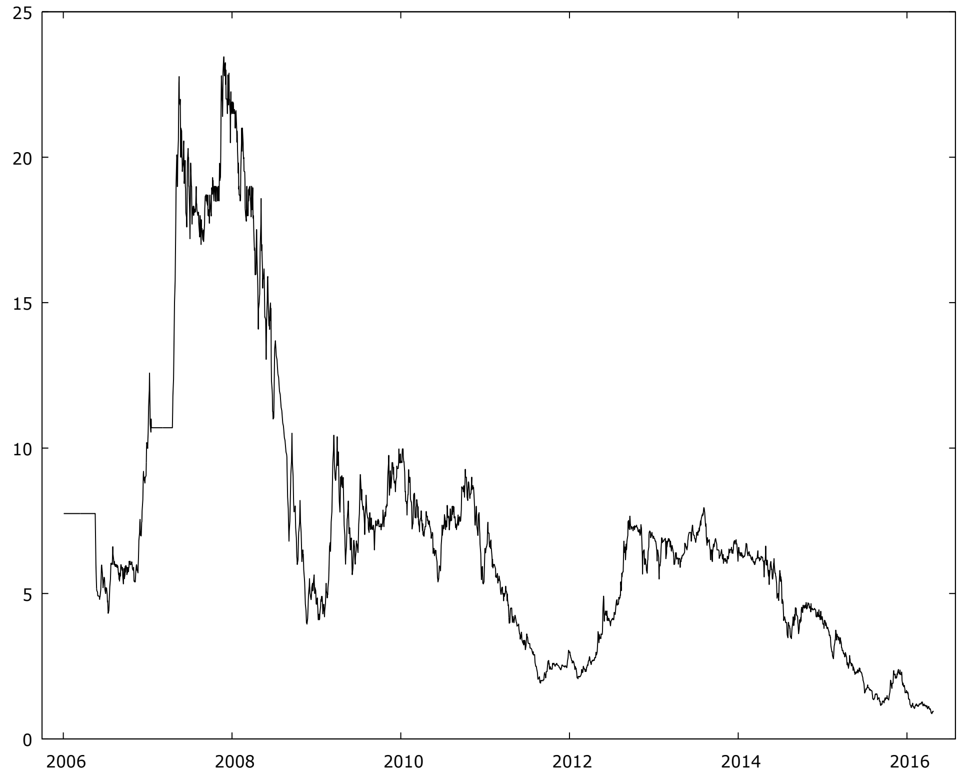

Figure 1. Share Price Series of Diamond Bank

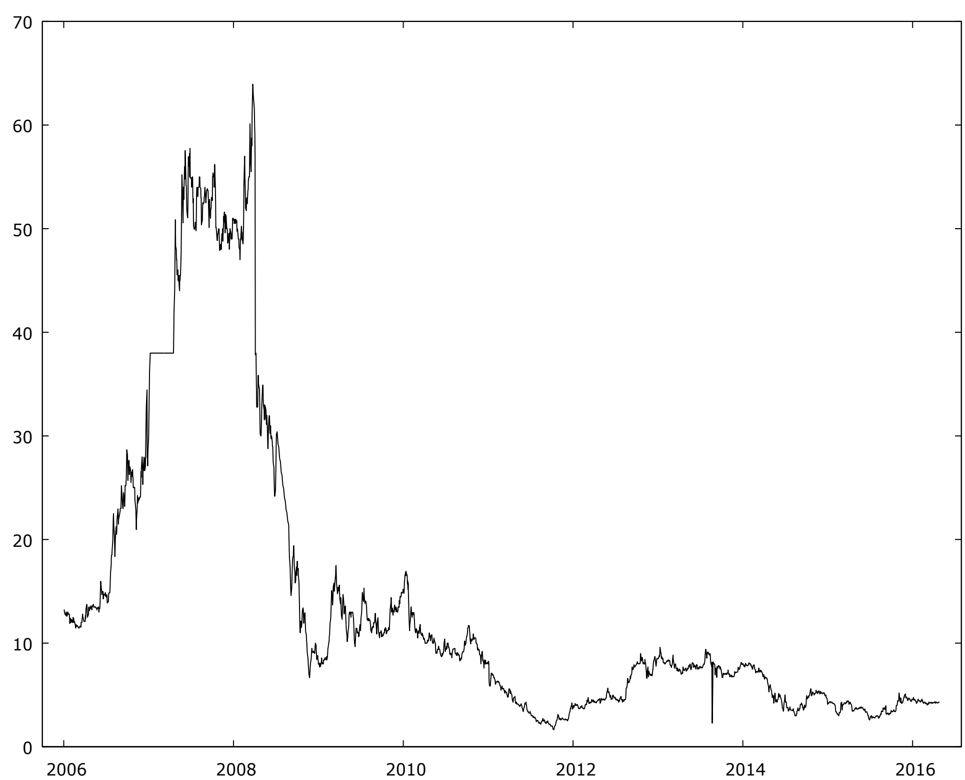

Figure 2. Share Price Series of United Bank for Africa

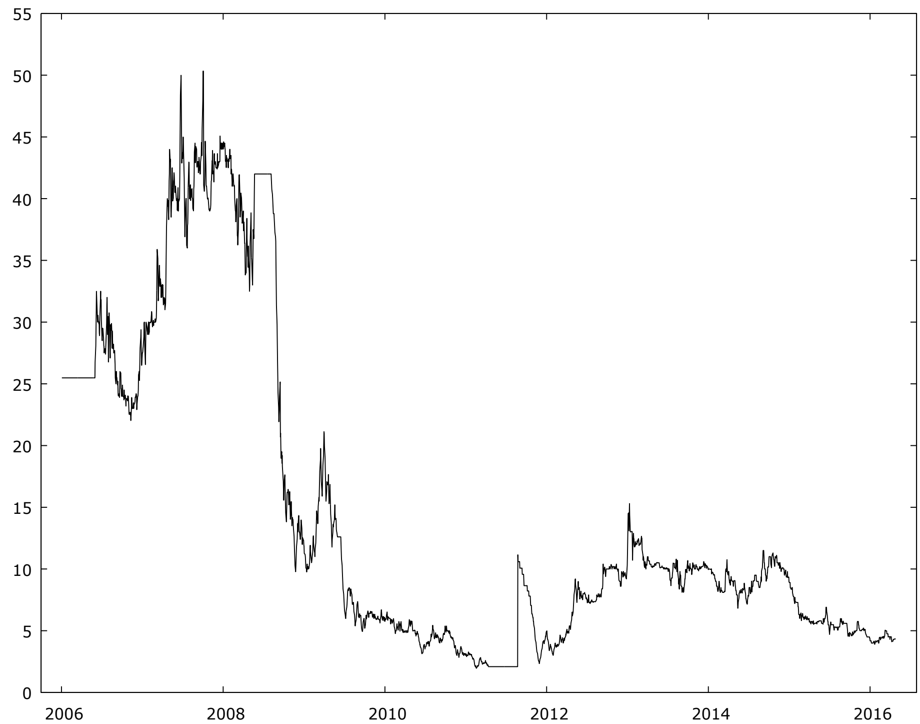

Figure 3. Share Price Series of Union Bank of Nigeria

However, to obtain stationarity, equation (1) was applied and the series (returns) in Figures 4-6 were found to cluster round a common mean.

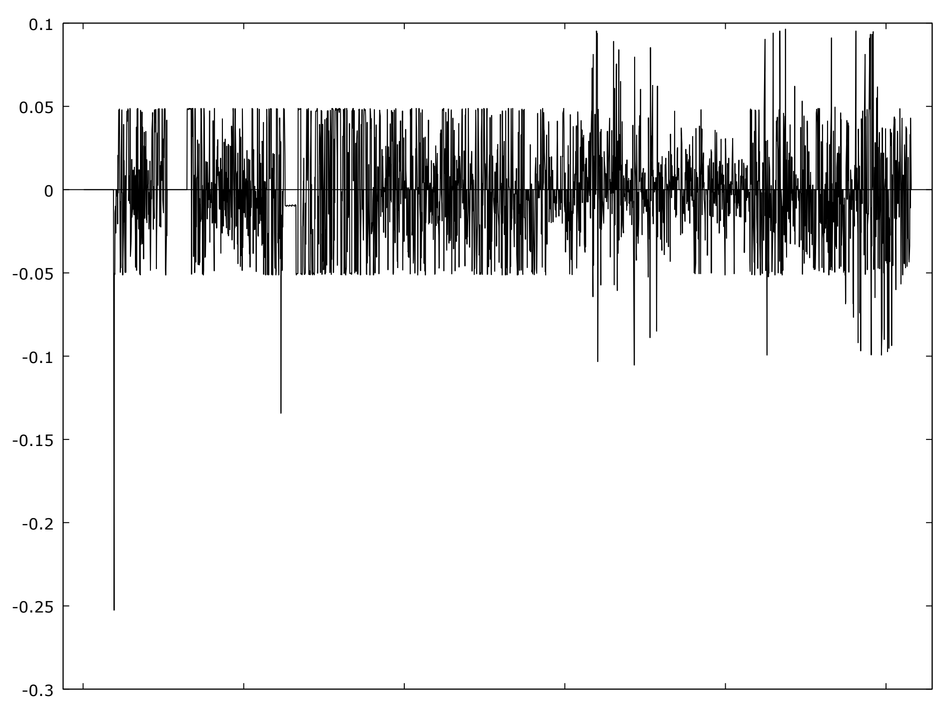

Figure 4. Return Series of Diamond Bank

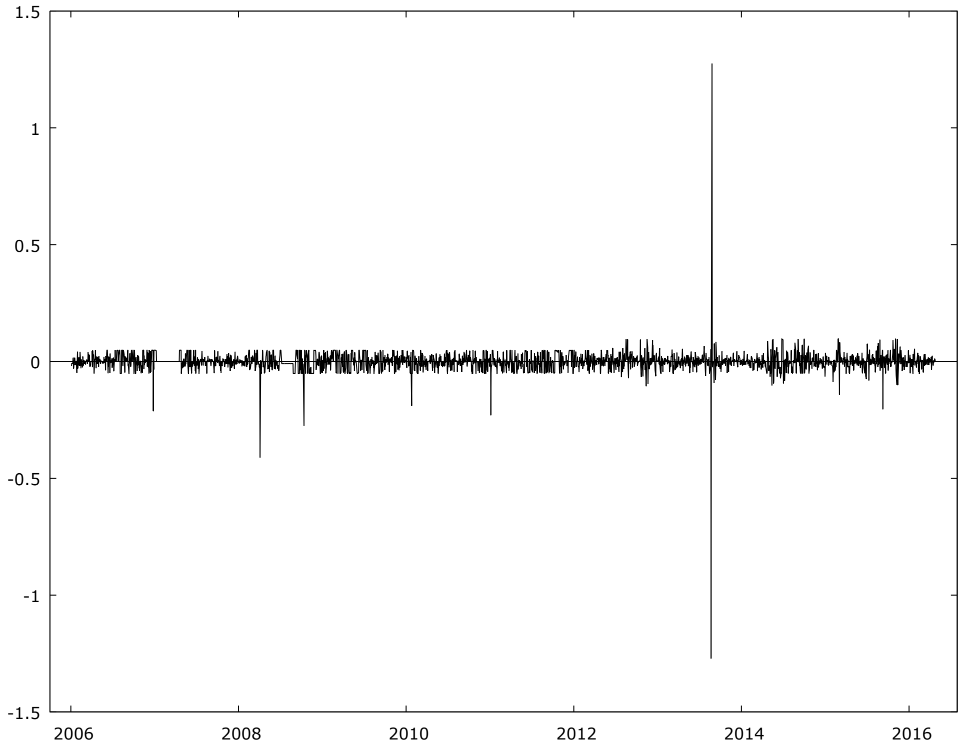

Figure 5. Return Series of United Bank for Africa

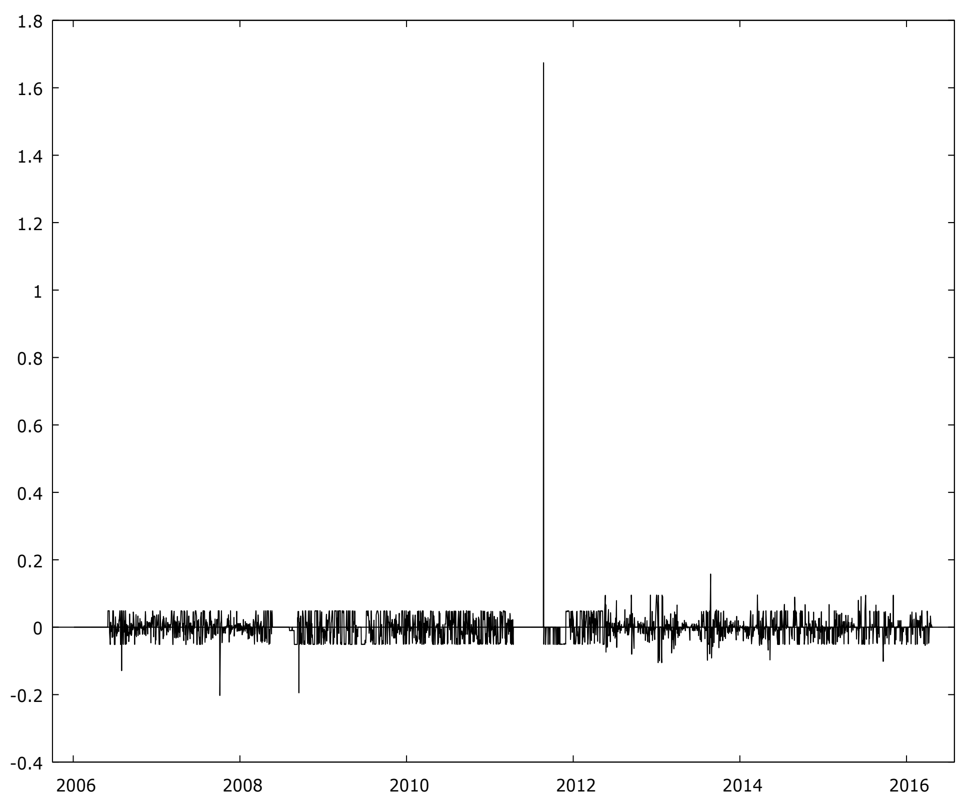

Figure 6. Return Series of Union Bank of Nigeria

3. 2. ARIMA-GARCH-type Modeling of Returns Series of Nigerian Banks

Several models with respect to normal (norm) and student-t (std) distributions were entertained tentatively. The models in Table 1 were found to be adequate for respective banks (see Table 2).

Table 1. ARIMA-GARCH-type Models of Returns Series of Nigerian Banks

Bank	Model
Diamond	ARIMA(2,1,1)-GARCH(2,0)-std
United Bank for Africa	ARIMA(0,1,2)-GARCH(1,1)-norm
Union	ARIMA(1,1,0)-GARCH(2,0)-std

Table 2. Diagnostic Checking for ARIMA-GARCH-type Models of Returns Series Of Nigerian Banks

Bank	Model	Standardized Residuals			Standardized Squared Residuals					
		Lag	Weighted LB	p-value	Lag	Weighted LB	p-value	Lag	Weighted ARCH – LM	p-value
Diamond	ARIMA (2,1,1)-GARCH (2,0)-std	1	0.0001	0.9903	1	0.0004	0.9835	3	0.0004	0.9835
		8	0.0007	1.0000	5	0.0012	1.0000	5	0.0010	1.0000
		14	0.0011	1.0000	9	0.0021	1.0000	7	0.0015	1.0000
United Bank for Africa	ARIMA (0,1,2)-GARCH (1,1)-norm	1	0.0853	0.7702	1	0.0028	0.958	3	0.0093	0.9233
		5	3.7001	0.1357	5	0.0158	1.0000	5	0.0136	0.9992
		9	5.8713	0.2781	9	0.0284	1.0000	7	0.0222	1.0000
Union	ARIMA (1,1,0)-GARCH (2,0)-std	1	0.0615	0.8042	1	0.0008	0.9768	3	0.0008	0.9770
		2	0.0726	1.0000	5	0.0025	1.0000	5	0.0020	1.0000
		5	0.1900	0.9998	9	0.0040	1.0000	7	0.0027	1.0000

LB = Ljung-Box, LM = Lagrange Multiplier

3. 3. Identification of Outliers in the Residual Series of ARIMA Models Fitted to the Return Series of Nigerian Bank

To identify the potential outliers in the residuals series of the fitted ARIMA models the returns series of the bank, those statistics that are in absolute value higher than a critical value (cval) are used in identifying the time point of a potential outlier. In this study, cval = 4 is chosen on the condition that the number of observations, $T \geq 450$ and where cval = 4 is not sufficient, cval = 5 would be considered.

3. 3. 1. Identification of Outliers in the Residual Series of ARIMA (2, 1, 1) Model Fitted to the Return Series of Diamond Bank

About seventeen (17) different outliers were detected to have contaminated the residuals series of ARIMA(1, 1, 0) model using the critical value, cval = 4; four (4) innovation outliers (IO), ten (10) additive outliers (AO) and three (3) temporary change (TC) as shown in (Table 3).

Table 3. Outliers Identified in the Residual Series of ARIMA (2, 1, 1) Model fitted to Return Series of Diamond Bank

Type	Observation index	Location	Estimate	T-statistic
IO	99	05/06/2006	-0.25260558	-10.126263
IO	642	14/08/2008	-0.14020952	-5.620614
IO	1671	12/10/2012	-0.09994872	-4.006669
IO	1791	10/04/2013	0.10771031	4.317810
AO	1656	20/09/2012	0.10786904	4.453514
AO	1723	31/12/2012	0.09917004	4.094364
AO	1739	23/01/2013	0.09980816	4.120710
AO	1790	09/04/2013	-0.09871167	-4.075440
AO	1843	25/06/2013	0.09746866	4.024121
AO	2263	05/03/2015	0.10332795	4.266029
AO	2281	31/03/2015	0.10065028	4.155478
AO	2562	19/05/2016	0.10020395	4.137050
AO	2626	23/08/2016	-0.10386669	-4.288272
TC	98	02/06/2006	-0.09207813	-4.344299
AO	2559	16/05/2016	-0.09636362	-4.043664
TC	1667	08/10/2012	0.09378011	4.497073
TC	2554	09/05/2016	0.08697699	4.190962

3. 3. 2. Identification of Outliers in the Residual Series of ARIMA (0, 1, 2) Model Fitted to the Return Series of United Bank for Africa

About thirty one (31) different outliers were detected to have contaminated the residuals series of ARIMA(1, 1, 0) model using the critical value, cval = 5; one (1) innovation outlier (IO), seven (11) additive outliers (AO) and Nineteen (19) temporary change (TC) as shown in (Table 4).

Table 4. Outliers Identified in the Residual Series of ARIMA (0, 1, 2) Model fitted to Return Series of United Bank for Africa

Type	Observation index	Location	Estimate	T-statistic
IO	1992	31/01/2014	-1.28502933	-50.180476
AO	255	19/01/2007	-0.22086607	-9.215670
AO	588	30/05/2008	-0.42240432	-17.624884
AO	590	03/06/2008	-0.18565429	-7.746453
AO	724	15/12/2008	-0.26195514	-10.930118
AO	1059	23/04/2010	-0.21501218	-8.971416
AO	1306	21/04/2011	-0.20172914	-8.417179
AO	1990	29/01/2014	-0.30146512	-12.578678
AO	1994	04/02/2014	0.85624927	35.727130
AO	2391	09/09/2015	-0.12980376	-5.416082
AO	2526	29/03/2016	-0.18732654	-7.816228
TC	258	24/01/2007	0.08938516	5.150659
TC	586	27/05/2008	-0.17371521	-10.010027
TC	720	05/12/2008	-0.09820632	-5.658963
TC	722	11/12/2008	-0.12146766	-6.999356
TC	743	14/01/2009	-0.08686036	-5.005173
TC	745	16/01/2009	-0.08974508	-5.171399
TC	747	20/01/2009	-0.08980803	-5.175027

TC	1057	21/04/2010	-0.10201697	-5.878545
TC	1507	16/02/2012	0.08901146	5.129125
TC	1727	07/01/2013	0.09046794	5.213053
TC	1988	27/01/2014	-0.14516399	-8.364815
TC	1993	03/02/2014	0.32569376	18.767519
TC	2212	18/12/2014	0.09073975	5.228715
AO	2001	13/02/2014	-0.12316896	-5.194574
TC	2217	29/12/2014	-0.08968031	-5.223304
TC	2386	02/09/2015	0.08645455	5.035424
TC	817	04/05/2009	0.08559737	5.031276
TC	1986	23/01/2014	0.08566229	5.035092
TC	726	17/12/2008	0.08504413	5.007596
TC	1989	28/01/2014	0.12691902	6.052727

3. 3. 3. Identification of Outliers in the Residual Series of ARIMA (1, 1, 0) Model Fitted to the Return Series of Union Bank

About nineteen (19) different outliers were detected to have contaminated the residuals series of ARIMA(1, 1, 0) model using the critical value, cval = 5; four (4) innovation outliers (IO), eight (8) additive outliers (AO) and seven (7) temporary change (TC) as shown in (Table 5).

Table 5. Outliers Identified in the Residual Series of ARIMA (1, 1, 0) Model fitted to Return Series of Union Bank

Type	Observation index	Location	Estimate	T-statistic
IO	458	16/11/2007	-0.20259320	-9.867965
IO	1472	23/12/2011	-0.22031597	-10.731210
IO	1831	07/06/2013	0.10533493	5.130683
IO	1843	25/06/2013	0.10590627	5.158511

AO	150	15/08/2006	-0.13856541	-6.783874
AO	705	14/11/2008	-0.20086454	-9.833910
AO	1471	22/12/2011	1.67935140	82.217553
AO	1830	06/06/2013	-0.11483241	-5.621956
AO	1842	24/06/2013	-0.10581300	-5.180384
AO	1984	21/01/2014	-0.10648119	-5.213098
AO	1994	04/02/2014	0.16239480	7.950512
TC	691	27/10/2008	-0.08071046	-5.129738
TC	901	31/08/2009	-0.08274861	-5.259278
TC	1470	22/12/2011	0.53378545	33.925958
TC	1523	09/03/2012	-0.08218825	-5.223663
TC	1541	04/04/2012	0.07869209	5.001456
TC	1824	28/05/2013	0.11353246	7.215815
TC	2534	08/04/2016	-0.08059290	-5.122266
AO	1748	06/02/2013	-0.11923771	-5.160464

3. 4. ARIMA-GARCH-type Modeling of Outlier Adjusted Returns Series of Nigerian Banks

Having cleaned the series of the detected outliers, the models in Table 6 were found to be adequate for the outlier adjusted returns series of the respective banks (see Table 7).

Table 6. ARIMA-GARCH-type Models of Outlier Adjusted Returns Series Of Nigerian Banks

Bank	Model
Diamond	ARIMA(2,1,1)-EGARCH(1,1)-std
United Bank for Africa	ARIMA(2,1,1)-EGARCH(1,1)-std
Union	ARIMA(1,1,0)-GARCH(1,1)-norm

Table 7. Diagnostic Checking for ARIMA-GARCH-type Models of Outlier Adjusted Returns Series of Nigerian Banks

Bank	Model	Standardized Residuals			Standardized Squared Residuals					
		Lag	Weighted LB	p-value	Lag	Weighted LB	p-value	Lag	Weighted ARCH – LM	p-value
Diamond	ARIMA (2,1,1)-EGARCH (1,1)-std	1	0.0665	0.7966	1	0.0039	0.9500	3	0.0041	0.9493
		8	0.2464	1.0000	5	0.0120	1.0000	5	0.0097	0.9995
		14	0.8309	1.0000	9	0.0200	1.0000	7	0.0144	1.0000
United Bank for Africa	ARIMA (2,1,1)-EGARCH (1,1)-std	1	0.9151	0.3388	1	0.0152	0.9020	3	0.0230	0.8636
		8	2.1899	1.0000	5	0.0658	0.9992	5	0.0651	0.9928
		14	2.9300	0.9976	9	0.1140	1.0000	7	0.0909	0.9995
Union	ARIMA (1,1,0)-GARCH (1,1)-norm	1	1.147	0.2841	1	3.333	0.0679	3	0.3721	0.5419
		2	1.764	0.3053	5	3.948	0.2606	5	0.4175	0.9077
		5	5.729	0.0586	9	5.867	0.3138	7	2.5696	0.5982

LB = Ljung-Box, LM = Lagrange Multiplier

Moreover, it is obvious that the model specifications for the outlier adjusted series in Table 6 are quite different from those of the outlier contaminated series in Table 1. These differences in model specifications are said to be associated with biased parameters.

3. 5. Forecasts Evaluation

The carry-over effect of biased parameters caused by the presence of outliers can be seen to slightly impact on the forecasts of the return series of Diamond bank. From Table 8, it is found that, in the presence of outliers, the forecasts are biased negatively by 0.0069 as indicated by ME and the accuracy is reduced by 0.002243, 0.000139 and 0.002626 as suggested by MAE, MSE and RMSE, respectively.

Table 8. Carry-over Effect of Biasness on Forecasts for Diamond Bank

Evaluation Criteria	GARCH(2,0)-std Model fitted to Returns Series of Diamond Bank	EGARCH(1,1)-std Model fitted to Outlier Adjusted Return Series of Diamond Bank	Carry-over Effect of Biasness
ME	– 0.005555	0.001819	– 0.003374
MAE	0.022278	0.020035	0.002243

MSE	0.000772	0.000633	0.000139
RMSE	0.027785	0.025159	0.002626

The carry-over effect of biased parameters caused by the presence of outliers can be seen to slightly impact on the forecasts of the return series of United Bank for Africa. From Table 9, it is found that, in the presence of outliers, the forecasts are biased by 0.000282 as indicated by ME and the accuracy is reduced by 0.000061, 0.000001 and 0.000037 as specified by MAE, MSE and RMSE, respectively.

Table 9. Carry-over Effect of Biasness on Forecasts for United Bank for Africa

Evaluation Criteria	GARCH(1,1)-norm Model fitted to Returns Series of United Bank for Africa	EGARCH(1,1)-std Model fitted to Outlier Adjusted Return Series of United Bank for Africa	Carry-over Effect of Biasness
ME	0.002387	0.002105	0.000282
MAE	0.008662	0.008601	0.000061
MSE	0.000179	0.000178	0.000001
RMSE	0.013379	0.013342	0.000037

The carry-over effect of biased parameters caused by the presence of outliers can be seen to slightly impact on the forecasts of the return series of Union bank. From Table 10, it is found that, in the presence of outliers, the forecasts are biased by 0.000309 as indicated by ME and the accuracy is reduced by 0.000200, 0.000007 and 0.000118 as specified by MAE, MSE and RMSE, respectively.

Table 10. Carry-over Effect of Biasness on Forecasts for Union Bank

Evaluation Criteria	GARCH(2,0)-std Model fitted to Returns Series of Union Bank	GARCH(1,1)-norm Model fitted to Outlier Adjusted Return Series of Union Bank	Carry-over Effect of Biasness
ME	0.011119	0.010810	0.000309
MAE	0.017603	0.017403	0.000200
MSE	0.000875	0.000868	0.000007
RMSE	0.029580	0.029462	0.000118

Therefore, it would be fair to say that there is marginal improvement in the forecasts of fitted models of outlier adjusted series over those of outlier contaminated series irrespective of the choice of the distribution of the innovations and of course, due to the fact that in returns, all the outliers are treated as innovations and they occurred far behind the forecast origin. In addition, these findings are in tandem with the works of [5]; [6] and [7] that innovation outliers especially when occurring before the forecast origin have little or no effect on the forecasts but differ from that [8] by investigating the carry-over effect of biased parameters on the forecasts.

4. CONCLUSION

So far, this study has shown that the carry-over effect of biased parameters on forecasts is minimal with slight improvement in the forecasts of the models fitted to the outlier adjusted returns series given that all the outliers were considered as innovations and occurred before the forecast origin. By extension, further study could be carried out to include the carry-over effect of biased parameters when the outliers occur near the forecast origin.

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