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SHORT COMMUNICATION

Distribution of the lengths of tree paths of gas molecules

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ABSTRACT

It is shown that the formula available in the literature for the distribution of the mean free paths of gas molecules, independent of pressure and temperature, produces obviously incorrect results. Accordingly, a conclusion is given for another formula for a given distribution, which depends on the size of the gas molecules, as well as temperature and pressure. In accordance with this formula, the distribution curve for oxygen at 1000 K and pressures 1 and 0.001 atm has been calculated.

Keywords: distribution of molecules, mean free path, gas, pressure, temperature, radius of the molecule

1. INTRODUCTION

Recently, there has been increased interest in the theoretical study of the possible properties of "dark matter" [1], which can be particles with negative mass (negatons). Since such particles, like any others, can be considered as a molecular gas that obeys Maxwell's distributions over the velocities of molecules and their energies, then, accordingly, works have appeared on the development and analysis of the theory of such distributions [2-6].

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The purpose of this work was to obtain the distribution of the free path lengths of gas molecules depending on their size, temperature and gas pressure.

Known formula for the distribution of free paths of l molecules as a function of the most probable mean free path l_0

$$dn = \frac{4n}{\sqrt{\pi}} \cdot \frac{l^2}{l_0^3} e^{-\left(\frac{l}{l_0}\right)^2} dl \,, \tag{1}$$

where $n -$ is the total number of gas molecules. The graph of the function $\frac{dn}{ndl}$ following from (1) has, as it should be, a dome-shaped form, but this formula unfortunately did not prove to be quantitative.

To clarify this fact, we divide formula (1) by n and multiply by 100%, then the following inequality must hold:

$$100 \frac{dn}{ndl} \% = \frac{400}{\sqrt{\pi}} \cdot \frac{l^2}{l_0^3} e^{-\left(\frac{l}{l_0}\right)^2} \leq 100 \,\% \tag{2}$$

We choose the case when $l = l_0$. Substituting this value of l into inequality (2), we obtain:

$$\frac{400}{e\sqrt{\pi} l_0} \leq 100 \,\%, \tag{3}$$

from where $l = l_0$. It follows from this that for all the most probable mean free paths for which $l_0 < 0{,}8302$ the fraction of molecules satisfying the equation $l = l_0$, will be greater than 100%, which is impossible.

2. DERIVATION OF A NEW FORMULA

As known the gas pressure equal to the force acting per unit area in the direction of the X-axis is equal to

$$P = n\sqrt{\frac{\alpha}{\pi}} \int_0^\infty (2mv_x) e^{-\alpha v_x^2} \, v_x dv_x, \tag{4}$$

where

$$\alpha = \frac{m}{2kT}, \tag{5}$$

m and v_x – mass and velocity of molecules in the direction of the X axis, $k -$Boltzmann's constant, $T -$ temperature. After substituting (5) into (4), we have

$$P = 2n\sqrt{\frac{m}{2\pi kT}} \int\limits_0^\infty e^{-\frac{mv_x^2}{2kT}} (mv_x^2) dv_x =$$

$$= \frac{4n}{3}\left[\begin{array}{l} \left(\frac{m}{2\pi kT}\right)^{1,5} \int\limits_{0}^{\infty} \frac{mv_x^2}{2}\, e^{-\frac{mv_x^2}{2kT}} dv_x \iint\limits_{-\infty}^{\infty} e^{-\frac{m(v_y^2+v_z^2)}{2kT}} dv_y\, dv_z + \\ + \left(\frac{m}{2\pi kT}\right)^{1,5} \int\limits_{0}^{\infty} \frac{mv_y^2}{2}\, e^{-\frac{mv_y^2}{2kT}} dv_y \iint\limits_{-\infty}^{\infty} e^{-\frac{m(v_x^2+v_z^2)}{2kT}} dv_x\, dv_z + \\ + \left(\frac{m}{2\pi kT}\right)^{1,5} \int\limits_{0}^{\infty} \frac{mv_z^2}{2}\, e^{-\frac{mv_z^2}{2kT}} dv_y \iint\limits_{-\infty}^{\infty} e^{-\frac{m(v_x^2+v_y^2)}{2kT}} dv_x\, dv_y \end{array}\right] =$$

$$= \frac{2n}{3}\left(\frac{m}{2\pi kT}\right)^{1,5} \int\limits_{-\infty}^{\infty} \frac{mv^2}{2}\, e^{-\frac{mv^2}{2kT}} dv_x dv_y dv_z = \frac{2N}{3V}\left(\overline{\frac{mv^2}{2}}\right) = \frac{2N\overline{\varepsilon}}{3V} = \frac{2}{3}\cdot\frac{E}{V}\,, \qquad (6)$$

where $\overline{\varepsilon} -$ the average kinetic energy of the translational motion of gas molecules, $N = 6{,}02 \cdot 10^{23} -$ is Avogadro's number. Equation (6) shows that the ideal gas pressure is numerically equal to 2/3 of the kinetic energy of the translational motion of gas molecules referred to a volume containing N gas molecules.

This equality can be written in the form:

$$P = \frac{2}{3}\cdot\frac{E/N}{V/N} = \frac{2}{3}\cdot\frac{\varepsilon}{w}\,, \qquad (7)$$

where $\varepsilon -$ the kinetic energy of the translational motion of the gas molecule:

$$\varepsilon = \frac{mv^2}{2}\,; \qquad (8)$$

$w -$ the volume of the gas cell in which the gas molecule of radius r is located [8, 9]:

$$w = \pi r^2 l\ . \qquad (9)$$

Substituting the quantities ε and w from (8) and (9) in (7), we obtain:

$$\pi r^2 P l = \frac{mv^2}{3}\,, \qquad (10)$$

whence we have:

$$v^2 = \frac{3\pi r^2 P l}{m} \qquad (11)$$

$$dv = \sqrt{\frac{3\pi r^2 P}{2ml}}\, dl\ . \qquad (12)$$

Let us write down the velocity distribution of the Maxwell molecules:

$$dn_v = 4\pi n \left(\frac{m}{2\pi kT}\right)^{1,5} e^{-\frac{mv^2}{2kT}} v^2 dv\,. \quad (13)$$

After substituting (11) and (12) into the distribution (13), we obtain the required distribution of gas molecules along their free paths:

$$dn_l = \pi n r^3 \left(\frac{3P}{kT}\right)^{1,5} e^{-\frac{3\pi r^2 Pl}{2kT}} \sqrt{l} dl\ . \quad (14)$$

For convenience of perception and decision-making, let us write down the expression obtained for estimating the proportion of the corresponding free paths in percentages:

$$100\frac{dn_l}{n} = 100\pi r^3 \left(\frac{3P}{kT}\right)^{1,5} e^{-\frac{3\pi r^2 Pl}{2kT}} \sqrt{l} dl\ \%. \quad (15)$$

From the expressions (14, 15) obtained it is clear that the form of the distribution depends, first of all, on the temperature and pressure of the gas, and also on the dimensions of its molecules.

Table 1. The quantitative content of the free paths of oxygen molecules at 1000 K.

$P = 1\ atm$		$P = 0{,}001\ atm$	
$l\ cm$	$100\frac{\Delta n_l}{n}\%$	$l\ cm$	$100\frac{\Delta n_l}{n}\%$
10^{-2}	10^{-17}	0,001	0,1
10^{-3}	21	0,01	10
10^{-4}	61	0,1	68
10^{-5}	3	1	24
10^{-6}	0,1	2	0,5
10^{-7}	0,003	3	0,005

Table 1 shows the calculated relative quantitative values of the mean free path in the distribution (15) at a temperature of 1000 K and various pressures for oxygen (the radius of its molecule is taken equal to $1{,}21 \cdot 10^{-8}\ cm$). As an amount dl taken $\Delta l = \pm l \sim 2l$, and the value dn_l for this choice is Δn_l. As can be seen from the data in Table 1, the results obtained correspond to a distribution whose curve has a dome-like appearance, and the values themselves correspond in order of magnitude to the values determined experimentally.

3. CONCLUSIONS

The length of the free path of gas molecules depends on the temperature and pressure of the gas, and also depends on the size of its molecules. The distribution curve itself is dome-shaped and is described by the formula:

$$100\frac{dn_l}{n} = 100\pi r^3 \left(\frac{3P}{kT}\right)^{1,5} e^{-\frac{3\pi r^2 Pl}{2kT}}\sqrt{l}dl\ \%.$$

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