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Double Diffusive MHD Flow of a Chemically Reacting Alumina Nanofluid Past a Semi-infinite Flat Plate

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ABSTRACT

An analytical Study of double diffusive MHD flow of a chemically reacting alumina nanofluid past an infinite flat plate is made. Laplace transform method is employed to determine the solution of the governing equation and its analysis showed that increase in both Reynolds number and Prandtl number bring about an increase in the rate of heat transfer coefficient. Skin friction is shown to reduce as the Reynolds number increases. Increase in Reynolds number and Schmidt number, increases the mass transfer coefficient of alumina nanofluid. Generally, comparison is made with other work and an appreciable trend is observed.

***Keywords*:** Alumina nanofluid, Flate Plate, Double diffusivity, Laplace transform method, Magnetohydrodynamics, Boussinesq's approximation, Buckinham - $\pi -$ theorem

1. INTRODUCTION

The concept of the influence of Soret and Dufour terms or double diffusion has many applications in geophysics, isotope separation and metallurgy [1]. Double diffusive flow is driven by buoyancy due to temperature and concentration gradients. In many technological and engineering processes, the application of the physics of nuclear science, chemical processes and petroleum reserviours also play a major role in heat and mass transfer systems. Although, the thermal-diffusion and the diffusion-thermo effects on natural convection is minimal on

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velocity, temperature and concentration fields [2]. Several works [3-6] and progress have been recorded on double diffusive MHD flows due to its applications in MHD devices. As a result of the high temperature involved, thermal radiation plays a major role in improved energy conversion systems. Owing to successes recorded in technological development and improved thermal conductivity of certain fluids, study of nanofluids and nanotechnology were vigorously pursued [7 and 8], nanofluids with dufour and soret effects were not exception. A review of several methods of preparation and stability of nanofluids were also highlighted by Muknerjee and Paria [9]. Aaiza et al [10], investigated nanofluid containing different shapes of nanoparticles and deduced that thermal conductivity plays a major role in nanofluids and base fluids and that the quantity of nanofluid not only depends on the type nanoparticles but also their shapes. Timofeera et al [11], examined a theoretical modeling together with experimental study to determine various shapes of alumina (Al_2O_3) nanoparticles in a base fluid mixture of ethylglycol and water of equal volumes. Few studies on mixed convection and nanofluids are also found in [12-17]. Xuan and Li [18], studied heat transfer enhancement of nanofluids and its stability. A theoretical model is also proposed to examine heat transfer effectiveness of the nanofluid flowing through a tube. Wasp [19], in his study, proposed an alternative expression to Hamilton and Crosser for calculating the effective thermal conductivity of solid-liquid mixtures. Singh et al. [20], used coupled approach in Fluent 63.26 to study non-Newtonian nanofluids flowing upward and verified that as Richardson's number increases at specific Reynolds number, the average Nusselt number increases. The study of Singh et al [20] was compared with the Newtonian fluid study of Srinivas et al [21], Badr [22] and wong el al [23] and reasonable agreement was observed. The aim of this study is to examine the (Al_2O_3) nanoparticle in water base fluid on double diffusive MHD flow of a chemically reacting fluid past a semi-infinite plate with a view to determining the skin friction, Nusselt number and Sherwood number on the fluid flow.

2. FORMALISM

We consider an unsteady two dimensional boundary layer flow of viscous, oscillatory, incompressible, radiating nanofluid along a semi-infinite flat vertical plate in the presence of thermal and concentration buoyancy effects. The $x'-$axis is taken along the vertical infinite plate in the upward direction and the $y'-$axis normal to the plate. The plate in the $x'-$direction is considered infinite, all the physical variables are independent of the coordinate (Aruna et al [24]), therefore, $\frac{\partial u'}{\partial x'} = 0$. The effect of soret and dufour are taken into account.

Using the Boussinesq's approximation, the governing equations of the nanofluid flow are given as

$$\frac{\partial u'}{\partial y'} = 0 \tag{1}$$

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = \frac{1}{\rho_{nf}}\frac{\partial p}{\partial x'} + \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2 u'}{\rho_{nf}} + g\beta_{nf}\left(T - T_\infty\right) + g\beta_{nf}^*\left(C - C_\infty\right) \tag{2}$$

$$\frac{\partial T}{\partial t'}+v'\frac{\partial T}{\partial y'}=\frac{k_{nf}}{(\rho C_p)_{nf}}\frac{\partial^2 T}{\partial y'^2}-\frac{1}{(\rho C_p)_{nf}}\frac{\partial q_r}{\partial y'}+\frac{D_m K_T \partial^2 C'}{C_s (C_p)_{nf}\partial y'^2} \tag{3}$$

$$\frac{\partial C'}{\partial t'}+v'\frac{\partial C'}{\partial y'}=\frac{D}{(\rho C_p)_{nf}}\frac{\partial^2 C'}{\partial y'^2}-k_r^2(C'-C_\infty)+\frac{D_m K_T \partial^2 T}{T_m \partial y'^2} \tag{4}$$

Subject to the boundary conditions

$$u'=u_0 \quad T=T_b+\varphi(T_b-T_\infty)e^{n't'} \quad C'=C_b+\varphi(C_b-C_\infty) \text{ at } y=0 \tag{5}$$

$$u'\to 0,\ T\to T_\infty,\ C'\to C_\infty \text{ as } y'\to\infty$$

where u' and v' are velocities in x' and y' directions respectively, t' is time, C is nanofluid concentration, p is pressure, ρ_{nf} is density of nanofluid, μ_{nf} is viscosity of nanofluid, σ is electrical conductivity of base fluid, B is imposed magnetic induction, g is acceleration due to gravity, β_{nf} is thermal expansion due to temperature, $\beta^*{}_{nf}$ is thermal expansion due to concentration, T is temperature of nanofluid, T_∞ is free stream temperature, C_∞ is free stream concentration, k_{nf} is thermal conductivity of nanofluid, $(C_p)_{nf}$ is specific heat at constant pressure, q_r is radiation term, K_T is thermal diffusion ratio, C_s is concentration susceptivity, T_m is mean nanofluid temperature k_r^2 is chemical reaction term, D is chemical molecular diffusivity, D_m is mass diffusivity

In this work, according to Hamilton and Crosser model [12], dynamic viscosity and thermal conductivity which is valid for both spherical and non spherical shapes nanoparticles is defined as

$$\mu_{nf}=\mu(1+a\phi+b\phi^2) \tag{6}$$

$$\frac{k_{nf}}{k_f}=\frac{k_s+(n-1)k_f+(n-1f)(k_s-k_f)\phi}{k_s+(n-1)k_f-(k_s-k_f)\phi} \tag{7}$$

According to the work of Tiwari and Das [25] and Asma et al. [26], density of nanofluid (ρ_{nf}), thermal expansion due to temperature of nanofluid (β_{nf}), thermal expansion due to concentration of nanofluid ($\beta^*{}_{nf}$), specific heat at constant pressure of nanofluid $(C_p)_{nf}$ are respectively

$$\rho_{nf}=(1-\phi)\rho_f+\phi\rho_s$$

$$\beta_{nf}=(1-\phi)\beta_f+\phi\beta_s \tag{8}$$

$$\beta^*_{nf} = (1-\phi)\beta^*_f + \phi\beta^*_s$$

$$(C_p)_{nf} = (1-\phi)(C_p)_f + \phi(C_p)_s$$

where ϕ is the nanoparticles volume fractions, ρ_f and ρ_s are the densities of the base fluid and solid nanoparticles, β_f and β_s are the thermal expansion due to temperature of base fluid and solid nanoparticles, β^*_f and β^*_s are the thermal expansion due to concentration of base fluid and solid nanoparticles and $(C_p)_f$ and $(C_p)_s$ are the specific heat at constant pressure due to base fluid and solid nanoparticles. a and b are constants that depend on the particle shape (Aaiza et al [10]). The thermo physical properties of alumina (Al_2O_3) nanoparticles and water (H_2O) as base fluid are presented in Table 1.

Table 1. Thermo-physical properties of water and alumina

Property	Base Fluid (water)	Alumina
Specific heat (J/kgK)	4179	765
Density (kg/m^3)	997.1	3970
Thermal conductivity	0.6	40

Taking the integral of equation (1) and the suction velocity which is normal to the plate can be written as

$$u' = -v_0\left(1 + \varepsilon A e^{n't'}\right) \tag{9}$$

where v_0 is characteristic plate velocity, ε and A are constants.

In order to consider the effect of radiation on an optically thick model in which the thermal layer becomes very thick or highly absorbing as described by Rosseland approximation, Cogley et al [27] as

$$\frac{\partial q_r}{\partial y'} = -\frac{4\zeta}{3\alpha}\frac{\partial^2 T^4}{\partial y'^2} \tag{10}$$

where ζ is the Stefan-Boltzmann constant and α is the absorption coefficient. If temperature difference within the flow of the nanofluid is sufficiently small, we can approximate T^4 using Taylor series expansion about 0 and obtain

$$T^4 = 4T_\infty^3 T - 3T_\infty^4 \tag{11}$$

3. DIMENSIONAL ANALYSIS

To effectively tackle the governing fluid flow equations, dimensional homogeneity of the governing equations using the Buckinham - π − theorem is stated

$$u = \frac{u'}{x't'}, y = \frac{y'}{x'}, t = \frac{t'}{u'y'}, \mathrm{Re} = \frac{\mu_{nf} C'}{v_0 \rho_{nf}}, \theta = \frac{T - T_\infty}{T_b - T_\infty}, C = \frac{C' - C_\infty}{C_b - C_\infty}$$

$$\Pr = \frac{v'(\rho C_p)_{nf}}{k_{nf}}, \frac{\partial p}{\partial x'} = \rho_{nf} e^{i\omega t}, Sc = \frac{\mu_{nf}}{D}, Du = \frac{D_m K_T (C' - C_\infty)}{\mu_{nf} v'^2 (C_p)_{nf} (C_s)_{nf} (T - T_\infty)}$$

$$Sr = \frac{D_m K_T (T - T_\infty)}{\mu_{nf} v' T_m (C' - C_\infty)}, k_0 = \frac{k_r^2 \mu_{nf}}{v'^2}, Ha = \frac{\sigma B_0^2 \mu_{nf}}{\rho_{nf} v'^2}, N = \frac{16\zeta T_\infty^3}{3\alpha (C_p)_{nf}}$$

$$Gt = \frac{g\beta_{nf}(T - T_\infty)\mu_{nf}}{v_0 u'^2}, Gc = \frac{g\beta_{nf}^*(C' - C_\infty)\mu_{nf}}{v_0 u'^2}$$

Using the dimensionless variables and equations (9) - (11), equations (2) – (4) can be written as

$$\frac{\partial u}{\partial t} - (1 + \varepsilon A e^{nt})\frac{\partial u}{\partial y} = -\rho_{nf} e^{i\omega t} + \mathrm{Re}_{nf}^{-1}\frac{\partial^2 u}{\partial y^2} - Hau + Gt\theta + GcC \tag{12}$$

$$\frac{\partial \theta}{\partial t} - (1 + \varepsilon A e^{nt})\frac{\partial \theta}{\partial y} = \left(\frac{1 + N}{\Pr}\right)\frac{\partial^2 \theta}{\partial y^2} + Du\frac{\partial^2 C}{\partial y^2} \tag{13}$$

$$\frac{\partial C}{\partial t} - (1 + \varepsilon A e^{nt})\frac{\partial C}{\partial y} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} - k_0 C + Sr\frac{\partial^2 \theta}{\partial y^2} \tag{14}$$

The boundary conditions also transform to

$$u = 1, \ \theta = 1 + \varepsilon e^{nt}, \ C = 1 + \varepsilon e^{nt} \ \text{at} \ y = 0$$

$$u^1 = 0, \ \theta^1 = 0, \ C^1 = 0 \ \text{at} \ y = 0$$

4. METHOD OF SOLUTION

The Laplace transform of equations (13) and (14) is taken, after imposing the boundary conditions and simplification, the results are

$$\bar{\theta}(s)\left[s-\left(1+\varepsilon Ae^{nt}\right)s-\left(\frac{1+N}{\Pr}\right)s^2\right]-s^2\bar{C}(s)=(1+\varepsilon)-\left(1+\varepsilon Ae^{nt}\right)\left(1+\varepsilon e^{nt}\right)$$
$$-\left(\frac{1+N}{\Pr}\right)s\left(1+\varepsilon e^{nt}\right)-Dus\left(1+\varepsilon e^{nt}\right) \tag{15}$$

$$\bar{C}(s)\left[s-\left(1+\varepsilon Ae^{nt}\right)s-s^2Sc^{-1}+k_0\right]-Srs^2\bar{\theta}(s)=(1+\varepsilon)-\left(1+\varepsilon Ae^{nt}\right)\left(1+\varepsilon e^{nt}\right)$$
$$-s\left(1+\varepsilon e^{nt}\right)-Sr\left(1+\varepsilon e^{nt}\right) \tag{16}$$

Equations, (15) and (16) can be written in matrix form as

$$\begin{pmatrix}\beta_1 & -Srs^2\\ -Dus^2 & \beta_2\end{pmatrix}\begin{pmatrix}\bar{C}(s)\\ \bar{\theta}(s)\end{pmatrix}=\begin{pmatrix}b_1\\ b_2\end{pmatrix} \tag{17}$$

where

$$b_1=(1+\varepsilon)-\left(1+\varepsilon Ae^{nt}\right)\left(1+\varepsilon e^{nt}\right)-s\left(1+\varepsilon e^{nt}\right)-Sr\left(1+\varepsilon e^{nt}\right)$$

$$\beta_1=\left[s-\left(1+\varepsilon Ae^{nt}\right)s-s^2Sc^{-1}+k_0\right]$$

$$\beta_2=\left[s-\left(1+\varepsilon Ae^{nt}\right)s-\left(\frac{1+N}{\Pr}\right)s^2\right]$$

$$b_2=(1+\varepsilon)-\left(1+\varepsilon Ae^{nt}\right)\left(1+\varepsilon e^{nt}\right)-\left(\frac{1+N}{\Pr}\right)s\left(1+\varepsilon e^{nt}\right)-Dus\left(1+\varepsilon e^{nt}\right)$$

Using the method of determinants (Gupta [28]), the result of equation (17) is given as

$$\bar{C}(s)=\frac{b_1\beta_2+b_2Srs^2}{\beta_1\beta_2-Dus^2Srs^2} \tag{18}$$

$$\bar{\theta}(s)=\frac{b_2\beta_1+b_1Dus^2}{\beta_1\beta_2-Dus^2Srs^2} \tag{19}$$

Substituting for the material parameters and constants

$\varepsilon=0.01, A=1.00, n=1.00, t=1.00, Sc=1.50, Sr=1.00, k_0=1.25,$
$N=1.00, \Pr=0.70, Du=1.30, \mathrm{Re}=10.00, Ha=0.30, \omega=1.00, Gr=1.00$
$Gc=1.00$

into equations (18) and (19) respectively, the equations transform approximately into

$$\bar{C}(s) = \frac{0.2s + 29s^2 + 13s^3}{-4s - 35s^2 + s^3 + 38s^4} \tag{20}$$

$$\bar{\theta}(s) = \frac{-42s - 26s^2 + 15s^3 - 0.5}{-4s - 35s^2 + s^3 + 38s^4} \tag{21}$$

Taking the inverse Laplace transform of equations (20) $L^{-1}(\bar{C})$ and (21) $L^{-1}(\bar{\theta})$ respectively, the solutions take the form

$$C(y,t) = 0.5e^{y} - 4e^{-1.026y}\left(Cos0.398\,y - 1.026\,Siny\right) + 2e^{-1.026y}Sin0.398\,y \tag{22}$$

$$\theta(y,t) = 0.4e^{y} - 1.4e^{-1.026y}\left(Cos0.398\,y - 1.026\,Siny\right) - 111e^{-1.026y}Sin0.398\,y$$

$$+ 3.16e^{-1.026y}\left(1 - Cos0.398\,y\right) \tag{23}$$

Similarly, the Laplace transform of equation (12) is taken and its inverse with the help of equations (22) and (23), the solution is given as

$$u(y,t) = 1.26e^{13.5y}Sin10.75\,y - 0.9e^{13.5y}Sin10.75\,y - 3970 +$$

$$0.4e^{y} - 1.4e^{-1.026y}\left(Cos0.398\,y - 1.026\,Siny\right) - 111e^{-1.026y}Sin0.398\,y$$

$$+ 3.16e^{-1.026y}\left(1 - Cos0.398\,y\right) + 0.5e^{y} - 4e^{-1.026y}\left(Cos0.398\,y - 1.026\,Siny\right) + 2e^{-1.026y}Sin0.398\,y$$

………… (24)

The mean Nusselt number (Nu) from equation (23) is given as

$$Nu = \left(\frac{\partial \theta}{\partial y}\right)_{y=0} = 40.9042 \tag{25}$$

Theoretically, the mean Nusselt number for flow along flat plates according to Bird et al [29] is given by

$$Nu = 2\sqrt{\frac{37}{1260}}\,\mathrm{Re}^{\frac{1}{2}}\,\mathrm{Pr}^{\frac{1}{3}} \tag{26}$$

The skin friction (τ) from equation (24), result in

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0} = 3.87 \tag{27}$$

The skin friction (τ) for flow along flat plates as reported by Bird et al [29] is

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0} = 0.664\,\mathrm{Re}^{-\frac{1}{2}} \tag{28}$$

The Sherwood number (sh) from equation (22), is given as

$$Sh = \left(\frac{\partial C}{\partial y}\right)_{y=0} = 0.3933 \tag{29}$$

The Sherwood number (Sh) for flow along flat plates as reported by Bird et al [29] is

$$Sh = \left(\frac{\partial C}{\partial y}\right)_{y=0} = 1.128(\mathrm{Re}\,Sc)^{0.5} \tag{30}$$

5. RESULTS AND DISCUSSION

Table 2. Comparison of the mean Nusselt number (Nu) for Flat Plates as Re increases

Re	Nu (Present)	Nu (Bird et al [29])
10	40.904200	0.962281
20	57.84725	1.360871
30	70.86849	1.666720
40	81.83185	1.924562
50	91.49079	2.151726

Table 3. Comparison of the Skin Friction (τ) for Flat Plates as Re increases

Re	τ (present)	τ (Bird et al [29])
10	3.870000	0.209975

20	2.736508	0.148475
30	2.234342	0.121229
40	1.935007	0.104988
50	1.730717	0.093904

Table 4. Comparison of the Sherwood number (Sh) for Flat Plates as Re increases

Re	Sh (present)	Sh (Bird et al [29])
10	0.393300	4.368725
20	0.556210	6.178309
30	0.681216	7.566853
40	0.786600	8.737449
50	0.879446	9.768765

Table 5. Comparison of the Nusselt number (Nu) for Flat Plates as Pr increases

Pr	Nu (Present)	Nu (Bird et al [29])
0.70	40.904200	0.962302
1.70	54.981668	1.293485
2.70	64.148927	1.509152
3.70	71.252843	1.676277
4.70	77.167470	1.815423

Table 6. Comparison of the Sherwood number (Sh) for Flat Plates as Sc increases.

Sc	Sh (Present)	Sh (Bird et al [29])
1.50	0.393300	4.368725
2.50	0.507748	5.639999
3.50	0.6007756	6.673338
4.50	0.681215	7.566854
5.50	0.753112	8.365471

From Table 2, the rate of heat transfer coefficient of the alumina nanofluid, increases as the Reynolds number increases. The same result is observed with increase in Prandtl number as depicted in Table 5. Comparison of this work with Bird et al [29], showed that the Nusselt number in this work is higher. The reason is the high thermal conductivity of alumina nanofluid than the conventional fluids. it is also shown on the Tables 2 and 5 that the trend of increase is similar. Table 3 showed that the skin friction decreases as a result of increase in Reynolds number. This is expected because as the Reynolds number increases, the velocity profile of the nanofluid increases thereby reducing the skin friction of the fluid. Comparison with work of Bird et al [29] showed similar trend. As depicted on Table 4, increase n Reynolds number, led to a corresponding increase in the Sherwood number of the nanofluid. As the concentration of the fluid increases, the rate of mass transfer is also enhanced. When compared with the work of Bird et al [29], it is observed that the rate of mass transfer is higher in conventional fluid than alumina nanofluid. However, the trend of increase is in agreement. Table 6 also showed that increase in Schmidt number brings about an increase in the Sherwood number. The Schmidt number is the ratio of the momentum diffusivity to the mass diffusivity. It relates the relative thickness of the hydrodynamic boundary layer and the mass transfer boundary layer. In the absence Dufour, Soret and chemical reaction terms, the results are almost in agreement with the work of Aaiza et al [10] and Timofeeva et al [11].

6. CONCLUSION

The traditional display and discussion of the effect of magnetic field and other parameters in our governing equations is ignored. The reason is that very many works cited, have discussed them. Generally, it is observed that alumina nanofluid and indeed nanofluids, owing to their high thermal conductivity and high viscosity compared to conventional base fluids makes it behave differently hence its suitability for scientific and engineering applications. In the absence of the added material parameters and for conventional fluid, the results are in reasonable agreement with that of Bird et al [29].

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