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A Two-step Hybrid Block Method for the Numerical Integration of Higher Order Initial Value Problems of Ordinary Differential Equations

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ABSTRACT

In this paper, a two-step implicit hybrid block multistep method is proposed for the approximate solution of higher order ordinary differential equations with a specification of fourth order. The study provides the use of both collocation and interpolation techniques to obtain the schemes. Direct form of power series is used as basis function for approximation solution. An order eight symmetric and zero-stable method is obtained. To implement our method, predictors of the same order of accuracy as the main method were developed using Taylor's series algorithm. This implementation strategy is found to be efficient and more accurate as the result has shown in the numerical experiments. The result obtained confirmed the superiority of our method over existing methods

***Keywords*:** Hybrid, Block method, Fourth order, Collocation, Higher Order, power series, approximate solutions, zero stability, symmetry

AMS Subject Classification: 65L02, 65L06, 65D30

1. INTRODUCTION

In this paper, we considered the method of approximate solution of the general fourth order initial value problem of the form:

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$$y^{(iv)} = f\left(x, y, y', y'', y'''\right), y^{(i)} = y_i, i = 0,1,2,3 \tag{1}$$

where: x_n , is the initial point, y_n is the solution at x_n,. f is continuous within the interval of integration.

Equation (1) is of interest to researchers because of its wide application in engineering, control theory and other real life problem, hence the study of the methods of its solution.

For instance, [1] stated that when a sinusoidal wave of frequency Ω passes along a ship or offshore structure, the resultant fluid actions vary with time t. A particular case study in [1], is the fourth order problem defined by

$$y^4 + 3y'' + y(2 + \varepsilon\cos(\Omega t) = 0, \quad t \succ 0 \tag{2}$$

This is subjected to the following initial conditions

$$y(0) = 1 \quad y'(0) = 0 \qquad y''(0) = 0 \qquad y'''(0) = 0 \quad \mathrm{h} = 0.1$$

where: $\varepsilon = 0$ for the existence of the theoretical solution, $y(t) = 2\cos t - \cos(t\sqrt{2})$.

The conventional method of solving (1) is to reduce it to a system of first order differential equation [2, 3]. The reduction of (1) to a system of first order equations leads to serious computational burden as well as wastage of computer and human efforts. However, these setbacks have been addressed by some researchers [4-7]. It has been reported in literature that the direct method of solving the above equation is more efficient in terms of speed and accuracy than the method of reduction to a system of first order ODES [3, 8]

Implicit linear multistep methods have better stability properties than explicit methods and are solved using predictor and corrector method. However, several authors have proposed multi-derivative multistep methods for the solution of (1).These methods were implemented in predictor-corrector mode [4, 5, 8] .Although these methods yield good results, but it has a major setback which includes computational burden and the reducing the order of accuracy of the predictors. To correct these set -backs scholars developed block methods [9-16]. Block method is found to be cost effective in terms of execution and saves time.

In Literature, some numerical methods for solving fourth order ODEs have been extended to solve the problem from ship dynamics (2). Instead of solving the fourth order ODEs directly, they considered the conventional approach of reduction to system of first order [17, 18]

In this paper, we proposed a two-step hybrid block method for the numerical integration of fourth order initial value problems with constant step –size which is implemented in block mode.

2. METHOD AND MATERIALS

We consider the simple power series as a basis function for approximation:

$$Y\left(x\right) = \sum_{j=0}^{r+s-1} a_j \phi_j\left(x\right),$$

where: $\phi_j\left(x\right)=x^j$ (2)

And $x\in\left[a,b\right]$, $a_j\text{'s}$ are coefficients to be determined and $\phi_j\left(x\right)$ is a polynomial of degree $r+s-1$. We construct a $k-$step multistep collocation method (MCM) by imposing the following conditions on (2)

$$Y\left(x_{n+j}\right)=y_{n+j},\ j=0,1,2,...,r-1 \tag{3}$$

$$DY\left(x_{n+j}\right)=f_{n+j},\ j=0,1,2,...,s-1 \tag{4}$$

Substituting (1) into (4) gives

$$f\left(x,y,y',y'',y'''\right)=\sum j(j-1)(j-2)(j-3)a_j x^{j-4} \tag{5}$$

We shall consider the grid point of step length two with constant step-size (h) , where $h=x_{n+i}-x_i, i=0,1,2$ and off–grid points at $x_{n+\frac{1}{2}}, x_{n+\frac{3}{2}}$.

Interpolating (2) at $x=x_n, x_{n+\frac{1}{2}}, x_{n+1}, x_{n+\frac{3}{2}}$ and collocating (5) at $x_n, x_{n+\frac{1}{2}}, x_{n+1}, x_{n+\frac{3}{2}}, x_{n+2}$ to give a system of non-linear equation of the form

$$AX=B \tag{6}$$

where: $A=\left[a_0,a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8\right]^T$, $B=\left[y_n, y_{n+\frac{1}{2}}, y_{n+1}, y_{n+\frac{3}{2}}, f_n, f_{n+\frac{1}{2}}, f_{n+1,} f_{n+\frac{3}{2}}, f_{n+2}\right]^T$

And

$$X=\begin{bmatrix} 1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 & x_n^6 & x_n^7 & x_n^8 \\ 1 & x_{n+\frac{1}{2}} & x_{n+\frac{1}{2}}^2 & x_{n+\frac{1}{2}}^3 & x_{n+\frac{1}{2}}^4 & x_{n+\frac{1}{2}}^5 & x_{n+\frac{1}{2}}^6 & x_{n+\frac{1}{2}}^7 & x_{n+\frac{1}{2}}^8 \\ 1 & x_{n+1} & x_{n+1}^2 & x_{n+1}^3 & x_{n+1}^4 & x_{n+1}^5 & x_{n+1}^6 & x_{n+1}^7 & x_{n+1}^8 \\ 1 & x_{n+\frac{3}{2}} & x_{n+\frac{3}{2}}^2 & x_{n+\frac{3}{2}}^3 & x_{n+\frac{3}{2}}^4 & x_{n+\frac{3}{2}}^5 & x_{n+\frac{3}{2}}^6 & x_{n+\frac{3}{2}}^7 & x_{n+\frac{3}{2}}^8 \\ 0 & 0 & 0 & 0 & 24 & 120x_n & 360x_n^2 & 840x_n^3 & 1680x_n^4 \\ 0 & 0 & 0 & 0 & 24 & 120x_{n+\frac{1}{2}} & 360x_{n+\frac{1}{2}}^2 & 840x_{n+\frac{1}{2}}^3 & 1680x_{n+\frac{1}{2}}^4 \\ 0 & 0 & 0 & 0 & 24 & 120x_{n+1} & 360x_{n+1}^2 & 840x_{n+1}^3 & 1680x_{n+1}^4 \\ 0 & 0 & 0 & 0 & 24 & 120x_{n+\frac{3}{2}} & 360x_{n+\frac{3}{2}}^2 & 840x_{n+\frac{3}{2}}^3 & 1680x_{n+\frac{3}{2}}^4 \\ 0 & 0 & 0 & 0 & 24 & 120x_{n+2} & 360x_{n+2}^2 & 840x_{n+2}^3 & 1680x_{n+2}^4 \end{bmatrix}$$

Solving (6) for the a_j's and substituting back into (2) and after much algebraic simplification, we obtained the hybrid multistep method of the form

$$y(x)=\alpha_0 y_n+\alpha_{1/2} y_{n+1/2}+\alpha_1 y_{n+1}+\alpha_{3/2} y_{n+3/2}+h^4\left[\sum_{j=0}^{2}\beta_j f_{n+j}+\beta_{1/2} f_{n+1/2}+\beta_{3/2} f_{n+3/2}\right] \qquad (7)$$

Equation (7) is of the form:

$$y(x)=\sum_{j=0}^{k-1}\alpha_j(x)y_{n+j}+h^4\left(\sum_{j=0}^{k}\beta_j(x)f_{n+j}+\beta_v(x)f_{n+v}\right) \qquad (8)$$

where: α_j's and β_j's are continuous coefficients expressed as functions of t, where

$$t=\frac{x-x_{n+1}}{h},\frac{dt}{dx}=\frac{1}{h} \qquad (9)$$

The coefficients of y_{n+j} and f_{n+j} are obtained as:

$$\alpha_0(t)=\frac{-1}{3}(-t+4t^3),\alpha_{1/2}(t)=2(-t+t^2+2t^3),$$

$$\alpha_1(t)=-(-1-t+4t^2+4t^3),\alpha_{3/2}(t)=\frac{2}{3}(t+3t^2+2t^3)$$

$$\beta_0(t)=\frac{h^4}{483840}\left[55t+11t^2-364t^3+672t^5-224t^6-384t^7+192t^8\right]$$

$$\beta_{\frac{1}{2}}(t)=\frac{-h^4}{120960}\left[493t+53t^2-2296t^3+1344t^5-896t^6-192t^7+192t^8\right] \qquad (10)$$

$$\beta_1(t)=\frac{h^4}{80640}\left[-553t-773t^2+2212t^3+3360t^4-1120t^6+192t^8\right]$$

$$\beta_{\frac{3}{2}}(t)=\frac{-h^4}{120960}\left[-59t+53t^2+560t^3-1344t^5-896t^6+192t^7+192t^8\right]$$

$$\beta_2(t)=\frac{h^4}{483840}\left[-41t+11t^2+308t^3-672t^5-224t^6+384t^7+192t^8\right]$$

Evaluating (10) at $t=1$ gives the main method below

$$y_{n+2}+y_n-4y_{n+1/2}+6y_{n+1}-4y_{n+3/2}=\frac{h^4}{11520}[-f_n+124f_{n+1/2}+474f_{n+1}+124f_{n+3/2}-f_{n+2}] \qquad (11)$$

To obtain additional methods, we use (7) and formulae for the derivatives which are expressed as follows:

$$y'(x)=\frac{1}{h}\left[\sum_{j=0}^{k-1}\alpha'_j(x)y_{n+j}+h^4\left(\sum_{j=0}^{k}\beta'_j(x)f_{n+j}+\beta'_v(x)f_{n+v}\right)\right] \tag{12}$$

$$y''(x)=\frac{1}{h^2}\left[\sum_{j=0}^{k-1}\alpha''_j(x)y_{n+j}+h^4\left(\sum_{j=0}^{k}\beta''_j(x)f_{n+j}+\beta''_v(x)f_{n+v}\right)\right] \tag{13}$$

$$y'''(x)=\frac{1}{h^3}\left[\sum_{j=0}^{k-1}\alpha'''_j(x)y_{n+j}+h^4\left(\sum_{j=0}^{k}\beta'''_j(x)f_{n+j}+\beta'''_v(x)f_{n+v}\right)\right] \tag{14}$$

2. 1. Formation of the block for two step hybrid method

Combining equations (12), (13) and (14), yields the block of the form

$$\mathbf{A}^{(0)}\mathbf{Y}_m^{(i)}=\sum_{i=0}^{3}\frac{(jh)}{i}e_i y_n^{(i)}+h^{(4-i)}\left[\ \mathbf{d}_i f\left(y_n\right)+\mathbf{b}_i\mathbf{F}(\mathbf{Y}_m)\ \right], \tag{15}$$

where:

$$\mathbf{Y}_m^{(i)}=\left[\ y_{n+1/2}^{(i)}\quad y_{n+1}^{(i)}\quad y_{n+3/2}^{(i)}\quad y_{n+2}^{(i)}\ \right]^T,$$

$$\mathbf{F}(\mathbf{Y}_m)=\left[\ f_{n+1/2}\quad f_{n+1}\quad f_{n+3/2}\quad f_{n+2}\ \right]^T,$$

$$\mathbf{Y}_n^{(i)}=\left[\ y_{n-1/2}^{(i)}\quad y_{n-1}^{(i)}\quad y_{n-3/2}^{(i)}\quad y_{n-2}^{(i)}\ \right]^T,$$

and $A^{(0)} = 4\times4$ identity matrix, $i=0(1)3$

when: $i=0$;

$$e_0=\begin{bmatrix}0&0&0&1\\0&0&0&1\\0&0&0&1\\0&0&0&1\end{bmatrix}, e_1=\begin{bmatrix}0&0&0&1/2\\0&0&0&1\\0&0&0&3/2\\0&0&0&2\end{bmatrix}, e_2=\begin{bmatrix}0&0&0&1/8\\0&0&0&1/2\\0&0&0&9/8\\0&0&0&2\end{bmatrix}, e_3=\begin{bmatrix}0&0&0&1/48\\0&0&0&1/6\\0&0&0&9/16\\0&0&0&4/3\end{bmatrix}$$

$$d_0 = \begin{bmatrix} 0 & 0 & 0 & \frac{3373}{1935360} \\ 0 & 0 & 0 & \frac{37}{1890} \\ 0 & 0 & 0 & \frac{5319}{71680} \\ 0 & 0 & 0 & \frac{176}{945} \end{bmatrix}, b_0 = \begin{bmatrix} \frac{131}{96768} & \frac{-283}{322560} & \frac{179}{483840} & \frac{-131}{96768} \\ \frac{59}{1890} & \frac{-1}{72} & \frac{11}{1890} & \frac{-1}{945} \\ \frac{2889}{17920} & \frac{-1539}{35840} & \frac{81}{3584} & \frac{-297}{71680} \\ \frac{64}{135} & \frac{-16}{315} & \frac{64}{945} & \frac{-2}{189} \end{bmatrix}$$

when: $i = 1$

$$e_1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad e_2 = \begin{bmatrix} 0 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 3/2 \\ 0 & 0 & 0 & 2 \end{bmatrix}, \quad e_3 = \begin{bmatrix} 0 & 0 & 0 & 1/8 \\ 0 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 1/8 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

$$d_1 = \begin{bmatrix} 0 & 0 & 0 & \frac{113}{8960} \\ 0 & 0 & 0 & \frac{331}{5040} \\ 0 & 0 & 0 & \frac{1431}{8960} \\ 0 & 0 & 0 & \frac{31}{108} \end{bmatrix}, \quad b_1 = \begin{bmatrix} \frac{107}{8064} & \frac{-103}{13440} & \frac{113}{13440} & \frac{-47}{80640} \\ \frac{63}{630} & \frac{-1}{21} & \frac{13}{630} & \frac{-19}{5040} \\ \frac{1863}{4480} & \frac{-243}{4480} & \frac{45}{896} & \frac{-81}{8960} \\ \frac{272}{315} & \frac{4}{105} & \frac{16}{105} & \frac{-1}{63} \end{bmatrix}$$

when: $i = 2$

$$e_2 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, e_3 = \begin{bmatrix} 0 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 3/2 \\ 0 & 0 & 0 & 2 \end{bmatrix}, d_2 = \begin{bmatrix} 0 & 0 & 0 & \frac{367}{5760} \\ 0 & 0 & 0 & \frac{53}{360} \\ 0 & 0 & 0 & \frac{147}{640} \\ 0 & 0 & 0 & \frac{14}{15} \end{bmatrix}, b_2 = \begin{bmatrix} \frac{3}{32} & \frac{-47}{960} & \frac{29}{1440} & \frac{-7}{1920} \\ \frac{2}{5} & \frac{-1}{12} & \frac{2}{45} & \frac{-1}{120} \\ \frac{117}{160} & \frac{27}{320} & \frac{3}{32} & \frac{-9}{640} \\ \frac{16}{15} & \frac{4}{15} & \frac{16}{45} & 0 \end{bmatrix}$$

when: $i=3$

$$e_3=\begin{bmatrix}0&0&0&1\\0&0&0&1\\0&0&0&1\\0&0&0&1\end{bmatrix}, d_3=\begin{bmatrix}0&0&0&\frac{251}{1440}\\0&0&0&\frac{-29}{180}\\0&0&0&\frac{27}{160}\\0&0&0&\frac{7}{45}\end{bmatrix}, \quad b_3=\begin{bmatrix}\frac{323}{720}&\frac{-11}{60}&\frac{53}{720}&\frac{-19}{1440}\\\frac{-31}{45}&\frac{-2}{15}&\frac{-1}{45}&\frac{1}{180}\\\frac{51}{80}&\frac{9}{20}&\frac{21}{80}&\frac{-3}{160}\\\frac{32}{45}&\frac{4}{15}&\frac{32}{45}&\frac{7}{45}\end{bmatrix}$$

3. BASIC PROPERTIES OF TWO- STEP METHOD

3. 1. Order and Error constant of the Block

Let the linear operator defined on the method be:

$$\mathscr{L}\left[y(x);h\right],$$

where:

$$\Delta\left[y(x);h\right]= \mathbf{A}^{(0)}\mathbf{Y}_m^{(i)}-\sum_{i=0}^{3}\frac{(jh)}{i}y_n^{(i)}-h^{(4-i)}\left[\ \mathbf{d}_i f\left(y_n\right)+\mathbf{b}_i\mathbf{F}(\mathbf{Y}_m)\right] \tag{15}$$

Expanding the form $\mathbf{Y}_m$ and $\mathbf{F}(\mathbf{Y}_m)$ in Taylor series and comparing coefficients of h, we obtained

$$\Delta\left[y(x);h\right]=C_0y(x)+C_1hy'(x)+...+C_ph^py^p(x)+C_{p+1}h^{p+1}y^{p+1}(x)+C_{p+2}h^{p+2}y^{p+2}(x)+...$$

Definition: The linear operator Δ and the associated block method are said to be of order p if $C_0=C_1=...=C_p=C_{p+1}=0,\quad C_{p+2}\neq 0.\quad C_{p+2}$ is called the error constant. It implies that the local truncation error is given by $T_{n+k}=C_{p+2}h^{p+2}y^{p+2}\left(x\right)+O\left(h^{p+3}\right)$

Expanding the block in Taylor series expansion gives

$$\begin{bmatrix} \sum_{q=0}^{\infty} \frac{(\frac{1}{2}h)^q}{q!} y^q - y_n - \frac{1}{2}hy_n' - \frac{1}{8}h^2y_n'' - \frac{1}{48}h^3y_n''' - \frac{3373}{1935360}h^4y_n^{(iv)} - \\ \sum_{q=0}^{\infty} \frac{h^{q+4}}{q!} y^{q+4}\left(\frac{139}{96768}(\frac{1}{2})^q - \frac{283}{322560}(1)^q + \frac{179}{483840}(\frac{3}{2})^q - \frac{131}{1935360}(2)^k\right) \\ \sum_{q=0}^{\infty} \frac{(h)^q}{q!} y^q - y_n - hy_n' - \frac{1}{2}h^2y_n'' - \frac{1}{48}h^3y_n''' - \frac{37}{1890}h^4y_n^{(iv)} - \\ \sum_{q=0}^{\infty} \frac{h^{q+4}}{q!} y^{q+4}\left(\frac{59}{1890}(\frac{1}{2})^q - \frac{1}{72}(1)^q + \frac{11}{1890}(\frac{3}{2})^q - \frac{1}{9450}(2)^k\right) \\ \sum_{q=0}^{\infty} \frac{(\frac{3}{2}h)^q}{q!} y^q - y_n - \frac{3}{2}hy_n' - \frac{9}{8}h^2y_n'' - \frac{9}{16}h^3y_n''' - \frac{5319}{71680}h^4y_n^{(iv)} - \\ \sum_{q=0}^{\infty} \frac{h^{q+4}}{q!} y^{q+4}\left(\frac{2889}{17920890}(\frac{1}{2})^q - \frac{1539}{35840}(1)^q + \frac{81}{3584}(\frac{3}{2})^q - \frac{297}{71680}(2)^k\right) \\ \sum_{q=0}^{\infty} \frac{(2h)^q}{q!} y^q - y_n - 2hy_n' - 2h^2y_n'' - \frac{4}{3}h^3y_n''' - \frac{176}{945}h^4y_n^{(iv)} - \\ \sum_{q=0}^{\infty} \frac{h^{q+4}}{q!} y^{q+4}\left(\frac{64}{135}(\frac{1}{2})^q - \frac{16}{315}(1)^q + \frac{64}{645}(\frac{3}{2})^q - \frac{2}{189}(2)^k\right) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{16}$$

Comparing the coefficients of h, the order of the block is p = 8

With error constant $C_{p+2} = \left[\frac{3619321}{3715891200}, \frac{-2111309}{3628800}, \frac{2210409}{45875200}, \frac{-716}{14175}\right]^T$

3. 2. Consistency

In numerical analysis, it is necessary that the method satisfies the necessary and sufficient conditions. (Lambert 1973)

A numerical method is said to be consistent if the following conditions are satisfies

i. The order of the scheme must be greater than or equal to 1 i.e. p≥1.

ii. $\sum_{j=0}^{k} \alpha_j = 0$

iii. $\rho(r) = \rho'(r) = 0$

iv. $\rho^{iv}(r) = 4!\sigma(r)$

where: $\rho(r)$ and $\sigma(r)$ are the first and second characteristics polynomials of our method. According to [13], the first condition is a sufficient condition for the associated block method to be consistent. Our method is order p = 8≥1. Hence it is consistent

3. 3. Zero stability of the method

Substituting (3.2.6) into (4.0.3) gives

$$\left\| \left[\lambda A^{(0)} - A^{(i)} \right] \right\| = \left\| \left[\lambda \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \right] \right\| = 0$$

$\lambda^4 - \lambda^3 = 0$, $\lambda = 0,0,0,1$. Hence the block is zero stable.

4. IMPLEMENTATION OF THE METHOD

In order to implement our method, we proposed a predictor equation of the form:

$$Y_m^{(0)} = \sum_{i=0}^{3} \frac{(jh)}{i!} y_n^{(i)} + h^3 \sum_{\lambda=0}^{p} \frac{\partial^\lambda}{\partial x^\lambda} f\left(x, y, y', y'', y'''\right)_{(x_0, y_0, y_0', y_0'', y_0''')}$$

where: $Y_m^{(0)} = Y_{m(x_0, y_0, y_0', y_0'', y_0''')}$, $\frac{\partial^\lambda f}{\partial x^\lambda} = \left(\frac{\partial}{\partial x} + y' \frac{\partial}{\partial y} + y'' \frac{\partial}{\partial y'} + y''' \frac{\partial}{\partial y'} + f \frac{\partial}{\partial y'''} \right) f_j = Df_j$

$f_j = f\left(x_j, y_j, y_j', y_j'', y_j'''\right)$ and p is the order of our method

Substituting the above into (15) yield

$$\mathbf{A}^{(0)} \mathbf{Y}_m^{(i)} = \sum_{i=0}^{3} \frac{(jh)}{i} y_n^{(i)} + h^{(4-i)} \left[\mathbf{d}_i f\left(y_n\right) + \mathbf{b}_i \mathbf{F}(\mathbf{Y}_m) \right]$$

This is the block method that is implemented as a simultaneous integrator.

4. 1. Numerical experiments

The method is tested on some numerical problems to test the accuracy of the proposed methods and our results are compared with the results obtained using existing methods.

The following problems are taken as test problems:

Problem 1: Consider the linear differential equation of fourth order

$$y^{(iv)} + y'' = 0, 0 \le x \le \frac{\pi}{2}$$

$$y(0) = 0, y'(0) = \frac{-1.1}{72 - 50\pi}, y''(0) = \frac{1}{144 - 100\pi}, y'''(0) = \frac{1.2}{144 - 100\pi}, h = \frac{1}{320}$$

Exact solution: $y(x)=\dfrac{1-x-\cos x-1.2\sin x}{144-100\pi}$

Problem 2: Consider a special fourth order differential equation

$$y^{(iv)}=-\sin x+\cos x, y'''(0)=7, y''(0)=y'(0)=-1, y(0)=0$$

h = 0.003125

Exact solution: $y(x)=-\sin x+\cos x+x^3-1$

Problem 3: Consider the non-linear equation of fourth order below

$$y^{(iv)}=(y')^2-yy'''-4x^2+e^x(1-4x+x^2)$$

$$y(0) = 1 \quad y'(0) = 1 y''(0) = 3 y'''(0) = 1 \quad \mathrm{h} = \frac{0.1}{32}$$

Exact solution: $y(x)=x^2+e^x$

Problem 4: $y''''=\dfrac{-(8+25x+30x^2+12x^3+x^4)}{(1+x^2)}, \quad y(0)=0, \quad y'(0)=1, \quad y''(0)=0, \quad y'''(0)=-3$

$h=0.103125$

Exact solution: $y(x)=x(1-x)e^x$

The result of these problems is shown in Tables 1-4. The error is defined as Error = $|y(x)-y_n(x)|$, where y(x) is the exact solution and $y_n(x)$ is our computed result.

Problem 5:

The propose methods is applied to solve a physical problem from ship dynamics, As stated in [1], when a sinusoidal wave of frequency Ω passes along a ship or offshore structure, the resultant fluid actions vary with time t. In particular case study by [1], is the fourth order problems defined as

$$y^4+3y''+y(2+\varepsilon\cos(\Omega t)=0, \quad t\succ 0$$

Which is subjected to the following initial conditions:

$$y(0) = 1 \quad y'(0) = 0 \qquad y''(0) = 0 \qquad y'''(0) = 0 \quad \mathrm{h} = 0.1$$

where: ε =0 for the existence of the theoretical solution, $y(t)=2\cos t-\cos(t\sqrt{2})$.

Table 1. Result of test problem 1

X-value	Exact Result	Computed Result	Error in our Method (p=8)
0.103125	0.0000403745930229973248	0.00004037459302299943936	2.11488E-18
0.206250	0.00008069158007107026168	0.00008069158007108083798	1.05763E-17
0.306250	0.0001209507467709597913	0.0001209507467709724738	1.26825E-17
0.406250	0.0001611518793140582724	0.0001611518793140802351	2.19627E-17
0.506250	0.0002012947644584974134	0.0002012947644585230366	2.56232E-17
0.603125	0.0002413791895312307148	0.0002413791895312680113	3.72965E-17
0.703125	0.0002814049424301103471	0.0002814049424301544446	4.40975E-17
0.803125	0.0003213718116259584653	0.0003213718116260182277	5.97624E-17
0.903125	0.0003612795861646329173	0.0003612795861647042303	7.13130E-17
1.003125	0.0004011280556690873430	0.0004011280556691799326	9.25896E-17

Table 2. Result of test problem 2

X-value	Exact Result	Computed Result	Error in (k=2, p=8) in our method
0.103125	-0.003129847204687696	-0.00312984720468770183	5.8350E-18
0.206250	-0.006269246355772101	-0.00626924635577214780	4.6708E-17
0.306250	-0.009417983687528419	-0.0094179836875284719	5.2467E-17
0.406250	-0.012575845339462482	-0.0125758453394625761	9.3430E-17
0.506250	-0.015742617356611092	-0.015742617356611191	9.9220E-17
0.603125	-0.018918085689843284	-0.0189180856898434241	1.4019E-16
0.703125	-0.022102036196162510	-0.0221020361961626568	1.4613E-16
0.803125	-0.025294254639009744	-0.0252942546390099310	1.8712E-16
0.903125	-0.028494526688567489	-0.0284945266885676831	1.9324E-16
1.003125	-0.003129847204687696	-0.00312984720468770183	5.8350E-18

Table 3. The result of test problem 3

X-value	Exact solution	New result for p=8	Errors in (k=2, p=8)
0.103125	0.1300799589367158E-02	0.1300799589367158E-02	8.20372E-18
0.206250	0.2531773700195635E-02	0.2531773700195635E-02	5.42632E-17
0.306250	0.3652478978884993E-02	0.3652478978884993E-02	5.35801E-16
0.406250	0.46959532231804849E-02	0.46959532231804849E-02	4.82543E-15
0.506250	0.56576423608034461E-02	0.56576423608034461E-02	6.22932E-14
0.606250	0.6507754608034524E-02	0.6507754608034524E-02	8.98742E-14
0.703125	0.7298314767638522E-02	0.7298314767638522E-02	7.97451E-14
0.803125	0.7998520222728983E-02	0.7998520222728983E-02	1.66439E-13
0.903125	0.8607246703302495E-02	0.8607246703302495E-02	2.32848E-13
1.031250	0.9124283967030094E-02	0.9124283967030094E-02	4.58611E-13

Table 4. Result of Problem 4

X-value	Exact solution	New result for (k=2, p=8)	Errors in Olabode and Alabi 2013 (k=4, p=8)	Errors in our method (k=2, p=8)
0.003125	0.3124984756E-02	0. 3124984756E-02	1.990205E-14	2.4874E-14
0.006250	0.6249877513E-02	0.6249877512E-02	6.379298E-13	7.9720E-13
0.009375	0.9374585568E-02	0.9374585568E-02	4.852393E-12	6.3116E-14
0.012500	0.1249901545E-01	0.1249901545E-01	2.048206E-11	4.4102E-12
0.015625	0.1562307290E-01	0.1562307290E-01	6.261025E-11	5.7680E-12
0.018750	0.1874666289E-01	0.1874666288E-01	1.560543E-10	1.4918E-11
0.021875	0.2186968961E-01	0.2186968951E-01	3.378600E-10	9.1931E-11
0.025000	0.2499205643E-01	0.2499205616E-01	6.598189E-10	2.7786E-10
0.028125	0.2811366598E-01	0.2811366533E-01	1.191010E-09	6.4684E-10
0.031250	0.3123442003E-01	0.3123441873E-01	2.020367E-09	1.2977E-09

Table 5. Result of the real life problem, when E=1, h=0.01

X-value	Exact solution	New result for (k=2, p=8)	Errors in our method (k=2, p=8)
0.010000	0.99999999916667492	0.99999999953306984	3.6639e-10
0.020000	0.99999998666719991	0.99999998745011653	7.8292e-10
0.030000	0.99999993250607477	0.99999992604823429	6.4578e -09
0.040000	0.99999978670079770	0 .99999975168830968	3.5012e -08
0.050000	0.99999947929686150	0.99999937290694052	1.0639e -07
0.060000	0.99999892038874172	0.99999867233152195	2.4806e -07
0.070000	0.99999800014687490	0 .99999750481604655	4.9533e -07
0.080000	0.99999658885061771	0.99999569935586163	8.8949e -07
0.090000	0.99999453692718066	0.99999305725142940	1.4797e -06
0.100000	0.99999167499652852	0.99998935401854916	2.3210e -06

Table 6. Result of the real life problem, when E=1, h=1/1000

X-value	Exact solution	New result for (k=2 ,p=8)	Errors in our method (k=2, p=8)
0.001000	0.99999999999991673	0.99999999999995326	3.6526e-14
0.002000	0.99999999999866662	0.99999999999874500	7.8382e -14
0.003000	0.99999999999325007	0.99999999999260514	6.4493e -13
0.004000	0.99999999997866673	0.99999999997517031	3.4964e -12
0.005000	0.99999999994791677	0.99999999993729261	1.0624e -11
0.006000	0.99999999989200039	0.99999999986723065	2.4770e -11
0.007000	0.99999999979991772	0.99999999975045895	4.9459e -11
0.008000	0.99999999965866893	0.99999999956985797	8.8811e -11
0.009000	0.99999999945325435	0.99999999930552430	1.4773e -10
0.010000	0.99999999916667492	0.99999999893496117	2.3171e -10

5. CONCLUSION

We have proposed a two-step hybrid block method for the numerical solution of higher order initial value problems in ordinary differential equations in this paper. The method is consistent, convergent and zero stable. The method derived efficiently solved fourth order IVPS as can be seen in the low error constant and hence better approximation than the existing methods. We have also applied our method to the shipping problem and the results are as displayed in tables 5 and 6. The exact solution and the theoretical solution are also in good agreements.

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