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Estimating parameters of the Weibull diameter distribution for *Gmelina arborea* in Nimbia Forest Reserve, Nigeria

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ABSTRACT

Parameters of the Weibull diameter distribution were estimated for Gmelina (*Gmelina arborea*) plantation in Nimbia Forest Reserve. Stratified random sampling was employed to select five plots of 20 × 20 m from each forest bit that serves as management unit, thereby having a total of 20 plots for the study. Stump diameter (Dst), diameter at breast height (Dbh), diameter at middle and top positions (Dm and Dt) of trees, and tree height for all the selected trees were measured. Average Dbh measured was 16.33 cm and the mean tree height 6.85 m. Pseudo coefficient of determination (Pseudo R^2), residual mean square error, Anderson-Darling, and Kolmogorov-Smirnov goodness-of-fit statistics were considered as model selection criteria. Weibull cumulative density function (Weibull-CDF) was the best fit method for Gmelina compared with the Weibull probability density function based on the goodness-of-fit statistics generated. The diameter distribution models produced are considered only efficient for tree stands of seven years rotation age, while additional site factors like spacing, density and silvicultural treatments are required for better models, hence the need for establishing permanent sample plots (PSP) in order to get additional information from re-measurements of the plots.

Keywords**:** *Gmelina arborea*, Nimbia Forest Reserve, Weibull distribution

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1. INTRODUCTION

The structure of a forest consists of the distribution of trees and their respective size in relation to a unit area, being a result of species growth pattern, environmental conditions and forest management practices. In studying forest structure, diameter distribution is a basic, widely disseminated and applied tool that constitutes the simplest and most effective way to describe the characteristics of a given stand (Miguel, 2010). Diameter distribution is an efficient indicator of forest growth and the most powerful way to describe the properties of a stand. The diameter variable is derived from direct measurement of trees and is well correlated to other important variables, including volume, production quality and exploration costs (Miguel *et al.*, 2010; Adegbehin, 1988, 1987; Agbede, 1979; Walte, 1967).

According to Scolforo (1998), knowledge of diameter distribution in planted forests is a critical requirement to ensure that prediction or prognosis of yield is implemented. In analyzing diameter structure, there is an interest in describing diameter frequency distribution by using probability density functions. A probability density function defines the probability associated to each value of the variable in question, or else it can describe the relative and/or absolute frequency distribution of the various tree sizes (Campos and Leite, 2006). Diameter distribution models can estimate the number of trees per hectare per diameter class at present and future ages. Then the use of a volume, taper or volumetric ratio equation allows estimating yield per diameter class, its being an important tool in situations where multiple wood products are concerned (Miguel *et al.* 2010). Diameter-class distribution models have become a useful tool in forest management, growth and yield modeling, and forests inventories. Various cumulative density function (CDF) and probability density functions (pdf) such as normal, log-normal, gamma, beta, Johnson's S_{B1} and Weibull have been utilized to characterize the diameter, cumulative and frequency distributions of forest stands (Hoekstra, 1994; Carpenter, 2004)

In forestry, it is important to be able to make accurate future predictions of the mean values of growth variables based on repeated measurements through time made on units grouped hierarchically. In many forest management practices, decisions are based on yield projections that crucially depend on projections of plot level averages of tree height, basal area, and other morphometric variables (Hall and Bailey, 2001; Azeez, 2017).

Gmelina like many other tropical hardwood species is reasonably strong for its weight. Its timber is highly esteemed for door and window panels, joinery and furniture especially for drawers, wardrobes, cupboards, kitchen and camp furniture, and musical instruments because of its lightweight, stability and durability. Gmelina is easy to cultivate and grow at the smallholder level. It has been widely grown in plantations in south and Southeast Asia. Experience indicates that marketable small-diameter Gmelina timbers can be produced in 7 to 10 years (Roshetko *et al*, 2003). In a reasonably good site, it takes only three years to attain a merchantable timber size from 5.8-8.3m with a diameter from 10-15 cm. It grows to a height of 30 meters, a diameter of 60-100 cm and lives up to 40 years.

2. MATERIALS AND METHODS

Study Area

Nimbia forest reserve is located in the Northern Guinea Savanna zone of Nigeria but in the Derived savanna zone at the Eastern part of Jema'a Local Government Area of Kaduna state, 70 km south east of Jos, along Jos-Kafanchan road. It lies between latitudes 9º29' and 9º32'N and longitudes 8º30' and 8º36'E with an elevation of about 594m above mean sea level. Nimbia forest reserve has an undulating topography. The eastern end of Nimbia forest reserve is the last part of Assop escarpment. It descends in a series of steps with long level stretches interrupted by steep boulder-strewn descents. It descends more gently west wards to Jama'a-Jagindi plains. The forest reserve drains south-west into the Gimi River and west towards the stream that forms the western boundary. There is no permanent stream that flows through the reserve. The northern and southern parts of the reserve are bounded by Lioc stream and Gimi River respectively.

Nimbia forest reserve is within the Jema'a platform and is underlain predominantly by igneous and metamorphic rocks. The greater part of the reserve is underlain by the Newer Basalt of the late Tertiary and Quaternary periods which is composed of olivine, amygdales and zeolites. There are different soil types within the plantation, but most commonly found are the red deep freely draining loams and dark-brown loamy soils. According to FDALR (1990) soils of Nimbia area as indicated on the soil map of Nigeria as belonging to the unit 15F and are classified as Typical Dystrusteps (USDA)/ Dystric Cambisols (FAO). The soils also belong to the Nimbia series which developed from weathered olivine basalt and classified as Eutrophic Brown soil by D'Hoore (1964).

The climate is determined by altitude and its location in relation to the seasonal migration of the inter-tropical convergence zone. The position of Nimbia with respect to altitude (594m above sea level) induces orographic rain and has an annual rainfall of between 1500mm – 2000mm spread over a period of seven months (April - October) while the dry months are five months (November - March). Minimum temperatures range between 17-22 ºC (December - March) and the maximum ranges from 28-35 ºC (August - March). Relative humidity is between 30 – 36% in the dry season and 95% in the rainy season.

The natural vegetation of Nimbia ranges from the Southern Guinea Savanna to a dry type of rain forest, which consists mostly of savannah woodland with tall grasses and broadleaved trees usually with short boles (e.g. *Milicia excelsa*, *Guira senegalensis*, *Parkia biglobosa*, *Daniela olivera*, *Vitex doniana et cetra*). Over the years the reserve has been over exploited which resulted in the introduction of fast growing exotic species specifically *Tectona grandis* and *Gmelina arborea.* The trees were planted in rows of 3 × 3 m, both inter and intra row spacing attaining a height of about 13.6m and diameter of about 41 cm averagely at the age of 20 years (Adegbehin, 2002).

Sampling and Data Collection

Nimbia forest reserve is divided into four different forest bits divided into different compartments and sub-compartments as management units. Stratified Random Sampling Technique was employed, considering each forest beat as a stratum. Each stratum was further divided into 20 × 20 m sampling units out of which five plots were randomly selected, thereby having a total of 20 plots for the study. Within each randomly selected sample plot, measurements (Dbh, Height, Basal area and Volume) were carried out on healthy standing trees.

Data Analysis

Data collected were organized and screened for analysis. Descriptive statistical analysis was further carried out in order to summarize the data. All analysis carried out were conducted using SAS statistical package version 9.3, licensed to The University of British Columbia, Vancouver, Canada.

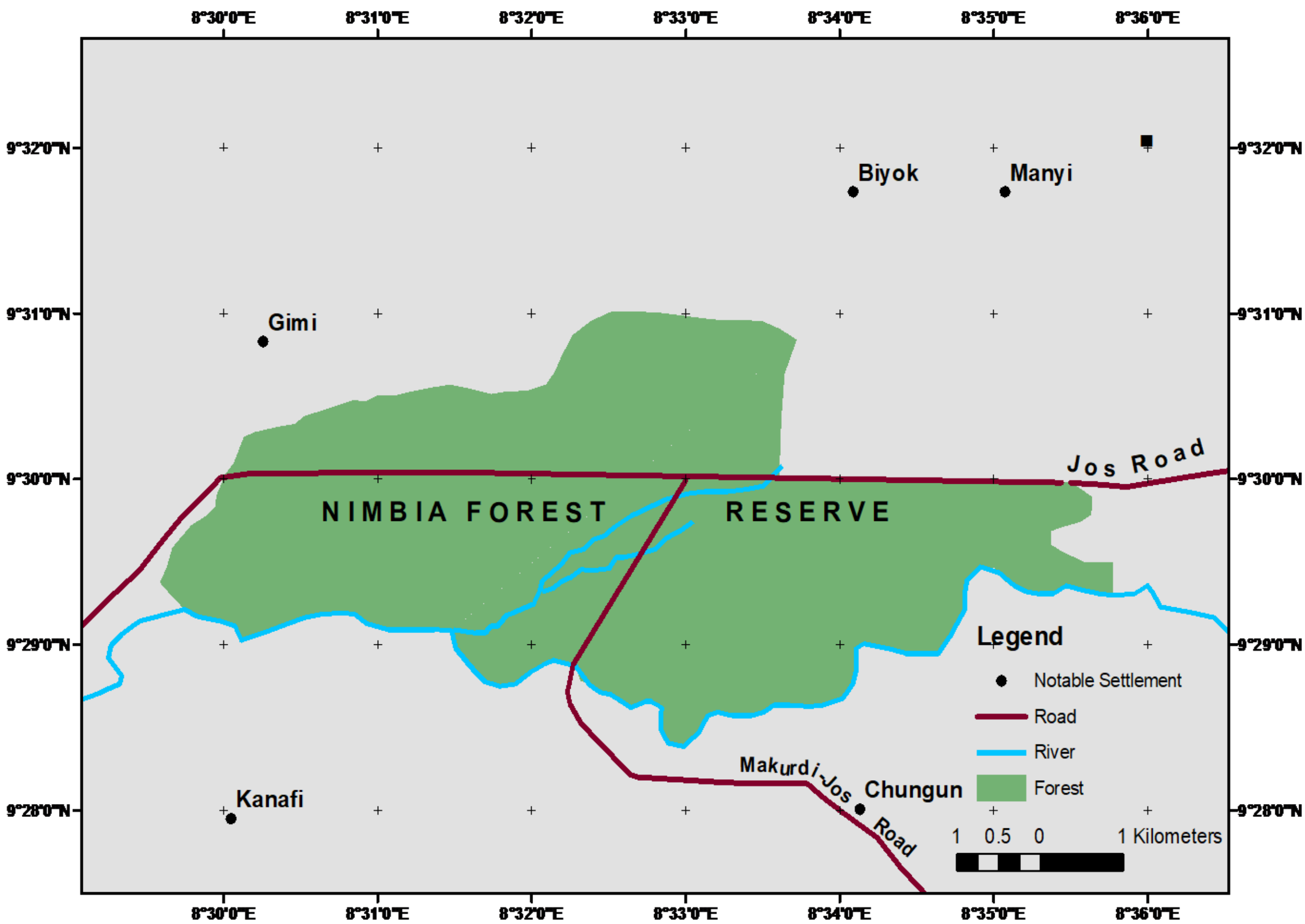

Figure 1. Map of the study area

Fitting Diameter Distribution Models

Table 1. Description of the diameter distribution models.

Model	Expression	References
Weibull CDF	$f(x) = 1 - exp\left[\left(\frac{x-a}{b}\right)^{c}\right]$	Bailey and Dell, 1973
Weibull PDF	$f(x) = \frac{c}{b}\left(\frac{x-a}{b}\right)^{c-1} exp\left[-\left(\frac{x-a}{b}\right)^{c}\right]$	Bailey and Dell, 1973

X = tree diameter; a, b and c = parameters to be estimated

Cumulative Density Function (CDF) and Probability Density Function (Pdf) of the Weibull were used to estimate the three parameter Weibull function. The parameters of the Weibull function are *a*, *b*, and *c* (location, scale, and shape) and the *a* parameter was considered to be lowest value of the Dbh measured in the data set, while *b* and *c* parameters were estimated and searched iteratively. Model forms are presented in Table 1.

Yield Computation and Goodness of Fit Test

In addition, basal area and volume of trees in each diameter class was computed using the Newton's formula. The mean tree method was then used to estimate the volume of trees in each class. This facilitates the computation of volume of products (timber, poles, and fuelwood) that can be obtained from the plantations.

Basal area was calculated using the following formula:

$$BA = \frac{\pi D^2}{4} \qquad 1$$

where: BA = basal area; D = tree diameter at breast height

Newton's formula used for volume calculation has the following form:

$$V = \frac{\pi h}{24}\left(Db^2 + 4Dm^2 + Dt^2\right) \qquad 2$$

where: H = tree height; Db, Dm, and Dt are diameters at base, middle, and top positions of the tree, respectively.

Each model was evaluated using some Goodness-of-Fit tests such as Pseudo R^2, RMSE, Anderson Darling test, and Kolmogorov Smirnoff test. Residual analysis was also carried out in order to compare the estimated and actual values of the model parameters.

3. RESULTS AND DISCUSSION

Summary Statistics

Data used were carefully obtained from the field and subjected to biological validation and the results indicated a normal distribution pattern as tree tapers from bottom to the top, so does the diameter decrease from bottom to the top as indicated in Table 2. This shows that the environmental factors are favorable for the normal growth and development of the species over time. Trees that are bigger in size also produce more volume compared with those with smaller diameters and this further confirms the biological validity of the data. The results of the descriptive statistics (Table 2) were found compatible with the works of Adegbehin (2002); Shamaki *et al.* (2011); Shamaki and Akindele (2013); and Shamaki and Ibrahim (2013).

One of the specific objectives of this study is to evaluate and project the number of multiple wood products to be obtained at the end of the scheduled age of silvicultural rotation (7years). From the results of diameter class distribution and the multiple products (Tables 3 and 4) it is evident that most trees fall within 10-20cm diameter classes. The results revealed that majority of the tree stands fall short of the required specification for sawn timber which is the most important product for managing the plantation, therefore the specified rotation age of seven years may not yield positive results in terms of timber quantity, unless if the stands are modified genetically and improve in silvicultural activities.

Table 2. Summary statistics of measured parameters.

Statistics	Dst (cm)	Dbh (cm)	Dm (cm)	Dt (cm)	Height (m)	BA (m^2)	Volume (m^3)
Mean	20.9	16.3	12.2	8.1	6.9	0.023	0.1130
Min	10.8	8.3	7.5	3.0	2.8	0.005	0.0156
Max	54.7	40.4	30.0	17.5	12.5	0.128	0.3237
SD	5.81	4.68	3.65	2.20	1.50	0.01	0.09
SE	0.25	0.20	0.15	0.09	0.06	0.00	0.00

Table 3. Diameter class distribution.

Dbh Class (cm)	Frequency	Proportion (%)
<10	16	2.9
10-15	245	43.8
15-20	198	35.4
20-25	73	13.0
25-30	19	3.4
>30	9	1.6

Table 4. Specifications for multiple wood products.

Diameter (cm)	Length (m)	Product
≥30	4	Sawn Timber
10-12	3	Fence Post
18-21	10	Pole
10-15	18	Scaffolds
<5	-	Fuelwood

Source: Kaduna State Forest Management Board

Notwithstanding the lesser proportion of the size class that will be suitable for timber production, the three species recorded good proportion of the other important products (fence post, poles, and scaffolds) within the specified rotation age. Miguel *et al.* (2010) observed a better yield for Eucalyptus in Brazil considering 7 years rotation age, in their study the diameter class distribution at the end of the rotation age favors timber size stands, and this could be as a result of environmental and other genetic variation across regions of the world.

Fitting of Weibull Function

Weibull function is characterized by three parameters (location, scale and shape). The smallest diameter measured in the data set was considered as the location parameter, while parameters *b* and *c* were iteratively searched in PROC NLIN module of SAS.

The standard error recorded for the parameter estimates (Table 5) was relatively low for the Weibull-CDF, but higher values of SE are recorded in Weibull-PDF and this indicates that Weibull-CDF is a better function in modeling the diameter distribution of the species. Clutter *et al.* 1983) stated that the *b* and *c* parameters in diameter modelling must be non-negative, but the Weibull-PDF parameter estimates showed negative values of *c* parameter and this further disqualified the Weibull-PDF for modelling diameter distribution of Gmelina stands. In order to validate the models, the observed and predicted values are plotted in the residual analysis (Figure 1) and compared to see if they are closely the same.

The observed values are represented by the scatter circles, while the line series represents the predicted values. Weibull-CDF has a smooth fitting of the Gmelina with a slight over prediction at about 95% of the distribution. This shows that the model can predict the parameters of the distribution with minimum error and is good enough for modelling the diameter distribution.

Four Goodness-of-Fit statistics (Pseudo R^2, RMSE, Anderson Darling, and Kolmogorov Smirnov) were used for comparison and validation of the models. From the results of Goodness-of-Fit (Table 6), Weibull-CDF showed a value of Pseudo R^2 that is near the unity which is an indication of a good model, the RMSE, A-D, and K-S values are much lower in Weibull-CDF than the values recorded by Weibull-PDF. In general, the Weibull-CDF fitted the diameter distribution of Gmelina better and the Weibull-PDF yielded poor results and considered not good on account of negative parameter estimate values.

Miguel *et al.* (2010) found that the fit of Weibull function was suitable to estimate the number of trees per diameter class by the Kolmogorov-Smirnov test while working with *Eucalyptus urophylla* stands in Brazil. To improve the volumetric estimators for the projected age of their study, the diameter distribution of the stand, as described by the Weibull function, was fitted according to site yield level. It is therefore, imperative to include site factors (e.g. stand age, spacing, density, and site index) in estimating the parameters of the diameter distribution.

Unfortunately, such information was not available for this study; as such the diameter distribution models developed can only be applicable to the seven year rotation age. In order to have a general model we need to have permanent sample plots (PSP) that will be re-measured at specified intervals. Many studies (e.g. Liu *et al.* 2001; Cao, 2004; Nord-Larsen and Cao, 2006; and Palahi *et al.* 2006) have used different methods to estimate parameters of the Weibull distribution using various kinds of transformations to linearize the function and subsequent estimation by linear regression, or by moment, or percentile estimation. Estimation of parameters by maximum likelihood have been found to produce consistently better goodness-of-fit statistics compared to other methods, but also put the largest demands on the computational resources (Cao, 2004).

Looking at the diameter distribution of the species (Figure 2), it is clear that the distribution is left skewed which is an evidence of selective harvest of the bigger trees as they grow bigger, as the stand matures at the peak, it become less skewed and the variation increases. The finding of this study is in concordant with the studies on even-aged beech in Denmark (Nord-Larsen and Cao, 2006) and eucalyptus in Brazil (Miguel *et al.* 2010).

Table 5. Parameter estimates.

Model	Parameters	Estimates	SE
Weibull-CDF	b	8.6930	0.0397
	c	1.8538	0.0229
Weibull-PDF	b	0.5495	0.2761
	c	-4.6091	2.4126

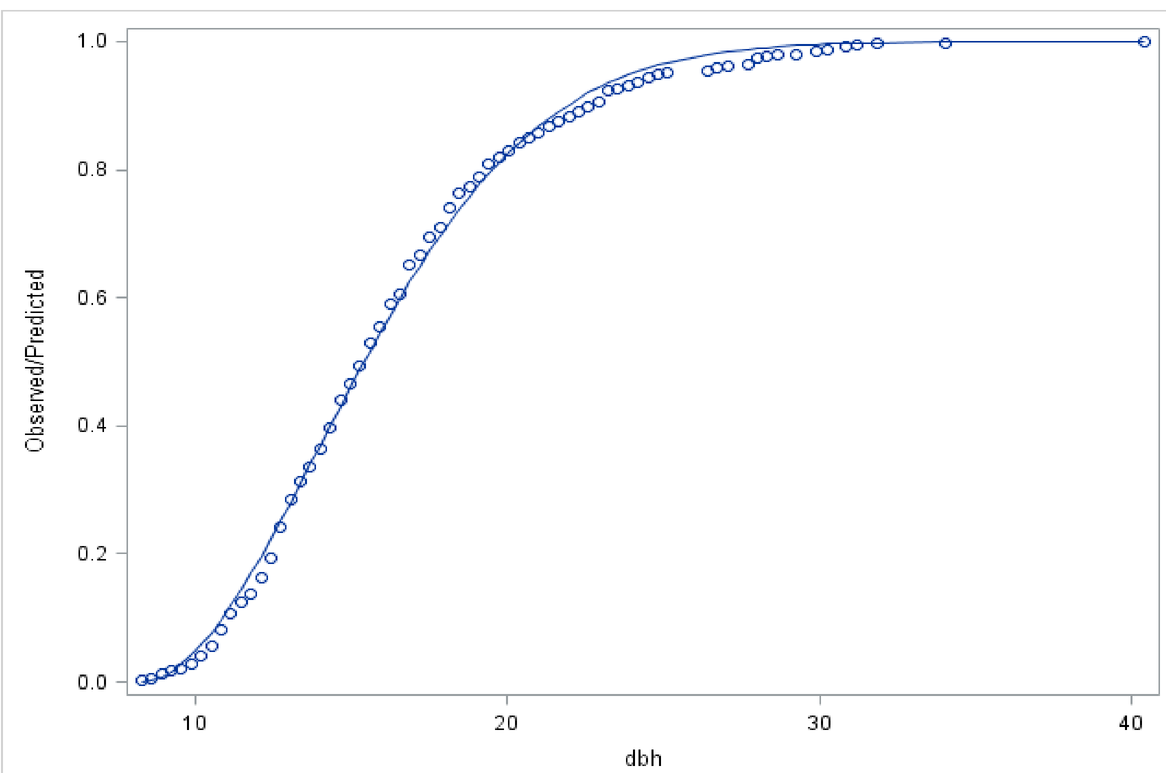

Figure 1. Observed and predicted Weibull cumulative distribution

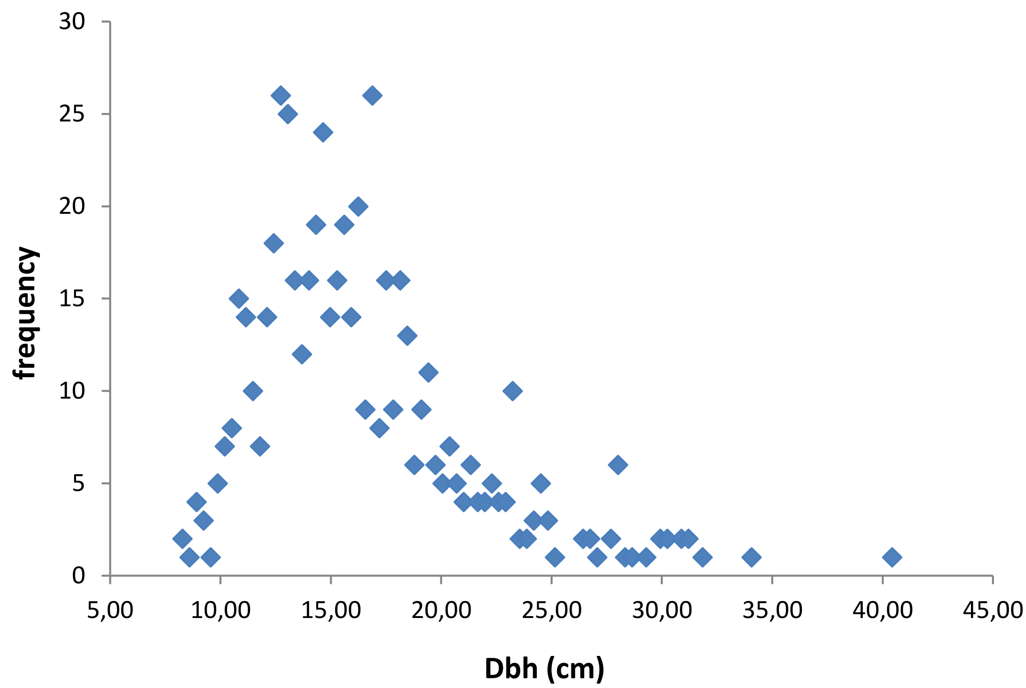

Figure 2. Shape of the diameter distribution for Gmelina stands

Table 6. Goodness of fit statistics.

Model	Pseudo R^2	RMSE	A-D	K-S
Weibull-CDF	0.998	0.0157	0.0665	0.1221
Weibull-PDF	0.036	6.9821	2.2447	0.1468

RMSE = Residual Mean Square Error, A-D = Anderson Darling, K-S = Kolmogorov Smirnov

4. CONCLUSIONS

This study revealed that the growth performance of trees in terms of tree taper has followed a normal distribution pattern, with diameter being reduced from bottom to the top of the trees. The frequency distribution of the size class is more concentrated in 10-20 cm Dbh class. Five different products are considered obtainable from the stands viz: Sawn timber, fence post, pole, scaffolds, and fuelwood. On the average, diameter class of ≥30 cm is considered suitable for sawn timber and other products fall within 10-20 cm Dbh with the exception of fuelwood that smaller sizes of less than 5 cm can be used. Diameter distribution is skewed towards the left for the stands. Weibull-PDF model was rejected on account of producing negative *c* parameter estimates and relatively higher sum of squares for error. Cumulative distribution function (CDF) of the Weibull function fitted better models as such is recommended for diameter distribution modelling.

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