Available online at www.worldscientificnews.com

World Scientific News

An International Scientific Journal

WSN 117 (2019) 183-188 EISSN 2392-2192

SHORT COMMUNICATION

# On new articulation of contra $\mu_\psi^+$-continuous in simple extended topological spaces

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**ABSTRACT**

This paper aims to study the new articulation of contra $\mu_\psi^+$-continuous functions under the canopy of simple extended topological spaces.

***Keywords*:** contra $\mu_\psi^+$-continuous function, $\mu_\psi^+$-locally indiscrete space and almost contra $\mu_\psi^+$-continuous function

**Subject classification:** 54A05; 54A99; 54C10; 54C20; 54F15

## 1. INTRODUCTION

In 1996, Dontchev [6] initiated the notion of contra continuous functions and an year later Dontchev, Ganster and Reilly studied a new class of functions called regular set connected

( Received 15 December 2018; Accepted 01 January 2019; Date of Publication 05 January 2019 )

functions. Dontchev and Noiri [5], Jafari and Noiri [7, 8] investigated the concepts of contra semi-continuous functions, contra pre-continuous functions and contra α- continuous functions between topological spaces respectively.

In 1963, Levine introduced the concept of simple extension of a topology τ by a non open set B as $\tau^+(B) = \{O\cup(O'\cap B) / O,O'\in \tau ,B \notin \tau \}$in simple extension as well. The authors [4] have already introduced $\mu_\psi^+$ -closed sets and some new fangled forms of $\mu_\psi^+$ -continuous in simple extended topological spaces.

This paper is an attempt to study a new articulation of contra contra $\mu_\psi^+$ -continuous functions under the ceiling of simple extended topological spaces. Throughout this paper X, Y and Z (or (X, τ+), (Y, σ+) and (Z, η+)) are simple extension topological space (SETS) in which no separation axioms are assumed unless and otherwise stated.

For any subset A of X, the interior of A is same as the interior in usual topology and the closure of A is newly defined in simple extension topological space.

## 2. PRELIMINARIES

**Definition 2.1 [4]:** A subset A of a topological space $(X, \tau^+)$ is called

(1) a $pre^+$-open set if $A \subseteq int(cl^+(A))$ and $semi^+$-open set if $A \subseteq cl^+(int(A))$.
(2) an $\alpha^+$-open set if $A \subseteq int(cl^+(int(A)))$ and $\beta^+$-open set if $A \subseteq cl^+(int(cl^+(A)))$.
(3) a $b^+$-open set if $A \subseteq cl^+(int(A)) \cup int(cl^+(A))$.

**Definition 2.2 [4]:** A subset A of a topological space $(X, \tau^+)$ is called

1. a semi - generalized$^+$ closed set (briefly $sg^+$-closed) if $s^+cl(A) \subseteq U$, whenever $A \subseteq U$ and U is $semi^+$-open in X.
2. a $\psi^+$-closed set if $s^+cl(A) \subseteq U$, whenever $A \subseteq U$ and U is $sg^+$-open in X.
3. a $g\alpha^{*+}$-closed set if $\alpha^+cl(A) \subseteq int(U)$ whenever $A \subseteq U$ and U is $\alpha^+$-open in X.
4. a $\mu^+$-closed set if $cl^+(A) \subseteq U$ whenever $A \subseteq U$ and U is $g\alpha^{*+}$-open in X.
5. a $\mu_\psi^+$ -closed set if $\mu^+cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\psi^+$-open in X.

**Definition 2.3 [4]:** A function $f: (X, \tau^+) \to (Y, \sigma^+)$ is called

1. contra -continuous if $f^{-1}(V)$ is closed in X for each open set V of Y.
2. contra pre-continuous if $f^{-1}(V)$ is pre-closed in X for each open set V of Y.
3. contra semi-continuous if $f^{-1}(V)$ is semi-closed in X for each open set V of Y.
4. contra α-continuous if $f^{-1}(V)$ is α-closed in X for every open set V of Y.
5. almost continuous (almost contra+-continuous) if $f^{-1}(V)$ is open (closed) in X for each regular open set V of Y.
6. $\mu_\psi^+$-continuous if every $f^{-1}(V)$ is $\mu_\psi^+$- closed in $(X, \tau^+)$ for every closed set V of Y.
7. $\mu_\psi^+$-irresolute if $f^{-1}(V)$ is $\mu_\psi^+$- closed in $(X, \tau^+)$ for every $\mu_\psi^+$ -closed set V in Y.

**Definition 2.4 [3]:** A function $f:(X, \tau^+) \to (Y, \sigma^+)$ is called

1. contra+ -continuous if $f^{-1}(V)$ is closed in X for each open set V of Y.

2. contra pre+ -continuous if $f^{-1}(V)$ is $pre^+$-closed in X for each open set V of Y.
3. contra semi+ -continuous if $f^{-1}(V)$ is $semi^+$-closed in X for each open set V of Y.
4. contra α+ -continuous if $f^{-1}(V)$ is $\alpha^+$-closed in X for every open set V of Y.
5. contra-b+ -continuous if $f^{-1}(V)$ is $b^+$-closed in X for each open set V of Y.
6. contra-sg+ -continuous if $f^{-1}(V)$ is $sg^+$-closed in X for each open set V of Y.
7. contra $\psi^+$ -continuous if $f^{-1}(V)$ is $\psi^+$-closed in $(X, \tau^+)$ for each open set V of Y.

**Definition 2.5 [3]:** A function f: $(X, \tau^+) \rightarrow (Y, \sigma^+)$ is called

1. almost+ continuous (almost contra+-continuous) if $f^{-1}(V)$ is open (closed) in X for each $regular^+$ open set V of Y.
2. almost contra $pre^+$-continuous if $f^{-1}(V)$ is $pre^+$- closed in X for each $regular^+$ open set V of Y.
3. almost contra $semi^+$-continuous if $f^{-1}(V)$ is $semi^+$- closed in X for each $regular^+$ open set V of Y.
4. almost contra $\alpha^+$-continuous if $f^{-1}(V)$ is $\alpha^+$-closed in X for each $regular^+$ open set V of Y.
5. almost contra $b^+$-continuous if $f^{-1}(V)$ is $b^+$-closed in X for each $regular^+$ open set V of Y.
6. almost $\psi^+$continuous if $f^{-1}(V)$ is $\psi^+$ -closed in X for each $regular^+$ open set V of Y.

## 3. CONTRA $\mu_\psi^+$ -CONTINUOUS FUNCTIONS

**Definition 3.1:** A function f: $(X, \tau^+) \rightarrow (Y, \sigma^+)$ is called contra $\mu^+$-continuous if $f^{-1}(V)$ is $\mu^+$-closed in $(X, \tau^+)$ for each open set V of (Y, σ+).

**Definition 3.2:** A function f: $(X, \tau^+) \rightarrow (Y, \sigma^+)$ is called contra $\mu_\psi^+$-continuous if $f^{-1}(V)$ is $\mu_\psi^+$-closed in $(X, \tau^+)$ for each open set V of (Y, σ+).

**Example 3.3:** Let X = Y = {a, b, c} with topologies τ = {Φ, X, {a}, {b}, {a, b}}, B ={c}, $\tau^+$ ={ Φ, X, {a},{b}, {c},{a, b}, {b, c}, {a, c}}and σ ={Φ, Y, {c}}, B = {b}, $\sigma^+$ = {Φ, Y, {a}, {c}, {a, c}.And f be the function defined by f(a) = c, f(b) = b, f(c) = a, then f is contra $\mu_\psi^+$-continuous.

**Theorem 3.4:** Every contra $\mu^+$-continuous function is contra $\mu_\psi^+$-continuous but not conversely.

**Proof:** Let f: $(X, \tau^+) \rightarrow (Y, \sigma^+)$ be contra $\mu^+$-continuous. Let V be any open set in Y. By the property of contra $\mu^+$-continuity we have the inverse image $f^{-1}(V)$ to be $\mu^+$-closed in X. But we know that every $\mu^+$-closed set is $\mu_\psi^+$-closed. Hence $f^{-1}(V)$ is $\mu_\psi^+$-closed in X. Therefore f is contra $\mu_\psi^+$-continuous.

The converse of the above theorem need not be true as seen from the following example

**Example 3.5:** Let X = Y = {a, b, c} be two topology spaces with topologies τ = {Φ, X, {c}}, B = {a}, $\tau^+$ = {Φ, X, {a}, {c}, {a, c}} and σ = {Φ, Y, {a, c}}, B = {a}, $\sigma^+$ = {Φ, Y,{a}, {a, c}}.

Define f: $(X, \tau^+) \rightarrow (Y, \sigma^+)$ be an identity function. Then the function f is contra $\mu_\psi{}^+$-continuous but not contra $\mu^+$-continuous.

**Theorem 3.6:** Every contra $\psi^+$-continuous function is contra $\mu_\psi{}^+$-continuous but not conversely.

**Proof:** Let f: $(X, \tau^+) \rightarrow (Y, \sigma^+)$ be contra $\psi^+$-continuous. Let V be any open set in Y. By the property of contra $\psi^+$-continuity we have the inverse image $f^{-1}(V)$ to be $\psi^+$-closed in X. But we know that every $\psi^+$-closed set is $\mu_\psi{}^+$-closed. Hence $f^{-1}(V)$ is $\mu_\psi{}^+$-closed in X. Therefore f is contra $\mu_\psi{}^+$-continuous.

The converse of the above theorem need not be true as seen from the following example

**Example 3.7:** Let X = Y = {a, b, c} be two topology spaces with topologies $\tau$ = {Φ, X, {b, c}}, B = {a}, $\tau^+$ = {Φ, X, {a},{b, c}}and $\sigma$ = {Φ, Y, {a, b}}, B = {a}, $\sigma^+$ = { Φ,Y, {a}, {a, b}}. Define f: $(X, \tau^+) \rightarrow (Y, \sigma^+)$ by f(a) = c, f(b) = b, f(c) = a. Then the function f is contra $\mu_\psi{}^+$ -continuous but not contra $\psi^+$-continuous.

**Theorem 3.8:** Every contra$^+$-continuous function is contra $\mu_\psi{}^+$-continuous but not conversely.

**Proof:** Let f: $(X, \tau^+) \rightarrow (Y, \sigma^+)$ be contra$^+$-continuous. Let V be any open set in Y. By the property of contra$^+$-continuity we have the inverse image $f^{-1}(V)$ to be closed in X. But we know that every closed set is $\mu_\psi{}^+$-closed. Hence $f^{-1}(V)$ is $\mu_\psi{}^+$-closed in X. Therefore f is contra $\mu_\psi{}^+$-continuous.

The converse of the above theorem need not be true as it is clear from the example 3.7, the function defined in it is is contra $\mu_\psi{}^+$ -continuous but not contra $^+$-continuous.

**Theorem 3.9:** If a function f:$(X, \tau^+)\rightarrow(Y, \sigma^+)$ is contra $\mu_\psi{}^+$-continuous and X is $\mu_\psi{}^+$ space then f is contra$^+$-continuous.

**Proof:** Let V be any open set in Y. Then the inverse image $f^{-1}(V)$ is $\mu_\psi{}^+$-closed in X as f is contra $\psi^+$-continuous. By hypothesis, $f^{-1}(V)$ is closed in X. Hence f is contra $\mu_\psi{}^+$-continuous.

**Definition 3.10:** A space $(X, \tau^+)$ is called a $\mu_\psi{}^+$-locally indiscrete if every $\mu_\psi{}^+$-open set in it is closed.

**Example 3.11:** Let X = {a, b} with the topology $\tau$ = {X, ϕ, {b, c}} and B = {a}, $\tau^+$ = {X, Φ, {a},{b, c}}. Then $(X, \tau^+)$ is $\mu_\psi{}^+$-locally indiscrete space.

**Remark 3.18:** The composition of two contra $\mu_\psi{}^+$-continuous maps cannot be a contra $\mu_\psi{}^+$-continuous map as seen from the following example.

**Example 3.19:** Let X={a, b, c},$\tau$ = {X, ϕ,{c}}, and B = {a}, $\tau^+$={X,Φ,{a},{c},{a, c}}, $\sigma$ = {ϕ, X,{a, b}} and B = {a}, $\sigma^+$ = {X, Φ, {a},{b, c}}. H = {X, ϕ, {a, b}} and B = {a}, $\eta^+$ = {X, Φ, {a}, {a, b}}. Define f:$(X, \tau^+)\rightarrow(X, \sigma^+)$ by f(a) = a, f(b) = b, f(c) = c. Define g: $(X, \sigma^+)\rightarrow(X, \eta^+)$

by g(a) = c, g(b) = b, g(c) = a. Then f and g are contra $\mu_\psi^+$-continuous but g ∘ f is not a contra $\mu_\psi^+$-continuous.

## 4. ALMOST CONTRA $\mu_\psi^+$ -CONTINUOUS FUNCTION

**Definition 4.1:** A function f:(X, $\tau^+$) → (Y, $\sigma^+$) is called almost contra $\mu_\psi^+$-continuous if $f^{-1}(V)$ is $\mu_\psi^+$-closed in X for each regular$^+$ open set V of Y.

**Theorem 4.2:**

If a map f: (X, $\tau^+$) → (Y, $\sigma^+$) from a topological space X into a topological space Y, then the following statements are equivalent:

1. f is almost contra $\mu_\psi^+$-continuous.
2. for every regular$^+$ closed set F of Y $f^{-1}(F)$ is $\mu_\psi^+$- open in X.

**Proof:**

(a) ⇒ (b):

Let F be a regular$^+$ closed set in Y, then Y – F is a regular$^+$ open set in Y. By (a), $f^{-1}(Y – F)$ = X – $f^{-1}(F)$ is $\mu_\psi^+$-closed set in X. This implies $f^{-1}(F)$ is $\mu_\psi^+$-open set in X. Therefore (b) holds.

(b) ⇒ (a):

Let G be a regular$^+$ open set of Y. Then Y – G is a regular+ closed set in Y. By (b), $f^{-1}(Y – G)$ is $\mu_\psi^+$-open set in X. This implies X – $f^{-1}(G)$ is $\mu_\psi^+$-open set in X, which implies $f^{-1}(G)$ is $\mu_\psi^+$-closed set in X. Therefore (a) holds.

**Theorem 4.3:**

1. Every almost contra+-continuous function is almost contra $\mu_\psi^+$-continuous function.
2. Every almost contra semi+-continuous function is almost contra $\mu_\psi^+$-continuous function.
3. Every almost contra α+-continuous function is almost contra $\mu_\psi^+$-continuous function.
4. Every almost contra $\psi^+$-continuous function is almost contra $\mu_\psi^+$-continuous function.

**Proof:** The proof is obvious.

**Remark 4.4:** Converse of the above statements is not true as shown in the following example.

**Example 4.5:**

**i.** Let X = Y = {a, b, c} be two topology spaces with topologies τ = {Φ, X, {c}}, B={a, $\tau^+$ = {Φ, X, {a}, {c}, {a, c}} and σ = {Φ, Y, {b, c}}, B = {a}, $\sigma^+$ = {Φ, Y, {a}, {b, c}}.

Define f: $(X, \tau^+) \rightarrow (Y, \sigma^+)$ by f(a) = a, f(b) = b, f(c) = c. Then the function f is almost contra $\mu_\psi{}^+$-continuous but not almost contra semi+-continuous (and not almost contra α+-continuous, almost contra $\psi^+$-continuous).

**ii.** Let X = Y = {a, b, c} be two topology spaces with topologies τ = {Φ, X, {c}}, B = {a}, $\tau^+$ = {Φ, X, {a}, {c}, {a, c}} and σ = {Φ, Y, {b, c}}, B = {a}, $\sigma^+$ = {Φ, Y, {a}, {b, c}}. Define f: $(X, \tau^+) \rightarrow (Y, \sigma^+)$ by f(a) = b, f(b) = a, f(c) = c. Then the function f is almost contra $\mu_\psi{}^+$-continuous but not almost contra+-continuous.

## 5. CONCLUSION

This paper on new articulation of contra $\mu_\psi{}^+$-continuous functions is of much use in studying the characteristics of $\mu_\psi$+- closed sets under the ceiling of extended topology.

It can also be further extended in view of other branches of topology such as ideal topology, fuzzy topology and grill topology.

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