Available online at www.worldscientificnews.com

World Scientific News

An International Scientific Journal

WSN 116 (2019) 230-237 EISSN 2392-2192

SHORT COMMUNICATION

The proof integral inequalities the first order by using Florkiewicz uniform method

G. I. Xolmirzayeva[1], **S. Dadaboyev**[2]

Faculty of Physics and Mathematics, Andijan State University, Andijan, Uzbekistan

[1,2]E-mail address: Xolmirzayeva1991@mail.ru , sdusardor@mail.ru

ABSTRACT

The aim of this paper to get an integral inequalities Nehari's and Pokorniy's type with of the function and it is the first order derivatives by using the uniform method.

***Keywords*:** Nehari type, Pokorniy type, uniform method, Florkiewicz uniform method

1. INTRODUCTION

Integral inequalities are a branch of mathematics developing rapidly during the last years. In this paper by using uniform method have been obtained integral inequalities of Nehary's and Pokorniy's type for the function and its the first order derivatives. The uniform way was first introduced by Florkiewich and Rubarski in [1] to get the first order integral inequalities. In [2, 4], the method was applied to derive second-order integral inequalities of generalized Hardy type. By using this method has been getting the new an integral inequalities Opials, Hardy's and Bessackes type in the papers [3-7].

(Received 08 December 2018; Accepted 19 December 2018; Date of Publication 20 December 2018)

The Florkiewicz uniform method of obtaining integral inequalities as follows: Given any weight function r an auxiliary function φ, there is selected the weight function s for the inequality

$$\int_I rh'^2 dt \geq \int_I sh^2 dt\,, \quad h \in H \tag{1.1}$$

is determined by

$$s = -(r\varphi')'\varphi^{-1}\,, \tag{1.2}$$

where $I = (\alpha, \beta)$, $-\infty \leq \alpha < \beta \leq +\infty$, r and s are fixed real-valued functions of the variable t, H is class entirely continuously functions. Next, for such determined weight functions, the obtained differential identity is used integral inequalities hold. It is considered new examples of integral inequalities. The first order include some integral inequalities Nehari and V. V. Pokorniy type [6-30].

2. HARDY-TYPE INEQUALITIES

Let $I = (\alpha, \beta)$, $-\infty \leq \alpha < \beta \leq +\infty$, be an arbitrary open interval. We denote by $AC(I)$ the class of real functions defined and absolutely continuous on the interval I, and by $AC^1(I)$ the class functions $f \in AC(I)$ such that $f' \in AC(I)$. Let $r \in AC(I)$ and $\varphi \in AC^1(I)$ be given functions such that $r > 0, \varphi > 0$ on the interval I, and $r\varphi' \in AC^1(I)$.

Let us put

$$s = -(r\varphi')'\varphi^{-1} \tag{2.1}$$

Let us denote by H the class of functions $h \in AC^1(I)$ satisfying the integral conditions

$$\int_I h'^2 dt < \infty, \quad \int_I sh^2 dt > -\infty \tag{2.2}$$

and the limit conditions

$$\liminf_{t\to\alpha}(r\varphi'\varphi^{-1}h^2) < \infty, \quad \limsup_{t\to\beta}(r\varphi'\varphi^{-1}h^2) > -\infty \tag{2.3}$$

and

$$\liminf_{t\to\alpha}(r\varphi'\varphi^{-1}h^2) \leq \limsup_{t\to\beta}(r\varphi'\varphi^{-1}h^2) \tag{2.4}$$

Theorem. For every function $h \in H$ the inequality

$$\int_I rh'^2 dt \geq \int_I sh^2 dt \,, \tag{2.5}$$

holds. The inequality becomes an equality if and only if and the additional condition $h = C\varphi, C = const$ is satisfied.

Proof. Let $h \in AC(I)$. By virtue (2.3) and (2.4), and from assumptions we have $r\varphi'\varphi^{-1}h^2 \in AC(I)$. If we substitute $h = \varphi f$, where $f \in AC^1(I)$, in the expression rh'^2, then, after simple calculations, we get

$$rh'^2 = r(\varphi f)'^2 = r(\varphi' f + \varphi f')^2 = r\varphi'(\varphi' f^2 + \varphi(f^2)') + r\varphi^2 f'^2 = r\varphi'(\varphi f^2)' + r\varphi^2(\varphi^{-1} f)'^2$$

Then, using the apparent identity

$$r\varphi'(\varphi f^2)' = -(r\varphi')'\varphi f^2 + (r\varphi'\varphi f^2)'$$

we have

$$rh'^2 = -(r\varphi')'\varphi f^2 + r\varphi^2 f'^2 + \left(r\varphi\varphi' f^2\right)' \tag{2.6}$$

Now if we substitute $f = \varphi^{-1}h$ on the right-hand side of the above identity (2.6), then since by (2.1) $\varphi f^2 = \varphi^{-1}h$ and

$$-(r\varphi')'\varphi f^2 = -(r\varphi')'\varphi^{-1}(\varphi f)^2 = sh^2;$$

$$r\varphi^2 f'^2 = r\varphi^2\left(\varphi^{-1}h\right)'^2;$$

$$\left(r\varphi\varphi' f^2\right)' = \left(r\varphi'\varphi^{-1}h^2\right)' = g$$

then we get the following identity

$$rh'^2 = sh^2 + (r\varphi'\varphi^{-1}h^2)' + g \,, \tag{2.7}$$

where $s = -(r\varphi')'\varphi^{-1}$, $g = r\varphi^2\left(\varphi^{-1}h^2\right)'^2$.

Now let $h \in H$. The first condition of (2.2) implies that the function rh'^2 is summability on I, since $rh'^2 \geq 0$ on I. It follows from assumptions that the tasks sh^2 and $r\varphi'\varphi^{-1}h^2$ are summability on each compact interval $[a,b] \subset I$.

Thus by we get the summability of the duty g on $[a,b] \subset I$ and by integrating (2.6) we obtain the equality

$$\int_a^b rh'^2 dt = \int_a^b sh^2 dt + r\varphi'\varphi^{-1}h^2\Big|_a^b + \int_a^b g dt, \qquad (2.8)$$

for arbitrary $\alpha < a < b < \beta$. In view (2.3), there exist two sequences $\{a_n\}$ and $\{b_n\}$ such that

$$\alpha < a_n < b_n < \beta,\ a_n \to \alpha,\ b_n \to \beta \qquad (2.9)$$

and

$$\lim_{n\to\infty} r\varphi'\varphi^{-1}h^2\Big|_{a_n} < \infty, \qquad \lim_{n\to\infty} r\varphi'\varphi^{-1}h^2\Big|_{b_n} > -\infty \qquad (2.10)$$

Thus there exist is a constant C such that

$$-r\varphi'\varphi^{-1}h^2\Big|_{a_n}^{b_n} \le C < \infty$$

By the condition $g \ge 0$ a.e.on I and from the equality (2.8) we infer that

$$\int_{a_n}^{b_n} sh^2 dt + C \le \int_{a_n}^{b_n} rh'^2 dt + C \le \int_I rh'^2 dt + C$$

and from this, letting $n \to \infty$, we obtain

$$\int_I sh^2 dt \le \int_I rh'^2 dt + C < \infty.$$

By this estimate, and by the second condition of (2.2), we conclude that sh^2 is summable on I. Next, similarly, using (2.8) and summability of the function sh^2 on I, we prove that the function g is summable on I. Thus all the integrals in the inequality (2.8) have finite limits as $a \to \alpha, b \to \beta$ and hence both of the limits in (2.3) are proper and finite. Therefore the conditions (2.3) and (2.4) may be written in the equivalent form

$$-\infty < \lim_{t\to\alpha} \inf(r\varphi'\varphi^{-1}h^2) \le \lim_{t\to\beta} \sup(r\varphi'\varphi^{-1}h^2) < \infty \qquad (2.11)$$

Now by (2.8), as $a \to \alpha, b \to \beta$, we obtain the equality

$$\int_I rh'^2 dt - \int_I sh^2 dt = \lim_{t\to\beta}(r\varphi'\varphi^{-1}h^2) - \lim_{t\to\alpha}(r\varphi'\varphi^{-1}h^2) + \int_I g dt \qquad (2.12)$$

whence, given (2.11), the inequality (2.5):

$$\int_I rh'^2 dt \geq \int_I sh^2 dt$$

Follows since $g \geq 0$ a.e. on I. If the difference (2.5) becomes equality for a non-vanishing function $h \in H$, then by (2.11) and (2.12) we have

$$\int_I gdt = 0, \quad \lim_{t\to\beta}(r\varphi'\varphi^{-1}h^2) = \lim_{t\to\alpha}(r\varphi'\varphi^{-1}h^2) \tag{2.13}$$

As $g \geq 0$ a.e. on I, we obtain $g = 0$ a.e.on I. Given (2.7), it follows from assumptions that if $g = 0$ a.e. on I then, $r\varphi^2(\varphi^{-1}h)'^2 = 0$, or $(\varphi^{-1}h)'^2 = 0$ .a.e.on I.

From $\varphi^{-1}h = const$, and $h = C\varphi$. Theorem is proved.

3. THE INTEGRAL INEQUALITIES OF Z. NEHARI'S AND V. V. POKORNIY'S TYPE

Let us take $I = (-1,1)$ and the functions

$$r = (1-t^n)^\alpha, \quad \varphi = (1-t^n)^\beta, \quad m \in N$$

On $I = (-1,1)$, where α, β – are arbitrary constants? Then we have

$$\varphi' = -n\beta \cdot t^{n-1} \cdot (1-t^n)^{\beta-1} \text{ and } r\varphi' = -n\beta \cdot t^{n-1} \cdot (1-t^n)^{\alpha+\beta-1}$$

From by differentiating we get

$$(r\varphi')' = -n(n-1)\beta \cdot t^{n-2} \cdot (1-t^n)^{\alpha+\beta-1} + n^2\beta(\alpha+\beta-1)t^{2n-2}(1-t^n)^{\alpha+\beta-2}$$

or

$$s = -(r\varphi')'\varphi^{-1} = n\beta \cdot t^{n-2}(1-t^n)^{\alpha-2}\left[(n-1)(1-t^n) - n(\alpha+\beta-1)t^n\right],$$

or by using the equality

$$(n-1)(1-t^n) - n(\alpha+\beta-1)t^n = [n-1+n(\alpha+\beta-1)](1-t^n) - n(\alpha+\beta-1)$$

we have

$$s = n\beta[n(\alpha+\beta)-1]t^{n-2}(1-t^n)^{\alpha-1} - n^2\beta(1-\alpha-\beta)t^{n-2}(1-t^n)^{\alpha-2} \tag{3.1}$$

Now we consider the following case:

1) if $\alpha+\beta=\frac{1}{n}$, then from (3.1) by (2.5) we obtain an integral inequality

$$\int_{-1}^{1}(1-t^n)^{\alpha}h'^2dt \geq (n-1)(1-n\alpha)\int_{-1}^{1}\frac{t^{n-2}h^2}{(1-t^n)^{2-\alpha}}dt \tag{3.2}$$

If in (3.3) we passed $\alpha=0,\ m=1$, then we get the integral inequalities of Nehari [6] if the function $h\in C^1[-1,1]$ satisfies boundary conditions $h(\pm 1)=0$, then

$$\int_{-1}^{1}h'^2dt \geq \int_{-1}^{1}\frac{h^2}{(1-t^2)^2}dt \qquad (3.3).$$

2) if $\alpha+\beta=1$ then from (3.1) by (2.5) we obtain an integral inequality

$$\int_{-1}^{1}(1-t^n)^{\alpha}h'^2dt \geq n(n-1)(1-\alpha)\int_{-1}^{1}\frac{t^{n-2}dt}{(1-t^n)^{1-\alpha}} \tag{3.4}$$

If in (3.4) we past $\alpha=0, n=2$, then we get an integral inequality of V.Pokorniy[7]: if the function $h\in C^1[-1,1]$ satisfies boundary conditions $h(\pm 1)=0$, then

$$\int_{-1}^{1}h'^2dt \geq 2\int_{-1}^{1}\frac{h^2}{1-t^2}dt \tag{3.5}$$

4. CONCLUSION

The integral inequalities (3.3) and (3.5) had been used for the discongugates theory differential equations second order $\omega''+p(z)\omega=0$ in the complex domain in [6] and [7] respectively.

References

[1] Ahmad, I., Fink, G.A., Mahmoud, S.A. Improvements in sub-character HMM model based Arabic text recognition. In: ICFHR (2014)

[2] M.T Parvez and S.A Mahmoud. Offline Arabic handwritten text recognition: A survey. *ACM Comput Survvol* 45 no 2 pp 23-35 May 2013

[3] S. Azeem and H. Ahmed. Effective technique for the recognition of offline Arabic handwritten words using hidden Markov models. *Int. J. Doc. Anal. Recognit.* vol. 16, no. 4, pp. 399-412, 2013.

[4] B. Su, X. Ding, L. Peng, and C. Liu. A Novel Baseline independent Feature Set for Arabic Handwriting Recognition; in Document Analysis and Recognition (ICDAR), 12th International Conference on, 2013, pp. 1250–1254.

[5] C. Choisy, A. Belaïd. Apprentissage croisé en reconnaissance analytique de l'écriture manuscrite » colloque international francophone sur l'écrit et le Document 2000.

[6] S. Young, al., the HTK Book V3.4 Cambridge University Press, Cambridge UK, 2006 M. T. Parvez, A. M. Sabri, Arabic handwriting recognition using structural and syntactic pattern attributes. *Pattern Recognition* 46 (2013), pp. 141-154.

[7] Anne-laureBianne-Bernard, Fares Menasri, Laurence Likforman-sulem, ChaficMokbel, Christopher Kermorvant (2012) Variable length and context-dependent HMM letter form models for Arabic handwritten word recognition. In Document Recognition and Retrieval Conference (DRR)

[8] AL-Shatnawi, M. Atallah, AL-Salaimeh, Safwan, AL-Zawaideh, Farah Hanna, Omar, Khairuddin, Offline arabic text recognition an overview. *World Comput. Sci. Inform. Technol. J.* 1 (5), 184–192, 2011.

[9] J. H. Alkhateeb, O. Pauplin, J. Ren, J. Jiang, Performance of hidden Markov model and dynamic Bayesian network classifiers on handwritten Arabic word recognition. *Knowledge-Based Systems* 24 (2011), pp. 680-688.

[10] W. Chew, M.-S. Tong, and B. Hu, Integral Equation Methods for Electromagnetic and Elastic Waves. San Rafael, CA, USA: Morgan & Claypool, 2008.

[11] F. P. Andriulli, A. Tabacco, and G. Vecchi, "Solving the EFIE at low frequencies with a conditioning that grows only logarithmically with the number of unknowns. *Antennas Propag.* vol. 58, no. 5, pp. 1614–1624, May 2010.

[12] A. Lawgali. A Survey on Arabic Character Recognition. *International Journal of Signal Processing, Image Processing and Pattern Recognition.* Vol. 8, No. 2 (2015), pp. 401-426.

[13] Théodore Bluche, Hermann Ney, Christopher Kermorvant (2015). The LIMSI Handwriting Recognition System for the HTRtS 2014 Contest. In International Conference of Document Analysis and Recognition (ICDAR).

[14] J.-S. Zhao and W. C. Chew. Integral equation solution of Maxwell's equations from zero frequency to microwave frequencies. *IEEE Trans. Antennas Propag.* vol. 48, no. 10, pp. 1635–1645, Oct. 2000.

[15] D. R. Wilton and A. W. Glisson, On improving the stability of the electric field integral equation at low frequencies, in Proc. URSI Radio Sci. Meeting, Los Angeles, CA, USA, Jun. 1981, p. 24.

[16] J. R. Mautz and R. Harrington. An E-field solution for a conducting surface small or comparable to the wavelength. *Antennas Propag.* vol. AP-32, no. 4, pp. 330–339, Apr. 2015.

[17] R. J. Adams, Physical and analytical properties of a stabilized electric field integral equation. *Antennas Propag.* vol. 52, no. 2, pp. 362–372, Feb. 2004.

[18] Buffa and S. Christiansen, A dual finite element complex on the barycentric refinement. *Math. Comput.* vol. 76, pp. 1743–1769, May 2007.

[19] M. Taskinen and P. Ylä-Oijala. Current and charge integral equation formulation. *IEEE Trans. Antennas Propag.* vol. 54, no. 1, pp. 58–67, Jan. 2006.

[20] D. Gope, A. Ruehli, and V. Jandhyala, Solving low-frequency EM-CKT problems using the PEEC method, *IEEE Trans. Adv. Packag.* vol. 30, no. 2, pp. 313–320, May 2007.

[21] J. AlKhateeb, H., Ren, Jinchang, Jiang, Jianmin, Al-Muhtaseb, Husni, Offline handwritten arabic cursive text recognition using hidden markov models and re-ranking, *Pattern Recognition Lett.* 32 (8), 1081–1088, 2011.

[22] S. Gasiorowicz, Quantum Physics. New York, NY, USA: Wiley, 2007.

[23] N. A. Demerdash, F. A. Fouad, T. W. Nehl, and O. A. Mohammed. Three dimensional finite element vector potential formulation of magnetic fields in electrical apparatus, *IEEE Trans. Power App. Syst.* vol. PAS-100, no. 8, pp. 4104–4111, Aug. 1981.

[24] P. De Doncker, A volume/surface potential formulation of the method of moments applied to electromagnetic scattering, *Eng. Anal. Boundaryn Elements*, vol. 27, no. 4, pp. 325–331, 2003.

[25] F. Vico, L. Greengard, M. Ferrando, and Z. Gimbutas, The decoupled potential integral equation for time-harmonic electromagnetic scattering. *Commun. Pure Appl. Math.* vol. 69, no. 4, pp. 771–812, Apr. 2016.

[26] W. C. Chew, Vector potential electromagnetics with generalized gauge for inhomogeneous media: Formulation (invited paper)," Prog. Electromagn. Res., vol. 149, pp. 69–84, 2014.

[27] Q. S. Liu, S. Sun, and W. C. Chew, A vector potential integral equation method for electromagnetic scattering. Proc. Int. Rev. Prog. Appl. Comput. Electromagn. (ACES), Mar. 2015, pp. 1–2.

[28] Y. Kessentini, T. Paquet, A. Ben Hamadou, Off-line handwritten word recognition using multistream hidden Markov models. *Pattern Recognition Letters* 31 (2010), pp. 60-70

[29] J. D. Jackson, Classical Electrodynamics. San Rafael, CA, USA: Morgan & Claypool, 2008

[30] O. Ergül and L. Gürel. The Multilevel Fast Multipole Algorithm (MLFMA) for Solving Large-Scale Computational Electromagnetics Problems. New York, NY, USA: Wiley, 2014.