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## Stability of Micropolar Fluid in a Porous Medium Provoked by Heat Function and Radiation

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### ABSTRACT

The stability of micropolar fluid flow in a channel under the influence of heat function, magnetic field and radiation was investigated. Using the similarity solution and non-dimensionalizing, the governing partial differential equations (PDEs) were transformed to a set of non-linear differential equations (ODEs). We therefore perturbed the system and determined the stability of the system. Wolfram 9 software (Mathematica) was used to analyze the various stability effect of the material parameters on the fluid flow. The graphical results reflect the expected physical behavior of the flow configuration under consideration. The study reveals that the Prandtl (Pr), wave (a), Radiation (R) and heat function parameters at a certain critical value of the wave number hastens the onset of stability. While the Electroconductivity ( $\sigma$ ), Magnetic field (M), Micropolar (K) and Microrotational ( $\lambda$ ) parameters delay the onset of stability. The results are in agreement with the works of other literature cited.

**Keywords:** Micropolar Fluid, Stability, Electroconductivity, Rayleigh, Wave Number, Heat Function

## **Nomenclatures**

$S_c$	Schmidt
$R$	Radiation Parameter
$P_r$	Prandtl number
$Q$	Heat function
$K$	Micropolar parameter
$\lambda_0$	Microrotational parameter
$G_r$	Grashof Number
$G_m$	Modified Grashof number
$\sigma_\alpha$	Electroconductivity
$M$	Magnetic term
$k_r$	Chemical potential
$n$	Transformed Co-ordinate
$\varphi$	Stream function
$U \& V$	Velocity component
$N$	Angular velocity
$C_p$	Specific heat capacity
$a$	Wave number
$Ra$	Rayleigh number

## **1. INTRODUCTION**

The most popular rheological model for Non-Newtonian fluid is the micropolar fluid model. This fluid is a fluid with microstructure and belongs to a class of fluid with non-symmetric stress tensor [17]. Studying the hydrodynamics stability of a micropolar fluid is important for a wide range of situation, varying from chemical sciences and engineering science due to its applications. There is a gradual shift of focus to this area [5] an instance could be, the process of coating a surface on a spinning substrate, where a liquid coating material is dispersed radially from the center above the surface and after application of the coating material the coating is cured [5]. This process is termed as spin coating.

The stability of hydrodynamics of plane poiseuille flow of Non-Newtonian fluids in the presence of a transverse magnetic field was studied by [20]. The linear stability theories for various film flows was presented clearly by [13, 14]. Kapitza [12] carried out the first study on stability of a film flow over a plane that is inclined, various stability behaviors of the layer flow were analyzed. Also, Chao-Kuang and Ming-Che [5] investigated non-linear weakly hydrodynamics stability of the thin Newtonian fluid flow on a rotating circular disk using the

normal mode approach and exploring also the weakly nonlinear behavior by the method of multiple scales. The results they got indicates that the supercritical instability region increases, and the subcritical stability region decreases with the increases of the rotation number. Nicola [16] studied the non-linear stability bounds for a horizontal layer of a porous medium with an exothermic reaction on the lower boundary.

Furthermore, Ngiangia et al. [15] worked on the influence of radiation on the onset of instability of magnetohydrodynamics plane poiseuille flow in a porous medium, and observed an early onset of instability in the presence of the parameters considered. Dhiman [8] examined the convective stability analysis of a micropolar fluid layer by the method of variation. They observed the effects on wave number and micropolar parameter on the Rayleigh numbers for onset of stationary instability for each possible combination of the bounded surface. Rusin [22] investigated the Navier-Stokes equation, stability and minimal perturbations of global solutions.

Ahmadi and Shahinpor [1] studied the universal stability of magneto-micropolar fluid motions. Furthermore, Pu-Jen et al. [19] investigated nonlinear stability analysis of the thin micropolar liquid fluid flowing down on a vertical cylinder. Weng et al. [23] examined the stability of micropolar fluid flow between concentric rotating cylinders. While Bhattacharyy and Jena [2] worked on the stability of hot layer of micropolar fluid, their results indicates that when a disturbance exist in the marginal state the critical Rayliegh number is found to be the same.

Mehrjardi et al. [14] investigated the study of the stability performance of noncircular lobed journal bearing with micropolar lubricant using the finite element method. Their results shows that in the case of non-circular bearings, the critical mass to load carrying capacity ratio decreases with increasing of the preload factor so for a constant vertical external load according to them, the stability performance of rotating system can be improved by replacing the circular journal bearing with noncircular types. Das et al., [7] worked on the theoretical analysis of stability characteristics of hydrodynamics journal bearing lubricant with micropolar fluid using the method of perturbation.

Their findings where that compared to Newtonian fluid, the micropolar fluid exhibits better stability. Reena and Rena [21] looked at linear stability of themosolutal convection in a micropolar fluid saturating a porous medium. From their findings oscillatory modes are introduced due to the presence of the micropolar viscous effects, micro-inertia and stable solute gradient. Othman and Zaki [18] worked on thermal instability in a rotating micropolar viscoelastic fluid layer under the effect of electric field. Draynomirescu et al. [9] did a study on the influence of micropolar parameter on the stability domain in a Rayleigh-Bernard convection problem using a reliable numerical study. They found out that all micropolar effects increases stability domains of the problem and has a marginal influence on the cell size at the onset of convection. Motivated by the above reference work, the main objective of this work is to examine the stability of magnetic field on a micropolar fluid under the influence of heat function and radiation.

## **2. MATHEMATICAL FORMULATION OF THE PROBLEM**

Considering a laminar steady boundary layer flow of a micropolar fluid along a channel. It is assumed that  $T$  is the constant temperature of the wall and that of the surrounding fluid  $T_{\infty}$

where  $T > T_{\infty}$ . A magnetic field  $B$  Is applied normal to the channel. It is assumed that the fluid flow will be under a slip boundary condition which will be generated. Considering temperature dependent heat function in the flow region to get the effect of temperature difference between the plate and the ambient fluid. There exist a first order chemical reaction between the species and fluid concentration.

Following [11,17] our governing equations results in

$$U \frac{\partial u}{\partial x} + V \frac{\partial u}{\partial y} = \left(\frac{\mu+K}{\rho}\right) \frac{\partial^2 u}{\partial y^2} + \frac{k}{\rho} \frac{\partial N}{\partial y} - \frac{\sigma \beta^2}{\rho} u + g\beta_T(T - T_{\infty}) + g\beta_e(C - C_{\infty}) - \frac{\sigma_{\infty} u'}{U} \quad 1$$

$$u \frac{\partial N}{\partial x} + V \frac{\partial N}{\partial y} = \frac{\epsilon}{\rho j} \frac{\partial^2 N}{\partial y^2} - \frac{K}{\rho j} \left(\frac{\partial u}{\partial y} + 2N\right) \quad 2$$

$$U \frac{\partial T}{\partial x} + V \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{Q}{\rho c_p} (T - T_{\infty}) - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} + \mu \left(\frac{\partial u}{\partial y}\right)^2 \quad 3$$

$$U \frac{\partial C}{\partial x} + V \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - K_r(X)(C - C_{\infty}) \quad 4$$

were

$K_r(X)$  = Chemical potential

$\frac{Q}{\rho c_p}$  = Heat function

$\frac{1}{\rho c_p} \frac{\partial q}{\partial y}$  = Radiation term

$\mu \left(\frac{\partial u}{\partial y}\right)^2$  = Viscous dissipation term

$\frac{\sigma_{\infty} u'}{U}$  = Electroconductivity term

Using the similarity equation in equations (5) – (14)

$$\varphi = \sqrt{\frac{v}{D}} x f(n), \quad n = y \sqrt{\frac{D}{v}}, \quad \frac{\partial n}{\partial y} = \sqrt{\frac{D}{v}}, \quad 5$$

$$U = \frac{\partial \varphi}{\partial y}, \quad V = -\frac{\partial \varphi}{\partial x}, \quad \frac{\partial u}{\partial x} = \sqrt{\frac{v}{D}} f'(n) \quad 6$$

$$\frac{\partial u}{\partial y} = \sqrt{\frac{v}{D}} x f''(n) \frac{\partial n}{\partial y} + \sqrt{\frac{v}{D}} x f'(n) \frac{\partial^2 n}{\partial y^2} \quad 7$$

$$\frac{\partial^2 u}{\partial y^2} = \sqrt{\frac{v}{D}} x f'''(n) \frac{\partial n}{\partial y} + \sqrt{\frac{v}{D}} x f''(n) \frac{\partial^2 n}{\partial y^2} + \sqrt{\frac{v}{D}} x f'(n) \frac{\partial^3 n}{\partial y^3} \quad 8$$

$$h(n) = \frac{N}{D\sqrt{\frac{v}{D}x}} \quad \frac{\partial N}{\partial y} = \sqrt{\frac{v}{D}} x h'(n) \frac{\partial n}{\partial y} \quad 9$$

$$\frac{\partial^2 N}{\partial y^2} = \sqrt{\frac{v}{D}} x h''(n) \frac{\partial n}{\partial y} + \sqrt{\frac{v}{D}} x h'(n) \frac{\partial^2 n}{\partial y^2} \quad 10$$

$$\theta(n) = \frac{T - T_\infty}{T_w - T_\infty} \quad \phi(n) = \frac{C - C_\infty}{C_w - C_\infty} \quad 11$$

$$\frac{\partial T}{\partial x} = \frac{\partial \theta(n)}{\partial x} = \frac{\partial \theta}{\partial n} \cdot \frac{\partial n}{\partial x} = \theta'(n) \cdot 0, \quad \frac{\partial T}{\partial y} = \frac{\partial \theta(n)}{\partial y} = \frac{\partial \theta}{\partial n} \cdot \frac{\partial n}{\partial y} = \theta'(n) \cdot \sqrt{\frac{D}{v}} \quad 12$$

$$\frac{\partial C}{\partial x} = \frac{\partial \phi(n)}{\partial x} = \frac{\partial \phi}{\partial n} \cdot \frac{\partial n}{\partial x} = \phi'(n) \cdot 0 \quad 13$$

$$\frac{\partial C}{\partial y} = \frac{\partial \phi(n)}{\partial y} = \frac{\partial \phi}{\partial n} \cdot \frac{\partial n}{\partial y} = \phi'(n) \cdot \sqrt{\frac{D}{v}} \quad 14$$

we reduced the modelled equations (1) – (4)

By using Rosseland approximation  $q_r$  takes the form

$$q_r = - \frac{4\sigma_I}{3K_I} \frac{\partial T^4}{\partial y} \quad 15$$

where  $\sigma_I$  is the Stefan-Bolyzmann constant and  $K_I$  is the mean absorption coefficient. It is assumed that the temperature difference within the flow are sufficiently small. Temperature differences within the flow are sufficiently small such that  $T^4$  can be expressed as a linear function of temperature. This is accomplished by expanding  $T^4$ , it can be expressed as a linear function of temperature. This is accomplished by expanding  $T^4$  in a Taylor series about  $T_\infty$  and neglecting higher order terms thus [6].

$$T^4 \equiv 4T_\infty^3, T - 3T_\infty^4 \quad 16$$

Using equation (15) and (16), equation (3) takes the form

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{Q}{\rho c_p} (T - T_\infty) + \frac{16\sigma_I T_\infty^3}{\rho c_p K_I} \frac{\partial^2 T}{\partial y^2} + \frac{v}{c_p} \left(\frac{\partial u}{\partial y}\right)^2 \quad 17$$

Substituting equation (5 -14) into equation (1), (2), (4) and (17), we have

$$x f'(n) f'(n) - \sqrt{\frac{v}{D}} f(n) x f''(n) = \frac{\mu+k}{\rho} x(n) f'''(n) - \frac{\sigma B^2}{\rho} x f'(n) + g_{B_T} \theta(n) + g_{B_e} \phi(n) - \frac{\sigma_\alpha}{v} x f'(n) \quad 18$$

$$x f'(n) \left( D \sqrt{\frac{D}{v}} h(n) \right) - \sqrt{\frac{v}{D}} f(n) \left( Dx h'(n) \right) = \frac{\varepsilon}{\rho_j} Dx h''(n) - \frac{k}{\rho_j} x f''(n) + 2h(n)D \sqrt{\frac{D}{v}} x \tag{19}$$

$$- f(n) \theta'(n) = \sqrt{\frac{D}{v}} \alpha \theta''(n) + \frac{Q\theta}{\rho c_p} + \frac{16 \sigma_I T_\alpha^3}{3 \rho k c_p} \sqrt{\frac{D}{v}} + \frac{\mu}{c_p} x f'(n) \tag{20}$$

$$-f(n) \phi'(n) = D_m \phi''(n) \sqrt{\frac{D}{v}} - K_r(x)\phi(n) \tag{21}$$

Non-dimensional variables and parameters are introduced into equations (18) – (21)

$$S_c = D_m \sqrt{\frac{v}{D}} \quad \text{Schmidt}$$

$$R = \frac{16 \sigma T_\alpha^3}{3 \rho c_p} \sqrt{\frac{v}{D}} \quad \text{Radiation Parameter}$$

$$P_r = \alpha \sqrt{\frac{D}{v}} = \frac{\mu}{c_p} \quad \text{Prandtl number}$$

$$Q = \frac{Q_0}{\rho c_p} \quad \text{Heat function}$$

$$K = \frac{k}{\rho_j} \quad \text{Micropolar parameter}$$

$$xD \sqrt{\frac{D}{v}} = 1$$

$$\frac{\varepsilon}{\rho_j} = \lambda_0 \quad \text{Microrotational parameter}$$

$$G_r \theta = \frac{g B_t (T - T_\alpha)}{v^2} \quad \text{Grashof Number}$$

$$G_m = \frac{g B_e (T - T_\alpha)}{v^2} \quad \text{Modified Grashof number} \tag{22}$$

$$\sigma_\alpha = \frac{\sigma_\alpha}{v} \quad \text{Electroconductivity}$$

$$M = \frac{\sigma B^2}{\rho} \quad \text{Magnetic term}$$

Substituting equations (22) into equations (18) to (21), we have

$$k f'''(n) + K h'(n) - f'(n) f'(n) - \sigma f'(n) - f(n)M + f'(n)f(n) + G_r\theta(n) + G_m\phi(n) = 0 \quad \dots 23$$

$$\lambda h''(n) + h'(n)f(n) - k ( f''(n) + 2h(n) ) - f'(n)h(n) = 0 \quad 24$$

$$\theta''(n)(Pr - R) + f(n)\theta'(n) + Q\theta(n) + Pr f''(n) f''(n) = 0 \quad 25$$

$$Sc\phi''(n) + f(n)\phi'(n) - K_r\phi(n) = 0 \quad 26$$

### Stability of the System

Following the procedure of linear stability analysis adopted by [20], let the disturbed micropolar fluid flow be a steady basic flow and a small time dependent disturbance. We perturb the system and perform the standard normal mode analysis of the form.

$$F = f + f_e \quad \theta_0 = \theta + \theta_e \quad H = f + f_e \quad \phi_0 = \phi + \phi_e$$

We ascribe also to all quantities describing the perturbation a dependence on n and t in the form

$$e^{-i(a^*n - \varphi^*t)}$$

where:  $\varphi$  is a time constant and  $a^*n$  is the resultant wave number of the disturbance

We write

$$(F, H, \theta_0, \phi_0) = [F(n), H(n), \theta_0(n), \phi_0(n)] \exp i(a^*n - \varphi^*t) \quad 27$$

$$F = F(N) e^{-i(a^*n - \varphi^*t)} \quad 28$$

$$H = H(N) e^{-i(a^*n - \varphi^*t)} \quad 29$$

$$\theta = \theta(N) e^{-i(a^*n - \varphi^*t)} \quad 31$$

$$\phi = \phi(N) e^{-i(a^*n - \varphi^*t)} \quad 32$$

From equation (23) to (26)

Substitute equations (28)-(32) into equations (23) to (26) and make  $\frac{d}{dn} = D$

$$k(D^3 - ia^*^3)F + k(D + ia^*)H - ((D + ia^*)F)^2 - \sigma_\infty(D + ia^*)F - F(n)(D + ia^*)F - M(D + ia^*) + G_r\theta(n) + G_m\phi(n) = 0 \quad \dots 33$$

$$\lambda(D^2 - a^{*2})H + F(D + ia^*)H - K(D^2 - a^{*2})F - 2Hk - FH(D + ia^*) = 0 \quad \dots 34$$

$$(Pr - R)(D^2 - a^{*2})\theta + F(D + ia^*)\theta + Q\theta + Pr[(D^2 - a^{*2})F]^2 = 0 \quad \dots 35$$

$$Sc(D^2 - a^{*2})\phi + F(D + ia^*)\phi - Kr\phi = 0 \quad \dots 36$$

From equation (33)

$$\frac{1}{G_r} [k(D^3 - ia^{*3})F + k(D + ia^*)H - ((D + ia^*)F)^2 - \sigma_\infty(D + ia^*)F - F(D + ia^*)F - M(D + ia^*) + G_m\phi] = -\theta \quad \dots 37$$

From equation (34)

$$[\lambda(D^2 - a^{*2}) - 2k]H = K(D^2 - a^{*2})F \quad \dots 38$$

From equation (35)

$$[(Pr - R)(D^2 - a^{*2}) + F(D + ia^*) + Q]\theta = -Pr[(D^2 - a^{*2})F]^2 \quad \dots 39$$

From equation (36)

$$[Sc(D^2 - ia^{*2}) + F(D + ia^*) - Kr]\phi = 0 \quad \dots 40$$

Substitute equation (37) into equation (39), and let

$$\beta_1 = [(Pr - R)(D^2 - ia^{*2}) + F(D + ia^*) + Q]$$

$$\beta_2 = [k(D^3 - ia^{*3})F + k(D + ia^*)H - ((D + ia^*)F)^2 - \sigma_\infty(D + ia^*)F - F(D + ia^*)F - M(D + ia^*)]$$

We therefore have

$$\beta_1 [\beta_2 + G_m\phi] = G_r Pr [(D^2 - a^{*2})F]^2 \quad 41$$

$$\beta_1 G_m\phi = G_r Pr [(D^2 - a^{*2})F]^2 - \beta_1\beta_2 \quad 42$$

From equation (38)

Put equation (38) into equation (42)

$$\phi = \left[ \frac{1}{\beta_1 G_m} [G_r Pr \left[ \left( \frac{\lambda}{k} (D^2 - a^{*2}) - 2 \right) H \right]^2 - \beta_1\beta_2] \right] \quad 43$$



Substitute equation 41 into equation 43

$$[\text{Sc}(D^2 - ia^{*2}) + F(D + ia^*) - Kr] \left[ \frac{1}{\beta_1 G_m} [G_r Pr \left[ \left( \frac{\lambda}{k} (D^2 - a^{*2}) - 2 \right) H \right]^2 - \beta_1 \beta_2] \right] = 0 \quad \dots 44$$

Let

$$\beta_3 = [\text{Sc}(D^2 - ia^{*2}) + F(D + ia^*) - Kr]$$

$$\beta_3 \left[ \frac{1}{\beta_1 G_m} [G_r Pr \left[ \left( \frac{\lambda}{k} (D^2 - ia^{*2}) - 2 \right) H \right]^2 - \beta_1 \beta_2] \right] = 0 \quad 45$$

According to Bird et al. [3], the analysis of free convection heat transfer from a vertical plate, shows that  $G_r Pr = Ra$ , where Ra is the Rayleigh's number therefore rewrite equation (45) as

$$\beta_1 \beta_2 \beta_3 = \beta_3 \left[ Ra \left( \frac{\lambda}{k} (D^2 - a^{*2}) - 2 \right) H \right]^2 \quad 46$$

$$\begin{aligned} & [(Pr - R)(D^2 - a^{*2}) + F(D + ia^*) + Q] [(D^3 - ia^{*3})F + k(D + ia^*)H - ((D + ia^*)F)^2 - \\ & \sigma_\infty(D + ia^*)F - F(D + ia^*)F - M(D + ia^*)] [\text{Sc}(D^2 - a^{*2}) + F(D + ia^*) - Kr] = \\ & [\text{Sc}(D^2 - a^{*2}) + F(D + ia^*) - Kr] \left[ Ra \left( \frac{\lambda}{k} (D^2 - a^{*2}) - 2 \right) H \right]^2 \quad 47 \end{aligned}$$

Following the method adopted by Hocking [10], the proper solution for F(n) and H(n) appropriate for the lowest mode is

$$H(n) = A \sin \pi n$$

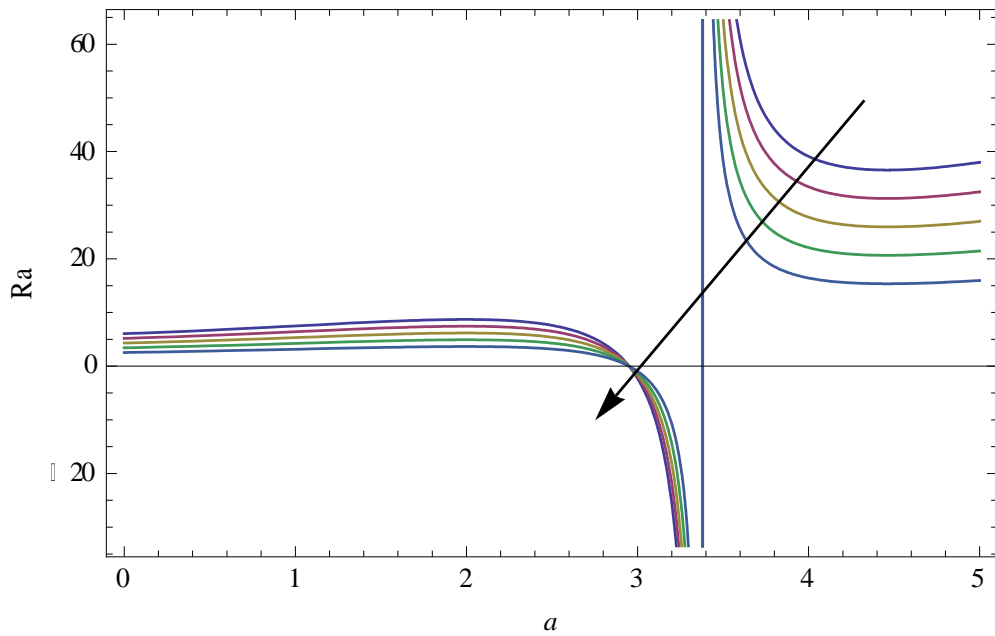
$$F(n) = A \sin \pi n$$

where: A is a constant

We substitute equation (48) into equation (47) and simplify, the conclusion will be that all the even derivatives of H(n) and F(n) must vanish for n = 0 or n = 1 and the result will be

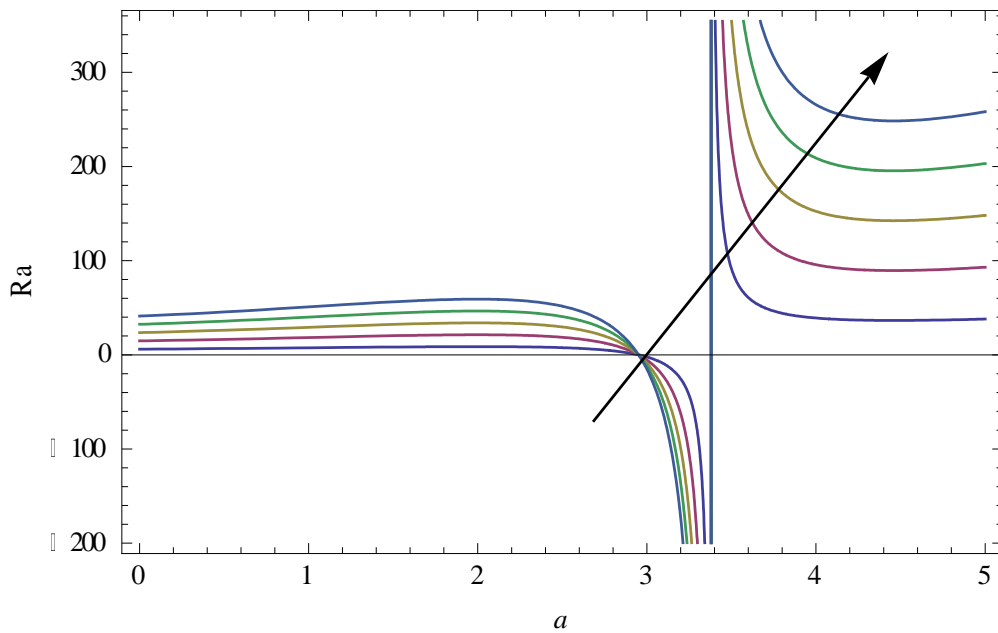
$$\begin{aligned} & [(Pr - R)(a^{*2} - \pi^2) + (ia^* + \pi) + Q] [(-ia^{*3} - \pi^3) + k(ia^* + \pi) - ((ia^* + \pi))^2 - \\ & \sigma_\infty(ia^* + \pi) - (ia^* + \pi) - M(ia^* + \pi)] = \left[ Ra \left( \frac{\lambda}{k} (a^{*2} - \pi^2) - 2 \right) \right]^2 \quad \dots 48 \end{aligned}$$

### 3. RESULTS



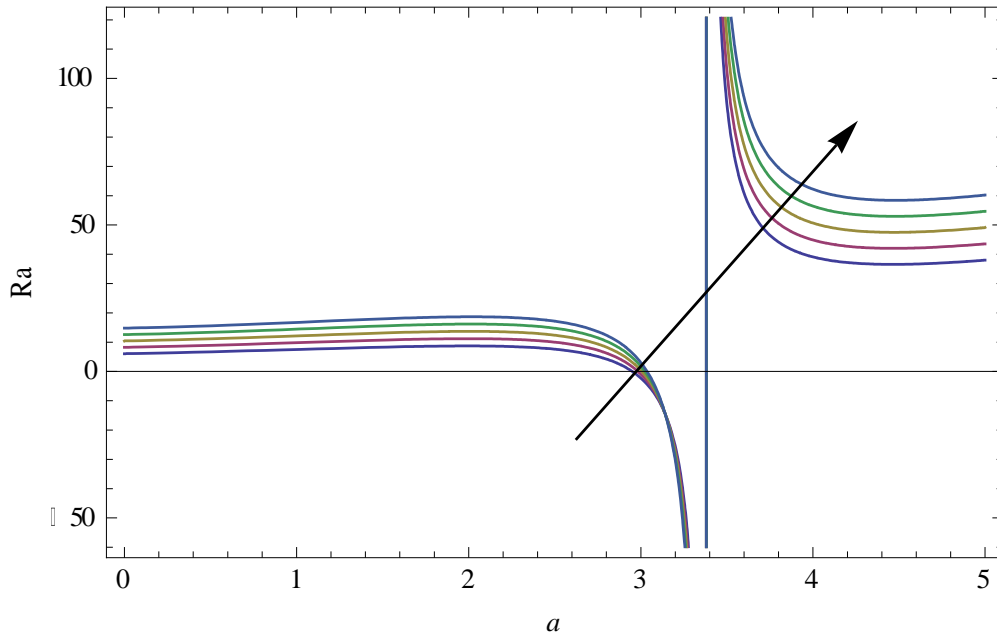
Pr = 0.31, 0.41, 0.51, 0.61, 0.71. For  $Q \geq 1.5$

**Figure 1.** Rayleigh number Ra against wave number (a) for varying Prandtl number Pr.

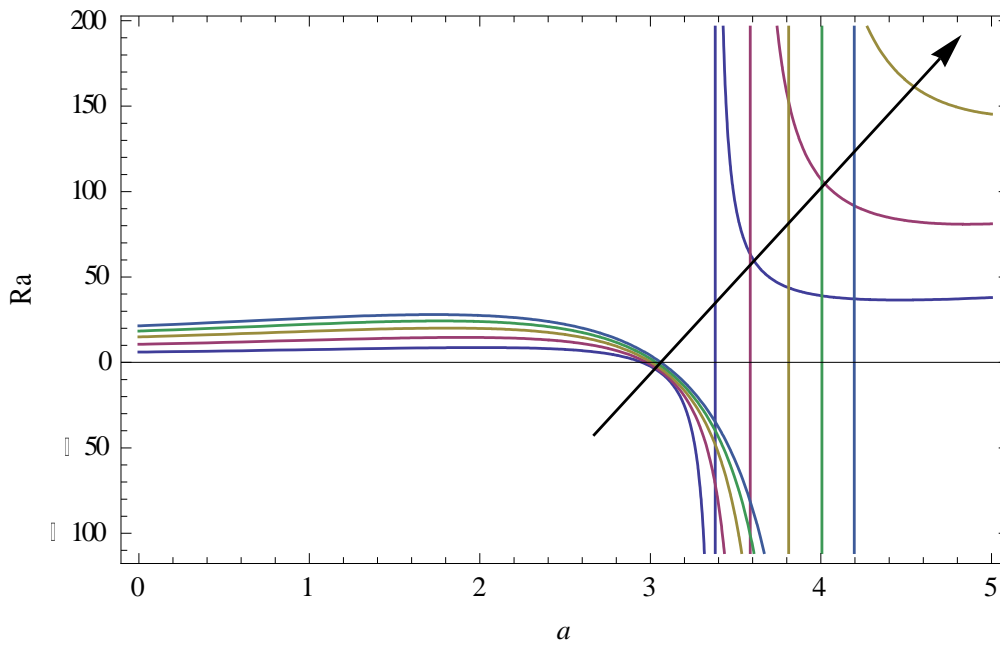


R = 1.0, 2.0, 3.0, 4.0, 5.0. For  $Q \geq 1.5$

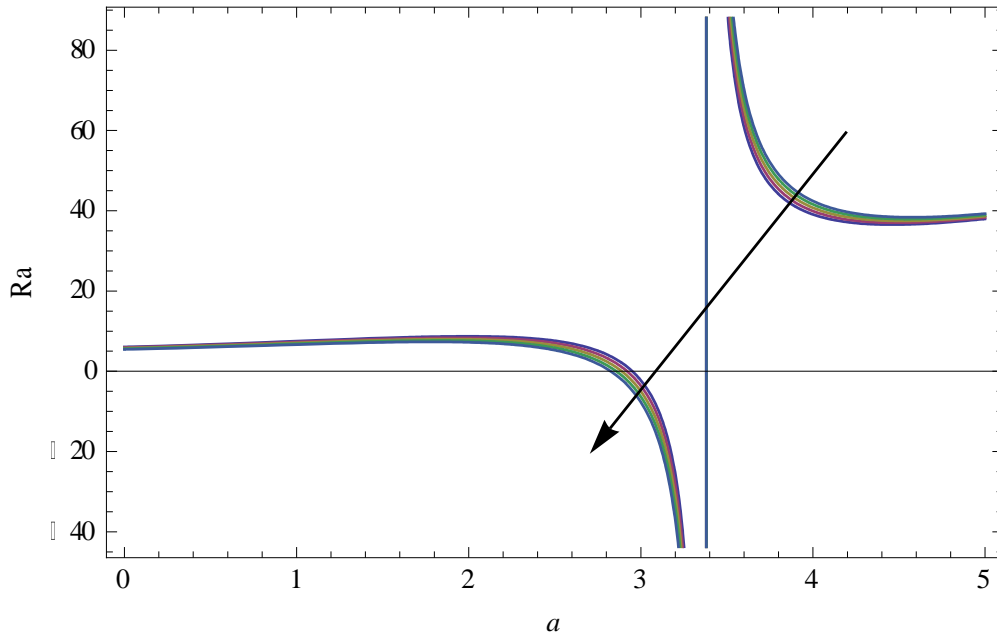
**Figure 2.** Rayleigh number Ra against wave number (a) for varying Radiation number.



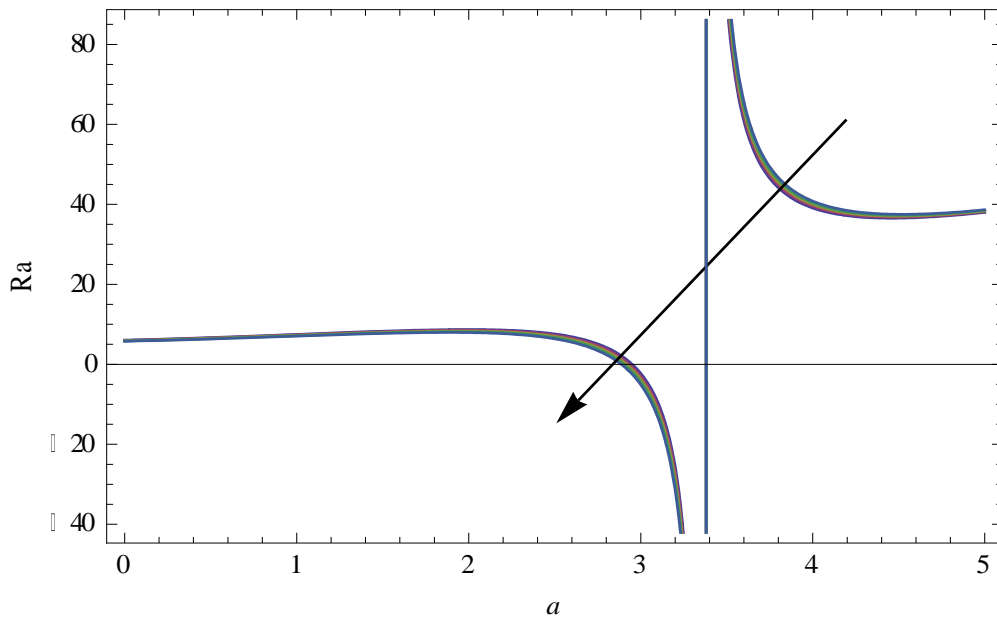
$Q = -1.5, -3.0, -4.5, -6.0, -7.5$ . For  $Q \geq 1.5$   
**Figure 3.** Rayleigh number Ra against wave number (a) for varying heat function Q.



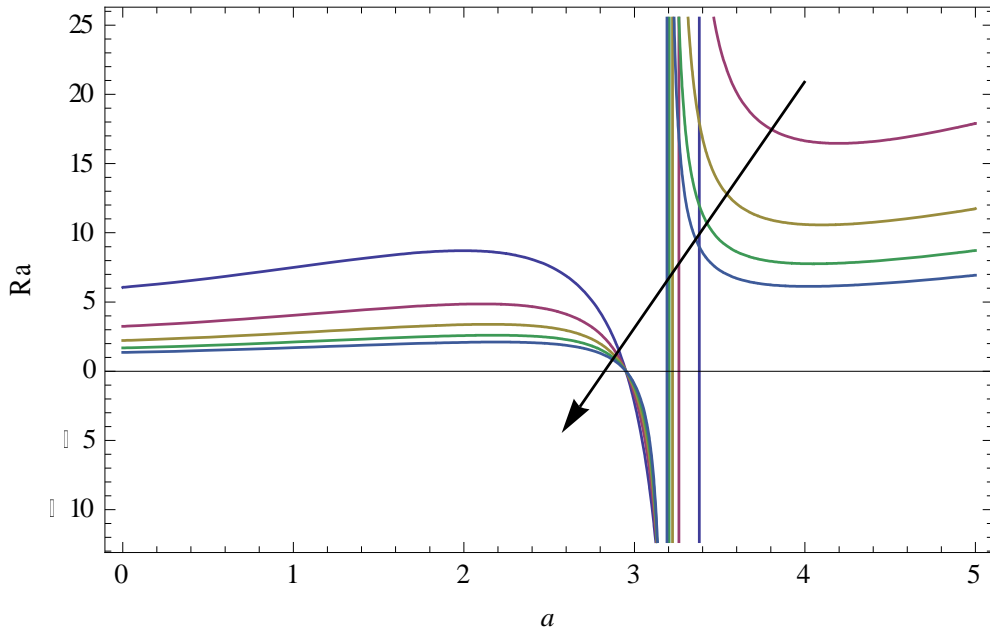
$k = 0.95, 1.90, 2.85, 3.80, 4.75$ . For  $Q \geq 1.5$   
**Figure 4.** Rayleigh number Ra against wave number (a) for varying Micropolar parameter K.



$\sigma = 1.91, 2.91, 3.91, 4.91, 5.91$ . For  $Q \geq 1.5$   
**Figure 5.** Rayleigh number  $Ra$  against wave number ( $a$ ) for varying Electroconductivity parameter  $\sigma$ .

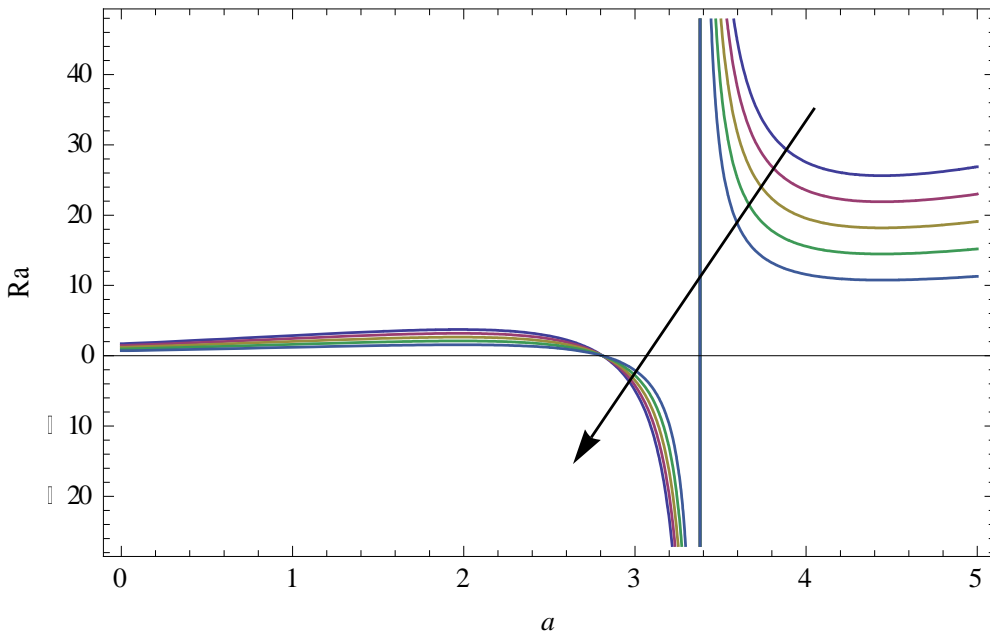


$M = 0.52, 1.04, 1.56, 2.08, 2.60$ . For  $Q \geq 1.5$   
**Figure 6.** Rayleigh number  $Ra$  against wave number ( $a$ ) for varying Magnetic parameter  $M$ .



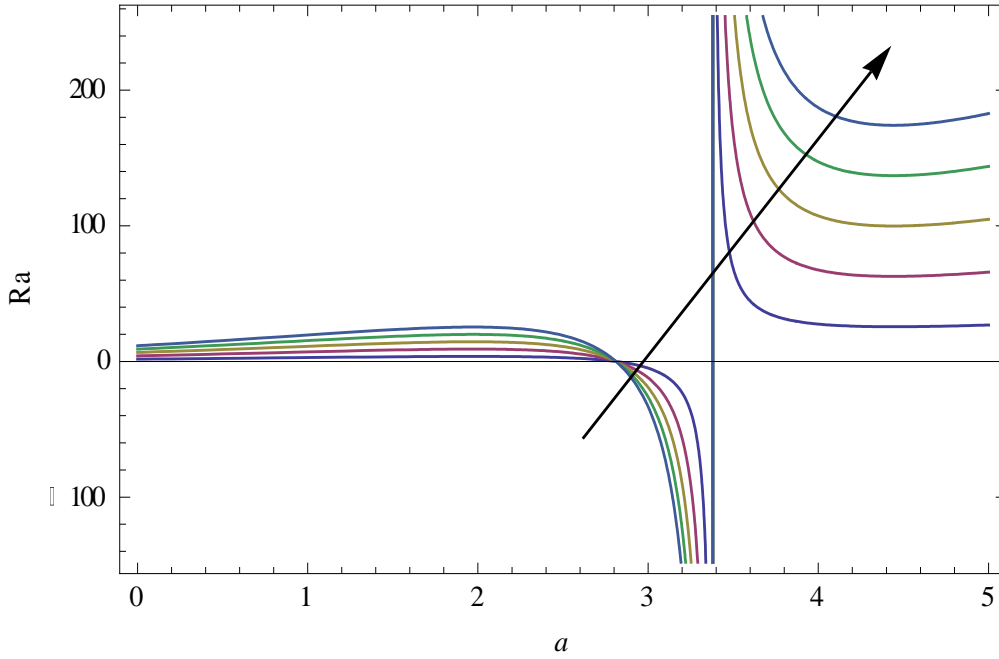
$\lambda = 1.25, 2.46, 3.69, 4.92, 6.15$ . For  $Q \geq 1.5$

**Figure 7.** Rayleigh number  $Ra$  against wave number ( $a$ ) for varying Microrotational density parameter  $\lambda$ .



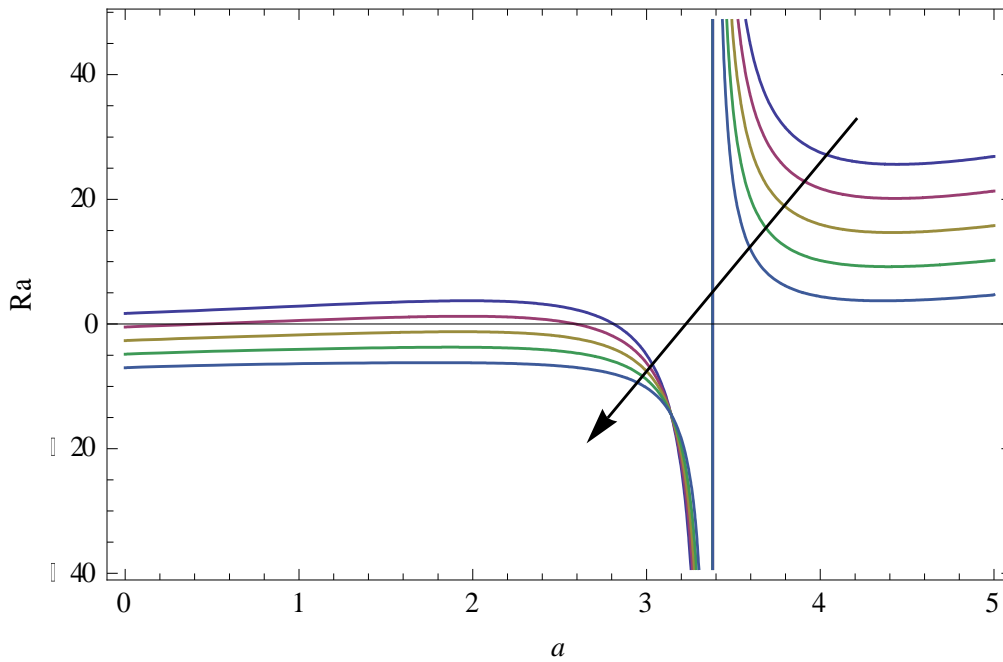
$Pr = 0.31, 0.41, 0.51, 0.61, 0.71$ . For  $Q \leq 1.5$

**Figure 8.** Rayleigh number  $Ra$  against wave number ( $a$ ) for varying Prandtl number  $Pr$ .



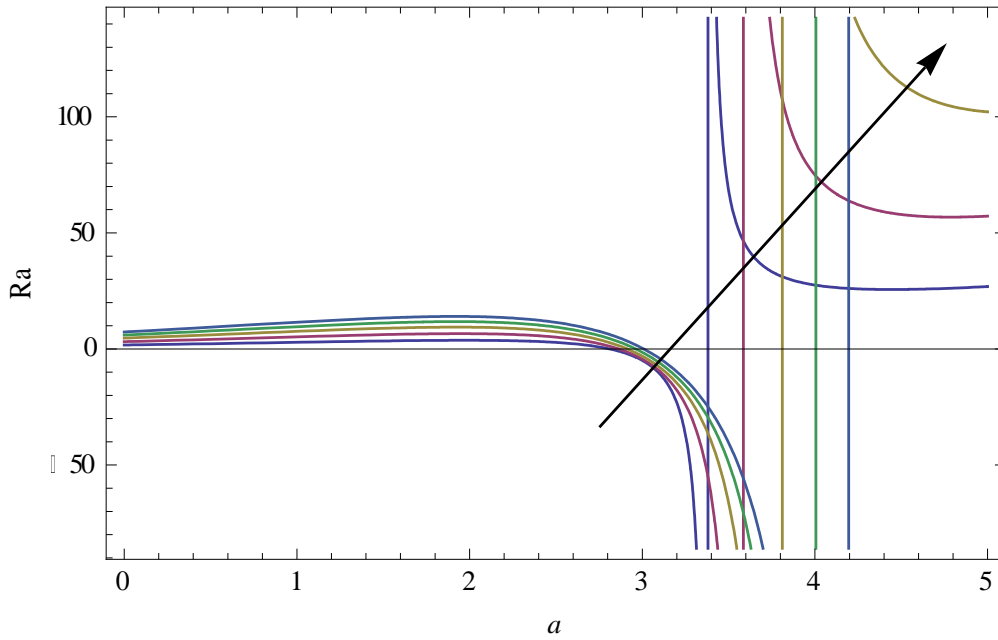
$R = 1.0, 2.0, 3.0, 4.0, 5.0$ . For  $Q \leq 1.5$

**Figure 9.** Rayleigh number Ra against wave number (a) for varying Radiation number.

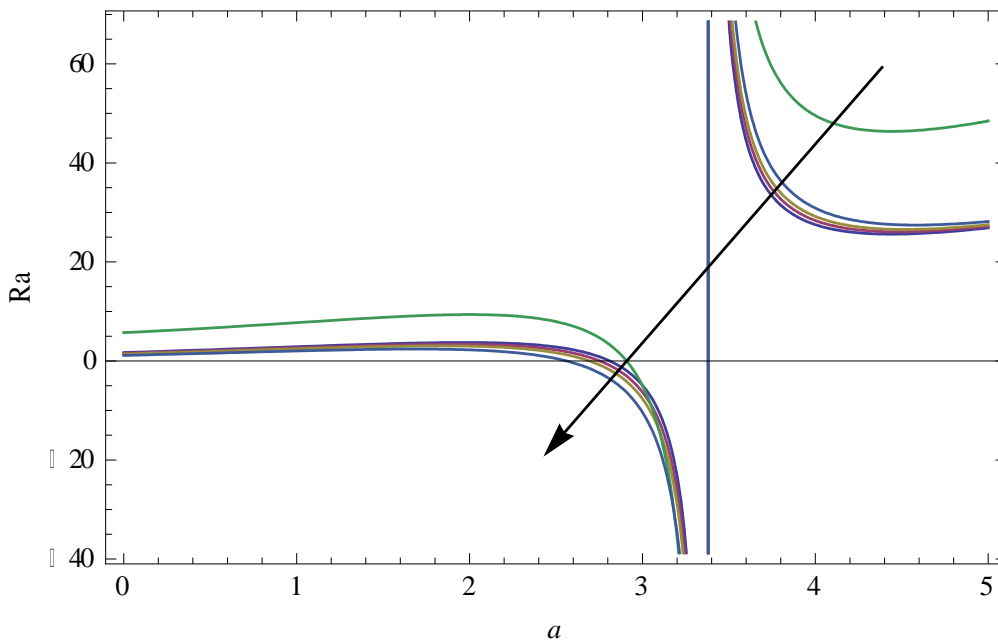


$Q = -1.5, -3.0, -4.5, -6.0, -7.5$ . For  $Q \leq 1.5$

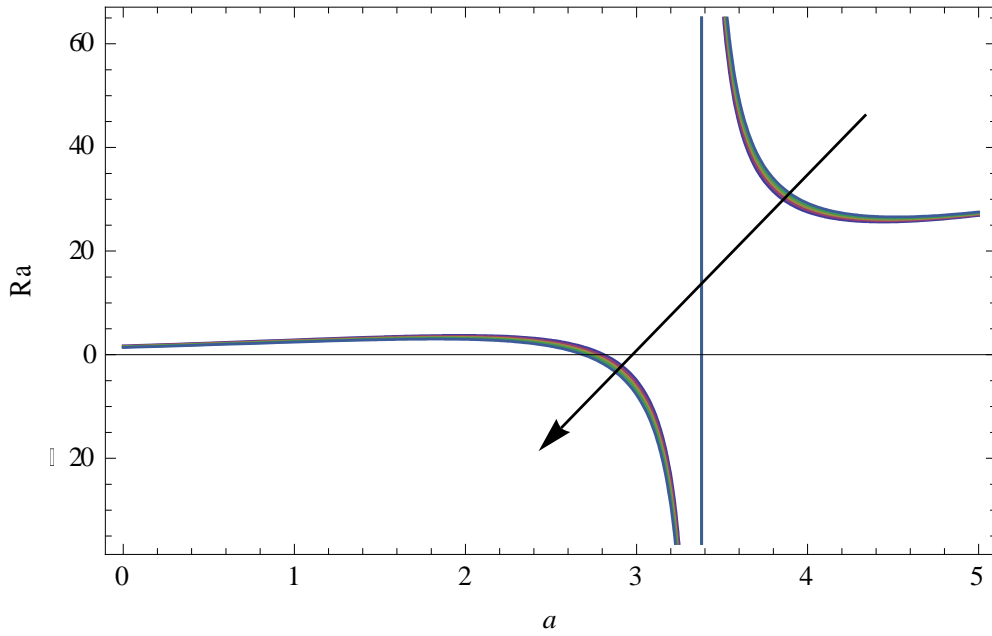
**Figure 10.** Rayleigh number Ra against wave number (a) for varying heat function Q.



$k = 0.95, 1.90, 2.85, 3.80, 4.75$ . For  $Q \leq 1.5$   
**Figure 11.** Rayleigh number  $Ra$  against wave number ( $a$ ) for varying Micropolar parameter  $K$ .

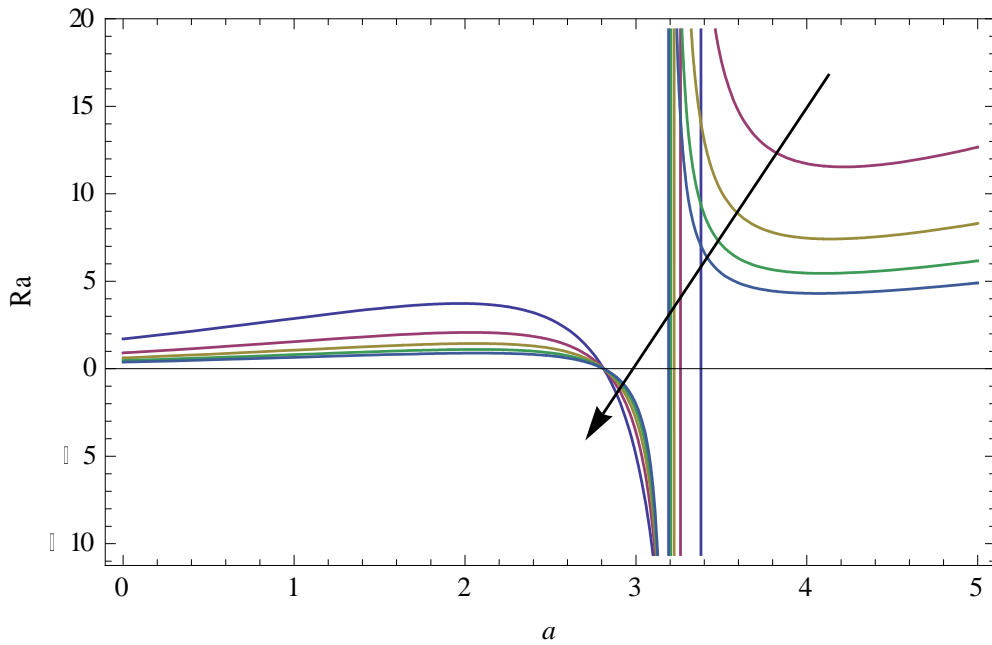


$\sigma = 1.91, 2.91, 3.91, 4.91, 5.91$ . For  $Q \leq 1.5$   
**Figure 12.** Rayleigh number  $Ra$  against wave number ( $a$ ) for varying Electroconductivity parameter  $\sigma$ .



$M = 0.52, 1.04, 1.56, 2.08, 2.60$ . For  $Q \leq 1.5$

**Figure 13.** Rayleigh number Ra against wave number (a) for varying Magnetic parameter  $M$ .



$\lambda = 1.25, 2.46, 3.69, 4.92, 6.15$ . For  $Q \leq 1.5$

**Figure 14.** Rayleigh number Ra against wave number (a) for varying Microrotational density parameter  $\lambda$ .



#### **4. DISCUSSION OF RESULTS**

From Fig. 1. We observed that at a critical wave number ( $a \geq 3.4$ ), increase in the Prandtl number for the heat source, results in an early onset of instability of the fluid flow, which corroborates the findings of Bhattacharyy and Jena [2].

Also, Fig. 2 shows clearly that at a critical wave number ( $a \geq 3.4$ ), increase in the radiation number for the heat source hastens the onset of instability. We see also in Fig. 3 that at a critical wave number of ( $a \geq 3.4$ ), increase in the heat source results in early onset of instability of fluid flow. In Fig. 4 we noticed that as the micropolar parameter  $K$  is increased there is a late onset of instability which becomes more pronounced as the micropolar parameter is increased, this negates the finding of Dhiman et al., [8]. From Fig. 5 and Fig. 6 we observed that at a critical wave number of ( $a \geq 3.4$ ), increase in the electroconductivity and that of the magnetic field term for the heat source leads to a late onset of instability in the fluid flow.

From Fig. 7 when the microrotational parameter  $\lambda$  is increase for the heat source there is a draw-back in the onset of the instability which becomes more pronounced when the microrotational parameter is increased. In Fig. 8 we noticed that at a critical wave number ( $a \geq 3.4$ ), increase in the Prandl number for heat sink hasten the instability of the fluid.

Fig. 9 shows clearly that at a critical wave number ( $a \geq 3.4$ ), increase in the radiation number  $R$  for heat sink result in early onset of instability of the fluid. Fig 10 depicts that at a critical wave number ( $a \geq 3.4$ ), increase in heat sink results in early onset of instability of the fluid. Fig. 11 shows clearly that as the micropolar parameter for the heat sink is increased there is a late onset of instability which becomes pronounced when the micropolar parameter is increased. From Fig. 12 and Fig. 13 indicates that at a critical value of the wave number given as ( $a \geq 3.4$ ), the magnetic term and that of the electroconductivity of the heat sink are increased there is a late onset of instability on the fluid. Also, Fig. 14 show that when the microrotational parameter  $\lambda$  for the heat sink increases there is a late onset of instability which becomes pronounced when the microrotational parameter is increased.

#### **5. CONCLUSIONS**

In this study the stability effect of magnetic field on micropolar fluid flow in a channel under the influence of radiation and heat function was investigated. The solution was obtained by the method of perturbation. The effect of the various material parameter on the stability of the micropolar fluid flow is observed through the graphs. The following are the observations from the study.

- The Prandtl  $Pr$  parameter for both the heat source and heat sink at a critical number of ( $a \geq 3.4$ ) hasten the instability of the fluid flow.
- There is a late onset of instability when micropolar parameter  $k$  and the microrotational parameter  $\lambda$  are increased.
- The magnetic field  $M$  term and the electroconductivity term  $\sigma$ , delays the onset of the stability, which becomes pronounced when their parameters are increased for the heat function.
- The wave number ( $a$ ) when increased leads to an early onset of instability in the fluid.

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