Polytropic Tachyon Scalar Field Model in 5-dimensional Universe

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ABSTRACT

Recent astrophysical observations show that the universe has entered a late-time accelerated expansion period. It is commonly accepted that the dark energy is responsible for this enigmatic behavior of our universe, because it makes up about 70 percent of the content of our universe and dominates over the matter part. Therefore, the dark energy will govern the speedy enlargement and determine the fate of our universe. Another interpretation of the dark energy is that this is a new kind of dynamical field or fluid, something mysterious which fills all of space but something whose influence on the accelerated enlargement nature of our universe is the opposite of that of baryonic matter and ordinary energy. Here, we focus on the polytropic gas unified dark matter-energy model in a 5-dimensional cosmology to reconstruct tachyonic scalar field proposal of the dark energy. It is known that the background dynamics of the polytropic gas is equivalent to that one for the dark energy interacting with the dark matter. On the other hand, using scalar field is one of the most significant methods helping us to understand the dark universe. There are many scalar field proposals, but generally it is hard to get exact relations for the scalar field function and its self-interacting potential. In this work, making use of the 5-dimensional Polytropic gas model, we calculate exact relations for the tachyonic scalar field. After that, we discuss our theoretical calculations graphically in order to interpret our theoretical results in a different way.

Keywords: Extra dimension, cosmology, scalar field, dark energy
1. INTRODUCTION

The current speedy expansion phase of our universe has been proved by many observational astrophysics data [1-6]. An exotic content which is responsible for this mysterious nature of the universe usually dubbed “dark energy”. It is worth noting that numerous theoretical candidates have been introduced to identify the dark energy, but its dynamical evolution still remains in the dark (for brief see [7-9] and references therein). Einstein’s cosmological constant idea [10], scalar fields models [9, 11-14], unified density prescriptions [15-18], modified theories of gravity [19-25] and even considering the existence of extra dimensions [26-30] are possible theoretical ideas given to interpret nature of the dark energy. Li and his/her collaborators [31] and Cai and his/her collaborators [32] prepared very useful briefs about the dark nature of our universe including a survey of some theoretical proposals.

Einstein’s a tiny positive time-independent cosmological constant model is the primordial description of the dark energy with the equation-of-state (EoS) $\omega = -1$ [7-8, 33-34]. For a cosmological framework including a minimally coupled scalar field, it was concluded [35-37] that value of the effective EoS parameter cannot cross the phantom divide line ($\omega = -1$). On the other hand, Vikman [38] showed that there is no possible transition from $\omega < -1$ to $\omega > -1$ (or the vice versa case) of dark energy defined by making use of a general scalar field Lagrangian density $\mathcal{L}(\phi, \partial_{\mu} \phi)$. Consequently, it should be supposed that the dark energy is identified by a nonlinear Lagrangian density including in the term $(\partial_{\mu} \phi)^2$ in order to interpret the transition under the shadow of minimal conjectures of non-kinetic interaction between the dark energy and other contents. Scalar field proposal with non-linear kinetic terms [39-43] or a non-linear form with higher derivative terms [38], brane world proposals [44-45], string-inspired ideas [46], modified theories of gravity and non-minimally coupled scalar field prescriptions have also been introduced to realize the phantom divide line crossing.

In the absence of a useful theoretical interpretation of proposals involving a self-interacting potential, an interesting approach which is known as the “reverse engineering” method and termed as the reconstruction of the self-interacting potential can be performed. After focusing on a given enlargement history, a potential reproducing the current observational evolution can be reconstructed. Ellis and Madsen [47] and Starobinsky [48] made the pioneering works in this direction. Later, Huterer and Turner [49-50] studied the reconstruction of dark energy’s EoS by considering cosmological distance observation. Additionally, Rubano and his/her collaborator Barrow [51] obtained an exact relation of the scalar field for a two fluid models.

In this work, we focus on dynamics of tachyon scalar field dark energy (TSFDE) definition in the 5-dimensional (5D) polytropic gas cosmology. Thus, the present study is an interesting attempt to redefine the tachyonic self-interacting potential from 5D form of the polytropic dark energy (PDE) description. An outline of the present work is as follows: In the next section, we give some preliminary relations briefly in order to introduce theoretical materials and method of our investigation. Then, in the third section, we derive exact expressions for the scalar field function and self-interacting potential describing the TSFDE model. Then, in the fourth section, we discuss our theoretical results obtained in the third section by performing a graphical analysis. Finally, the fifth section is devoted to the eventual conclusions.
2. MATERIALS AND METHODS

In this section of the work, we give some preliminary relations in order to introduce theoretical materials which will be used in further calculations. On this purpose, we discuss main features of the 5-dimensional PDE proposal briefly.

The PDE model is defined by the following EoS parameter [16]:

\[ p = \kappa \rho^{1 + \frac{1}{m}}, \]  

where: \( \kappa > 0 \) and \( m \neq 0 \) denote real constants, \( p \) is pressure and \( \rho \) indicates energy density. Moreover, here, \( m \) is also called as the polytropic index. Here, we can define two significant limiting conditions such as the original Chaplygin gas (OCG) proposal [15] and the generalized Chaplygin gas (GCG) model [52-54]:

- \[ \lim_{m \to -\frac{1}{2}} \kappa \rho^{1 + \frac{1}{m}} = -\frac{B}{\rho} \Rightarrow \text{the OCG model}, \]
- \[ \lim_{m \to \frac{1}{1+\alpha}} \kappa \rho^{1 + \frac{1}{m}} = -\frac{B}{\rho^\alpha} \Rightarrow \text{the GCG description}. \]

On the other hand, 5D Friedman-Robertson-Walker (FRW) universe is given by the following line-element [55]

\[ ds^2 = dt^2 - a^2(t) \left[ \frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) + (1-kr^2)dy^2 \right]. \]  

Next, according to Einstein’s theory of gravity, the general relativistic field equation is written as below

\[ R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G \left[ (\rho + p)u_\mu u_\nu - g_{\mu\nu}p \right]. \]  

where: \( R_{\mu\nu}, g_{\mu\nu}, R, G \) and \( u_\mu \) show the Ricci tensor, metric tensor, Ricci curvature scalar, gravitational constant and the four-velocity vector, respectively. Making use of the field equation (3) with the line-element (2) yields the following Friedmann equations:

\[ H^2 = \frac{4\pi G}{3} \rho, \]

\[ 2H^2 + \dot{H} = -\frac{8\pi G}{3} p. \]

where: \( H = \frac{\dot{a}}{a} \) is the cosmic Hubble parameter.

Subsequently, considering the conservation relation \( T_{\mu\nu;\nu} = 0 \), one can find the following result:
\[ \dot{\rho} + 4H(\rho + p) = 0. \]  \hspace{1cm} (6) 

The above equation can be rewritten in a more useful form:

\[ d(\rho a^4) + pd(a^4) = 0. \]  \hspace{1cm} (7) 

Now, substituting the relation (1) into the above result yields [56]

\[ \rho = \frac{1}{m} \sqrt{(c - \kappa)(1 - \xi + \frac{4}{\xi} a^m)}, \]  \hspace{1cm} (8) 

where: \( \xi = \frac{c}{c - \kappa}, \)  \hspace{1cm} (9) 

with the integration constant \( c = \kappa - \frac{m}{\sqrt{1.3}}. \)

At this step, the corresponding EoS parameter can be computed as [56]:

\[ \omega = \frac{p}{\rho} \approx -1 + \frac{\xi}{(1 - \xi)^2} \frac{4}{a^m}. \]  \hspace{1cm} (10) 

Now, we are in a position to mention about the method we perform in the next section. We start with discussions of the energy density and pressure relations of the tachyonic TSFDE model in order to understand main properties of the model. Making use of the corresponding energy density and pressure relations, it is easy to get a relation for the tachyonic EoS parameter. After defining this cosmological parameter, we equate it with the polytropic 5D EoS parameter and solve corresponding equations in order to find exact relations of the tachyonic scalar field function and self-interacting potential.

3. RESULTS

The TSFDE model is defined as one of the possible theoretical dark energy descriptions. It includes the following energy density and pressure definitions [9]

\[ \rho_T = \frac{V(\phi)}{\sqrt{1 - \phi^2}}, \]  \hspace{1cm} (11) 

\[ p_T = -V(\phi) \sqrt{1 - \phi^2}. \]  \hspace{1cm} (12) 

This significant model is described by an interesting EoS parameter that is taking values between -1 and 0 [57-58]. Consequently, the TSFDE proposal can be used as a source of dark energy as well as one of the possible theoretical descriptions explaining the inflation phase at high energy level [59-60]. The corresponding EoS parameter of the TSFDE model is written as below

\[ \omega = \frac{p_T}{\rho_T} = \dot{\phi}^2 - 1. \]  \hspace{1cm} (13)
Now, we are in a position to implement a correspondence between the PDE and the TSFDE models. Correspondence between these theoretical dark energy definitions can be obtained by assuming \( \rho_T = \rho, \ p_T = p \) and \( \omega_T = \omega \). Thus, the set of these assumptions leads to the following conclusions

\[
\dot{\varphi}^2 = 1 + \omega_T = 1 + \omega. \tag{14}
\]

\[
V(\varphi) = \rho_T \sqrt{1 - \dot{\varphi}^2} = \rho \sqrt{1 - \omega}. \tag{15}
\]

Here, it is seen that kinetic term \( \dot{\varphi}^2 \) and tachyon potential \( V(\varphi) \) may exist if it is found that \(-1 \leq \omega \leq 0\). This conclusion shows that the phantom line cannot be crossed with a speedy expansion. Making use of the relations (8) and (10) in the above results, it can be found that

\[
\varphi = \varphi_0 + \frac{\sqrt{\xi}}{|1 - \xi|} \int_{t_0}^t a^m dt, \tag{16}
\]

\[
V(\varphi) = \left( c - \kappa \right) \frac{1}{|1 - \xi|} \sqrt{-\xi a^m} \frac{1}{\sqrt{1 - \xi}} \sqrt{1 - \xi + \xi a^m}. \tag{17}
\]

Form the equation (4), it can be calculated that

\[
H = \sqrt{\frac{4\pi G}{3}} \rho = \sqrt{\frac{4\pi G}{3}} \left( ca^m - \kappa \right)^{\frac{m}{2}}. \tag{18}
\]

In the equation (16), we can write \( dt = \frac{1}{a} da \) to solve the integral. Hence, performing required mathematical steps leads to

\[
\varphi = \varphi_0 + \frac{\sqrt{\xi}}{|1 - \xi|} \int_{a_0}^a \frac{a^m - 1}{H} da. \tag{19}
\]

Substituting the relations (18) in to the above result, one can solve the corresponding integral.

Thus, it can be found that

\[
\varphi = \frac{m\sqrt{\xi} a^m}{2|1 - \xi|} \left[ -\kappa \right]^{\frac{m}{2}} \left[ \frac{1}{2} - \frac{m}{2}, \frac{3}{2}, ca^m \right]. \tag{20}
\]

with \( a_0 = \varphi_0 = 0 \). Here, \( _2F_1 \) is the the Kummer Confluent Hypergeometric function of the second kind which is given by

\[
_2F_1[A, B, C; x] = 1 + \frac{AB}{x} + \frac{A(A+1)B(B+1)}{C(C+1)} \frac{x^2}{2!} + \cdots = \sum_{i=1}^{\infty} \frac{A_i B_i x^i}{C_i \cdot i!}. \tag{21}
\]

with \( C_i \neq 0, -1, -2, -3, \ldots \) and \( |x| < 1 \).
Making use of the red shift parameter $z$ in cosmological calculations helps us to fix the auxiliary parameters given a model. It is known that $z = \frac{1}{a} - 1$. In an expanding space-time model such as the one we have experienced, the cosmic scale factor increases monotonically while time passes, therefore the cosmic red shift parameter takes positive values which means that distant galaxies appear red shifted. That’s why the red shift parameter plays important role for the astrophysical interpretations.

So, we can write

$$\varphi = \frac{m\sqrt{\xi(1+z)^{-\frac{2}{m}}} \left[ -\kappa \right]^{\frac{m}{2}}}{2[1-\xi]} \left[ \frac{1}{2}, -\frac{m}{2}, \frac{3}{2}, -\frac{\kappa}{c(1+z)^{-\frac{4}{m}}} \right] F_1 \left( \frac{1}{2}, -\frac{m}{2}, \frac{3}{2}, -\frac{\kappa}{c(1+z)^{-\frac{4}{m}}} \right), \quad (22)$$

$$V(\varphi) = \frac{(c-\kappa)^{-\frac{1}{m}}}{(1-\xi)} \sqrt{-\xi(1+z)^{-\frac{2}{m}} \left[ 1 - \xi + \xi(1+z)^{-\frac{4}{m}} \right]}. \quad (23)$$

4. DISCUSSIONS

In the previous section, we compute exact relations of the polytropic 5-dimensional tachyon scalar field dark energy model. As we mentioned before, there are many scalar field descriptions given in literature, but it is so hard to get exact formulations of the corresponding scalar field function and its self-interacting potential. On the other hand, there is no definite reason to select one of them as capable of the current observational data. That’s why our calculations may be useful for the further investigations.

![Figure 1. Relation between the scalar field function $\varphi$ and the cosmic red shift parameter $z$.](image-url)
Figure 2. The $V(\phi)\sim z$ relation.

Here, we want to discuss our results in a numerical way. In Figures 1 and 2, we illustrate the evolution of the polytropic tachyon scalar field function and its self-interacting potential as a function of the cosmic red shift parameter. Here, auxiliary parameters are chosen [56] as $\kappa = -0.5$, $m = -2.4$ and $\sqrt{4\pi G} = \sqrt{3}$. Due to negative values of the auxiliary parameters, we have an imaginary scalar field function. On the other hand, for the self-interacting potential, same values of the free parameters lead a different conclusion.

5. CONCLUSIONS

After focusing on the set including higher dimensional form of the PDE description and the TSFDE proposal, it is very attractive to discuss how the PDE density is taken into account to describe the higher dimensional tachyon field. Considering such higher dimensional framework, we have constructed a correspondence between the tachyonic scalar field idea and the PDE scenario. Making use of the higher dimensional formulation of the TSFDE description as an effective model of the PDE definition, the selected proposal should be considered to mimic the dynamical behavior of the polytropic energy and reconstruct the description of scalar field function using that evolutionary property. Thus, with the help of this motivation, we have calculated useful original descriptions for the tachyonic self-interacting potential and the corresponding scalar field function with the help of higher dimensional form of the polytropic model. Mainly, we have reconstructed the dynamics of higher dimensional tachyon field by working with the evolutionary nature of the PDE density.
Moreover, we want to emphasize here that the presented conclusions can be extended easily to other famous scalar fields formulations such as the quintessence, dilaton k-essence and the quintom. Also, one can generalize the aforementioned results presented in this study to the non-flat form of higher dimensional FRW type universe.

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References


