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Reliability, MTTF and Cost Analysis of a Bio Ethanol Plant by Orthogonal Matrix Method

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ABSTRACT

The development in the science, technology and modernization in lifestyle has raised many issues towards environment safety. One of the major issues is of air pollution due to excess use of Petrol and diesel in our daily use vehicles like two wheelers and four wheelers. So the importance of eco-friendly fuel is the necessity of time. Keeping this in mind in our proposed work, we have discussed production of popular green fuel (additive) called Bio-ethanol. The working of bio ethanol production plant under consideration has been divided in various sections on the basis of operational preferences the components. A mathematical model of plant has been developed and computations for various reliability parameters such as Reliability, MTTF and Cost etc. have been carried out. In this paper orthogonal matrix method (OMM) is used to solve the model and at last some numerical examples are taken to study the effect of these reliability parameters over the working of the system under consideration.

***Keywords*:** Bio ethanol, Reliability, MTTF, Orthogonal Matrix Method

1. INTRODUCTION

Ethanol is a clean burning, high-octane motor fuel that is produced by the fermentation of plants containing sugar. It reduces greenhouse house gas emissions by as much as 90%. Ethanol has been produced since pre-historic times, mainly used in alcoholic beverages.

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However, in the transport industry ethanol is either used as a vehicle fuel by itself, blended with petrol, or as a petrol octane enhancer and oxygenate. This blending was introduced to alleviate fuel shortages, to reduce environmental pollution and high petrol prices. Ethanol blends are the amount of ethanol percentage mixed with petrol i. e. E10 is 10% ethanol blended with 90% petrol. The most common blends are E10, E20, E50 and E85. The 'E' indicates that the fuel contains ethanol.

Bio-ethanol can be produced from any crop that contains fermentable sugars, which includes sugar cane, sugar beets and unused portion of other crops such that fruit waste. sugar beets have a composition of 16% to 18% sucrose, as compared to a 10% to 11% composition of sugarcane. Basically, the same amount of sugar beets will produce more sugar than the same amount of sugarcane. Tropical sugar beet (Beta vulgaris) is a biennial sugar producing tuber crop, grown in temperate countries. Now tropical sugar beet hybrids are gaining momentum in tropical and sub-tropical countries including Brazil, USA etc. India as a promising energy crop and alternative raw materials for the production of ethanol. Apart from sugar production, the value added products like ethanol can also be extracted from sugar beet. The ethanol can be blended with petrol or diesel to the extent of 10% and used as bio-fuel. The sugar beet waste material viz., beet top used as green fodder, beet pulp used as cattle feed and filter cake from industry used as organic manure.

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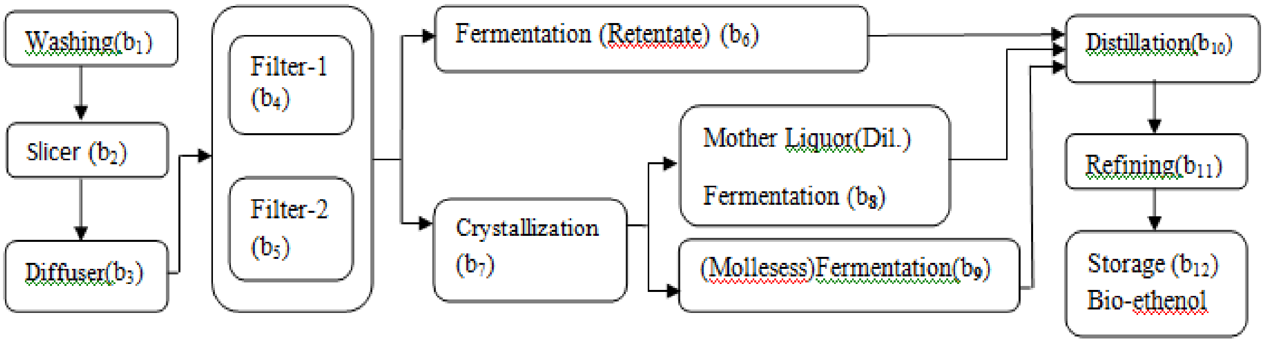
In last decade various researcher has reported their work and find out various reliability parameters. There are so many techniques available to solve mathematical model such as supplementary variable technique (SVT), Regenerative point technique (RPT) and Orthogonal Matrix Method (OMM) etc. Gupta P. P. and Agarwal S C [1] in their paper take the problem of three state repairable systems and find cost function for two types of failure. [3] In their paper study a standby system with delay in repair concept and use supplementary variable technique to solve the model, [6] also make use of supplementary variable technique to solve the model. [4] In their paper proposed a fuzzy logic based approach for obtaining value of component based system reliability. [11] proposed an algorithm to solve semi - markov process (SMP) and asses reliability of a phase mission system with non-exponential distributions. Other authors also submit their work and obtain expression for various reliability parameters using BFT, ANN and genetic algorithm concepts etc. Keeping all these factors in view we take a realistic model of Bio ethanol production plant and using OMM derived the expressions for Reliability, MTTF and Cost function. Also to study the effects of various failure patterns of components of the system, two cases of exponential and weibull distribution are included in this paper. At last numerical computations are performed and graphs for each Reliability, MTTF and Cost function are plotted.

2. MODEL DESCRIPTION

There are 12 components denoted by b_k, where k = 1, 2, 3, ….. 12, are configured in series-parallel configuration, in the bio-ethanol plant under consideration. Firstly the raw sugar beets are washed at b_1 to separate the soil, stone and other impurities, then sent to Slicer b_2 , where sugar beets are sliced into cassettes and forwarded to diffuser b_3, here raw juice extracted from cassettes and biogases is separated which is another useful by-product. Raw juice contains 15-20% of dry non-sugar solids. Now the sugar rich raw juice filtered using Nano-Filteration process. To avoid total breakdown due to failure in filters section which consists of two filtering units b_4, b_5 in redundant capacity. From filter section, Retentate sent for fermentation at b_6 and permeate is crystallized at b_7. During crystallization, sugar crystals, Mother liquor and Molasses are separated. Now Mother liquor and Molasses both sent for fermentation at b_8 , b_9 respectively. After fermentation from these three sections bio-ethanol is produced with some water and other volatile content therefore it is sent for distillation at b_{10}. Now bio-ethanol is refined at b_{11} and stored in storage tank at b_{12} . Finally bio-ethanol is ready to supply, as a green fuel, in market.

State configuration of Bio-ethanol plant under consideration is given below:



3. PRESUMPTIONS

The determination of reliability of model described in this paper, using Orthogonal Matrix Method, the following assumptions are made:

1. All the components/subsystems are in good and operable condition, initially.
2. The Failure times are arbitrary for all the components.
3. The state of all components is statistically independent, in the system.
4. In the system, each of the component is either in good or in failed state.
5. Reliability of every component or stage is known in advance.
6. The repair facility for any failed component is not available.
7. The whole system will fail if there is failure in any one of its units.

4. MATHEMATICAL NOTATIONS

b_1	: States of cleaning/washing unit
b_2	: States of slicing unit
b_3	: States of Diffuser
b_4, b_5	: States of Nano-Filtering units

b_6 : States of Fermentation (Retentate) unit

b_7 : State of Crystallization unit

b_8 : State of Mother Liquor (Dil.) Fermentation unit

b_9 : State of (Molasses) Fermentation unit

b_{10} : State of Distillation unit

b_{11} : State of Refining unit

b_{12} : State of storage unit.

b_k : State of components which is equal to 1/0 in good/failed state for all k =1, 2, 3, … , 12.

b_k' : Negation of b_k

$\wedge$: Conjunction

$|\ |$: Logical Matrix

R_k : Reliability of k^{th} component of the system.

Q_k : $1-R_k, \forall k = 1,2,3,.....12$

R_s **:** Reliability of the whole system.

$R_{SW}\,/\,R_{SE}$ **:** Reliability of k^{th} component of the system.

C_1 : Revenue cost per unit time

C_2 : Service cost per unit time

5. FORMULATION OF MATHEMATICAL MODEL

Incorporating Orthogonal Matrix Method, The possible successful operating conditions of juice plant in terms of logical matrix are:

$$F(b_1,b_2,b_3,.....b_{12}) = \begin{bmatrix} b_1 & b_2 & b_3 & b_4 & b_6 & b_{10} & b_{11} & b_{12} & \\ b_1 & b_2 & b_3 & b_4 & b_7 & b_8 & b_{10} & b_{11} & b_{12} \\ b_1 & b_2 & b_3 & b_4 & b_7 & b_9 & b_{10} & b_{11} & b_{12} \\ b_1 & b_2 & b_3 & b_5 & b_6 & b_{10} & b_{11} & b_{12} & \\ b_1 & b_2 & b_3 & b_5 & b_7 & b_8 & b_{10} & b_{11} & b_{12} \\ b_1 & b_2 & b_3 & b_5 & b_7 & b_9 & b_{10} & b_{11} & b_{12} \end{bmatrix} \qquad \ldots\ldots (1)$$

6. SOLUTION OF MODEL

By means of algebra of logics, Equation (1) can be expressed as:

$$F(b_1,b_2,b_3,......b_{12}) = b_1 \wedge b_2 \wedge b_3 \wedge b_{10} \wedge b_{11} \wedge b_{12} \wedge f(b_1,b_2,b_3,......b_{12}) \quad \ldots\ldots\ldots\ldots (2)$$

where,

$$f(b_1, b_2, b_3, \ldots\ldots b_{12}) = \begin{bmatrix} b_4 & b_6 & \\ b_4 & b_7 & b_8 \\ b_4 & b_7 & b_9 \\ b_5 & b_6 & \\ b_5 & b_7 & b_8 \\ b_5 & b_7 & b_9 \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \\ B_6 \end{bmatrix} \qquad \ldots\ldots (3)$$

where

$B_1 = |b_4 \;\; b_6|$
$B_2 = |b_4 \;\; b_7 \;\; b_8|$
$B_3 = |b_4 \;\; b_7 \;\; b_9|$
$B_4 = |\; b_5 \;\; b_6|$
$B_5 = |b_5 \;\; b_7 \;\; b_8|$
$B_6 = |b_5 \;\; b_7 \;\; b_9|$

By orthogonalization algorithm, we may write (3) as

$$f(b_1, b_2, b_3, \ldots\ldots\ldots b_{12}) = \begin{vmatrix} B_1 \\ B_1' \; B_2 \\ B_1' \; B_2' \; B_3 \\ B_1' \; B_2' \; B_3' \; B_4 \\ B_1' \; B_2' \; B_3' \; B_4' \; B_5 \\ B_1' \; B_2' \; B_3' \; B_4' \; B_5' \; B_6 \end{vmatrix} \qquad \ldots\ldots (4)$$

Now

$B_1 = |b_4 \;\; b_6|$
$B_1' \;\; B_2 = \begin{vmatrix} b_4' & \\ b_4 & b_6' \end{vmatrix} \wedge |b_4 \;\; b_7 \;\; b_8| = |b_4 \;\; b_6' \;\; b_7 \;\; b_8|$
$B_1' \;\; B_2' \;\; B_3 = |b_4 \;\; b_6' \;\; b_7 \;\; b_8' \;\; b_9|$
$B_1' \;\; B_2' \;\; B_3' \;\; B_4 = |\; b_4' \;\; b_5 \;\; b_6|$
$B_1' \;\; B_2' \;\; B_3' \;\; B_4' \;\; B_5 = |\; b_4' \;\; b_5 \;\; b_7 \;\; b_8 \;|$
$B_1' \;\; B_2' \;\; B_3' \;\; B_4' \;\; B_5' \;\; B_6 = |\; b_4' \;\; b_5 \;\; b_6' \;\; b_7 \;\; b_8' \;\; b_9|$

Putting all in (4), we get

$$f(b_1, b_2, b_3, \ldots\ldots\ldots b_{12}) = \begin{vmatrix} b_4 \; b_6 \\ b_4 \; b_6' \; b_7 \; b_8 \\ b_4 \; b_6' \; b_7 \; b_8' \; b_9 \\ b_4' \; b_5 \; b_6 \\ b_4' \; b_5 \; b_7 \; b_8 \\ b_4' \; b_5 \; b_6' \; b_7 \; b_8' \; b_9 \end{vmatrix} \qquad \ldots\ldots (5)$$

Therefore (2) becomes

$$F(b_1, b_2, b_3, \dots\dots\dots b_{12}) = \left| \begin{array}{llllllllllll} b_1 & b_2 & b_3 & b_4 & b_6 & b_{10} & b_{11} & b_{12} \\ b_1 & b_2 & b_3 & b_4 & b_6' & b_7 & b_8 & b_{10} & b_{11} & b_{12} \\ b_1 & b_2 & b_3 & b_4 & b_6' & b_7 & b_8' & b_9 & b_{10} & b_{11} & b_{12} \\ b_1 & b_2 & b_3 & b_4' & b_5 & b_6 & b_{10} & b_{11} & b_{12} \\ b_1 & b_2 & b_3 & b_4' & b_5 & b_7 & b_8 & b_{10} & b_{11} & b_{12} \\ b_1 & b_2 & b_3 & b_4' & b_5 & b_6' & b_7 & b_8' & b_9 & b_{10} & b_{11} & b_{12} \end{array} \right| \qquad (6)$$

This equation is the disjoint of disjoint conjunctions, therefore we can calculate the reliability of whole system which is given by

$$R_s = P_r\{F(b_1, b_2, b_3, \dots\dots\dots b_{12}) = 1\} \qquad \dots\dots\ (7)$$

$$R_s = R_1R_2R_3R_{10}R_{11}R_{12}\,\{R_4R_6 + R_4Q_6R_7R_8 + R_4Q_6R_7Q_8R_9 + Q_4R_5R_6 + Q_4R_5R_7R_8 + Q_4R_5Q_6R_7Q_8R_9\} \ \dots\dots\ (8)$$

where R_i is the reliability of i^{th} state of the system and $Q_i = 1 - R_i, \forall\, i = 1,2,3 \dots\dots 12$

$$\begin{aligned} R_s = R_1R_2R_3R_{10}R_{11}R_{12}\,\{ & R_4R_6 + R_4R_7R_8 - R_4R_6R_7R_8 + R_4R_7R_9 - R_4R_6R_7R_9 \\ & - R_4R_7R_8R_9 + R_4R_6R_7R_8R_9 + R_5R_6 - R_4R_5R_6 + R_5R_7R_8 - R_4R_5R_7R_8 \\ & + R_5R_7R_9 - R_4R_5R_7R_9 - R_5R_6R_7R_9 + R_4R_5R_6R_7R_9 - R_5R_7R_8R_9 \\ & + R_4R_5R_7R_8R_9 + R_5R_6R_7R_8R_9 - R_4R_5R_6R_7R_8R_9\} \end{aligned} \qquad \dots\dots\ (9)$$

$$\begin{aligned} R_s = x_1 + x_2 - x_3 + x_4 - x_5 - x_6 + x_7 - x_8 + x_9 - x_{10} + x_{11} - x_{12} - x_{13} + x_{14} - x_{15} \\ + x_{16} + x_{17} + x_{18} - x_{19} \end{aligned}$$

Case – (1) when reliability of each component is R, Then (9) gives:

$$R_s = 2R^8 + 4R^9 - R^9 - 7R^{10} + 4R^{11} - R^{12} \qquad \dots\dots\ (10)$$

Case – (2) When failure rates follow Weibull time distribution:

Let m_k is the rate of failure for kth state of system where k = 1, 2, 3, …… 12, then reliability of system at any time “t” is calculated by

$$R_{SW}(t) = \sum_{i\epsilon A} e^{-x_it^p} - \sum_{i\epsilon B} e^{-x_it^p} \qquad \dots\dots\ (11)$$

where p is a parameter (p > 0), A={1,2,4,7,9,11,14,16,17,18}, B={3,5,6,8,10,12,13,15,19} and

$x_1 = m_1 + m_2 + m_3 + m_4 + m_6 + m_{10} + m_{11} + m_{12}$, similarly $x_i = \sum m_k$, $i = 1,2,3 \dots 19, \textit{where k may be } 1,2,3 \dots 12.$

Case – (3) When failure rates follow Exponential time distribution:

Taking p =1 in case (2), it will become Exponential time distribution, then we have

$$R_{SE}(t) = \sum_{i\epsilon A} e^{-x_it} - \sum_{i\epsilon B} e^{-x_it} \qquad \dots\dots\ (12)$$

where x_i taken as in case (2).

In this case, the expression for another reliability parameter i.e. M.T.T.F. given by

$$\mathbf{M.T.T.F.} = \int_0^{\infty} R_{SE}(t)\,dt = \sum_{i\epsilon A}\left[\frac{1}{x_i}\right] - \sum_{i\epsilon B}\left[\frac{1}{x_i}\right] \qquad \ldots\ldots (13)$$

7. COST FUNCTION ANALYSIS

If C_1 and C_2 are the revenue and service cost per unit time, then expected cost is obtained by

$$C(t) = C_1 \int_0^t R_{SE}(t)\,dt - C_2 t$$

Or $C(t) = \frac{C_1}{m_k}\Big[\frac{1}{4}(1-e^{-8m_k t}) + \frac{4}{9}(1-e^{-9m_k t}) + \frac{4}{11}(1-e^{-11m_k t}) - \frac{1}{9}(1-e^{-9m_k t}) - \frac{7}{10}(1-e^{-10m_k t}) - \frac{1}{12}(1-e^{-12m_k t})\Big] - C_2 t$ …….(14)

8. NUMERICAL CALCULATIONS

For Numerical computation, let us setting the values as:

(i) $m_k = 0.001$, t = 0,1,2,…, p = 2 in equation (11) where k = 1,2,3…..12.
(ii) $m_k = 0.001$, t = 0,1,2,… in equation (12) where k = 1,2,3…..12.
(iii) $m_k = 0.0, 0.1, 0.2, \ldots 1.0$ in equation (13) where k = 1,2,3…..12.
(iv) $m_k = 0.001, 0.002, 0.003\ldots$, $C_1 = 1, C_2 = 0.1, 0.2, 0.3$, t = 0,1,2,… in equation (14)

where k = 1,2,3…..12. The computed results are given in Table-1 and Table-2 and graphically shown in fig.-2 and fig.-3 respectively.

Table 1. Reliability v/s Time.

m	t	R_{SW}	R_{SE}
0.001	0	1	1
0.001	1	0.995005532	0.995005532
0.001	2	0.980090013	0.990022254
0.001	3	0.955467805	0.985050354
0.001	4	0.921528545	0.980090013
0.001	5	0.878874953	0.975141411
0.001	6	0.828357036	0.97020472
0.001	7	0.771089708	0.965280112
0.001	8	0.708443642	0.960367753

0.001	9	0.642004362	0.955467805
0.001	10	0.573500942	0.950580427
0.001	11	0.504711653	0.945705774
0.001	12	0.437358297	0.940843997

Table 2. MTTF v/s Failure Rate

m	MTTF
0.01	16.36364
0.02	8.181818
0.03	5.454545
0.04	4.090909
0.05	3.272727
0.06	2.727273
0.07	2.337662
0.08	2.045455
0.09	1.818182
0.1	1.636364

Table 3. Cost Function v/s Time

m	t	C(t), C2=0.03	C(t), C2=0.06	C(t), C2=0.09
0.001	0	0	0	0
0.002	1	0.965007397	0.935007397	0.905007397
0.003	2	1.910135383	1.850135383	1.790135383
0.004	3	2.820831523	2.730831523	2.640831523
0.005	4	3.683168812	3.563168812	3.443168812
0.006	5	4.484200514	4.334200514	4.184200514
0.007	6	5.212372861	5.032372861	4.852372861
0.008	7	5.857966776	5.647966776	5.437966776
0.009	8	6.413521828	6.173521828	5.933521828

0.01	9	6.874186979	6.604186979	6.334186979
0.011	10	7.237945284	6.937945284	6.637945284
0.012	11	7.505673119	7.175673119	6.845673119
0.013	12	7.681015562	7.321015562	6.961015562
0.014	13	7.770083932	7.380083932	6.990083932
0.015	14	7.78100443	7.36100443	6.94100443
0.016	15	7.723364233	7.273364233	6.823364233

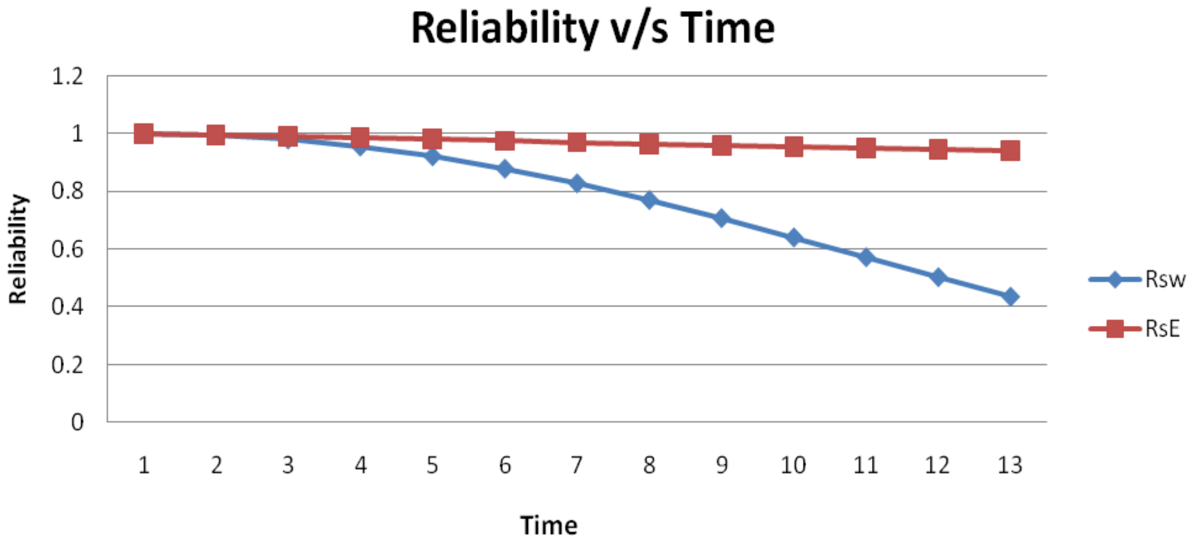


Figure 1. Reliability v/s Time

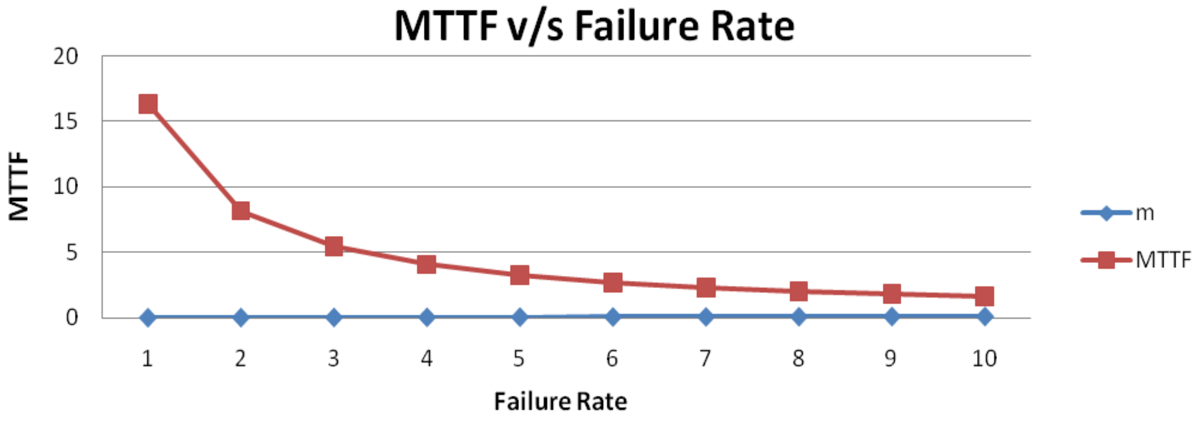


Figure 2. MTTF v/s Failure Rate

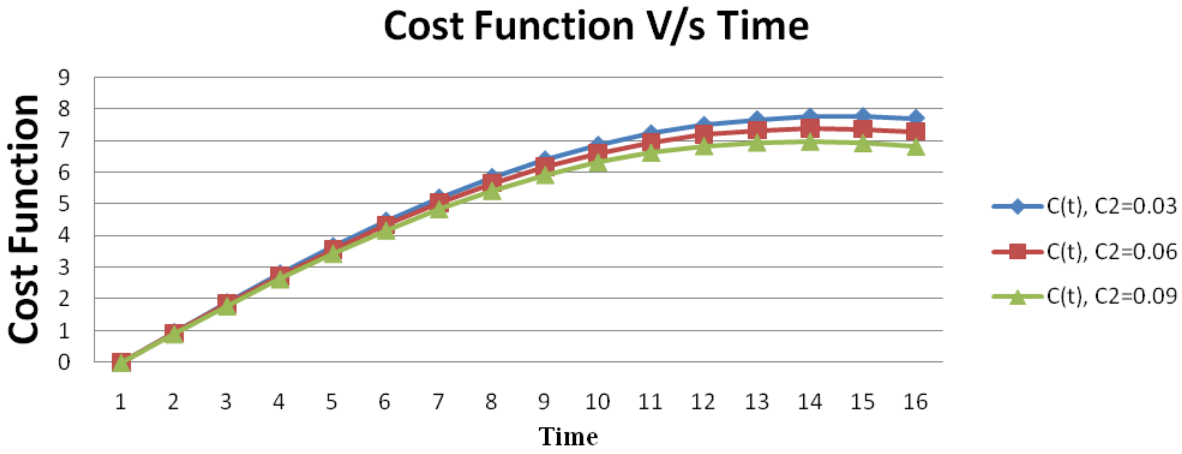


Figure 3. Cost Function v/s Time

9. CONCLUSIONS

The author has analyzed a Bio-Ethanol Plant for determining various reliability parameters by applying the Orthogonal Matrix Method & algebra of logics.

In Table 1, Reliability of system is computed with respect to time when failure rates follow Weibull time distribution and exponential time distribution. The graph of "Reliability v/s Time" in the Figure 1 depicts that the reliability of the complex system decreases approximately at a uniform rate in case of Exponential Time Distribution but it decreases very rapidly in case when failure rates follow Weibull Time Distributions.

In Table 2, MTTF is calculated and the graph "MTTF v/s Failure Rate" in Figure 2 depicts that MTTF of the system decreases catastrophically in beginning but later it decreases approximately at a uniform rate.

From Cost Function V/s Time graph, it is evident that initially there is minor difference between curves but later on effect of variation in service cost is visible.

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