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2-Repeated Solid Burst Error Detecting Cyclic Codes

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ABSTRACT

In modern era, coding theory has found various applications in almost every field whether it is theoretical or practical. Such as: digital data transmission, medical science, space science, geographical sciences etc. It is natural that bursts have different behavior in different channels. But the burst errors are found to occur mostly in various communication channels. In some of the systems, lightening and other short term irregular disturbances introduce various types of repeated burst errors. Usually they operate in such a way that over a specific length, some digits in a message are received correctly, while all other are corrupted. It is very common in some extra noisy channels that all the digits in a burst are corrupted. Such type of errors is called 'solid burst errors'. It may also be mentioned that cyclic codes play a significant role in error detection and correction. In this paper, we obtain results for cyclic codes that are capable of detecting 2-repeated solid bursts of length *b*.

Keyword: Repeated solid burst errors, cyclic codes, burst error detection, parity-check digits

1. INTRODUCTION

It is perceptible that from last few decades, communication devices and computing have become essential parts of human life. Although current communication devices are very efficient and reliable yet unlimited usage causes interrupted data transmission. There may be any cause of that e.g., server fading, call-drop, dynamic noise, jamming multi access interference etc. These problems arise due to the occurrence of various types of multiple burst errors in the channel in use.

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Among the various types of errors, known so far, repeated solid burst error is much more frequently occur in a very busy channel. The study of this particular error was introduced by Das [6]. Basically repeated solid burst error can be considered as an extension of the solid burst error [1] combined with repeated burst error [5]. Under specific consideration, repeated solid burst error is a generalization of random errors.

The study of burst errors, respective of cyclic codes, has always been centre of attraction for researchers as cyclic codes have very interesting mathematical structure and can easily be implemented using shift registers. Following this fact, Jain [4] obtained results for cyclic codes detecting Moderate-density open-loop burst error detection for cyclic codes.

In this paper, our study is précised to cyclic codes detecting 2-repeated solid burst errors.

The paper is organized as follows:

Section 2 presents basic definitions related to our study.

In section 3, results on cyclic codes for the detection of 2-repeated solid bursts are obtained.

In section 4 represents the conclusion of the paper. Some future scope for the study is also given in this section.

2. PRELIMINARIES

Definition 1: A *solid burst of length* b is a vector with non-zero entries in some b consecutive positions and zeros elsewhere.

Example 1: (000**111111**) is a solid burst of length 6 over $GF(2)$.

A 2-repeated burst of length b may be defined as follows:

Definition 2: A *2-repeated burst of length* b is a vector of length n whose only non-zero components are confined to two distinct sets of b consecutive components the first and the last component of each set being non zero.

Example 2: (0**110101**00**110111**) is a 2-repeated burst of length 6 over $GF(2)$.

Definition 3: A *2-repeated solid burst of length* b is a vector with non-zero entries in some b consecutive positions of two distinct sets and zeros elsewhere.

Example: (00**11**00000**11**) is a 2-repeated solid burst of length 2 over $GF(2)$.

3. 2-REPEATED SOLID BURST ERROR DETECTING CYCLIC CODES

Theorem 3.1: *Any 2-repeated solid burst-error, each of length up to n-k digits, can be detected by an (n, k) cyclic code.*

Proof: It is clear that a 2-repeated solid burst will have two segments each having b consecutive non-zero entries. Let, the first segment starts from *ith* position where $1 \leq i \leq n-2b$ and is of the form $x^i T_1(x)$, where $T_1(x)$ is a polynomial of degree $(b-1)$. The second segment then will start from $(i+j+b)th$ position where $1 \leq j \leq n-b$ and will be of the form $x^{i+j+b}T_2(x)$, where $T_2(x)$ is a polynomial of degree $(b-1)$.

So let,

$$T(x) = x^i T_1(x) + x^{i+j+b} T_2(x), \tag{3.1}$$

be a 2-repeated solid burst, each burst having length $b \geq 1$. If $\phi(x)$ is the codeword transmitted, then with $T(x)$ as error, the received n-tuple $\phi'(x)$ is given by

$$\phi'(x) = T(x) + \phi(x), \tag{3.2}$$

or

$$\phi'(x) = (x^i T_1(x) + x^{i+j+b} T_2(x)) + \phi(x)$$

Thus,

$$\{\phi'(x)\}_{g(x)} = \{(x^i T_1(x) + x^{i+j+b} T_2(x)) + \phi(x)\}_{g(x)}. \tag{3.3}$$

where: $\{\phi'(x)\}_{g(x)}$ is the remainder polynomial that is obtained after dividing the n-tuple by the generator polynomial $g(x)$.

Now the syndrome, obtained in (3.3) must be zero. For this $g(x)$ must divide $(x^i T_1(x) + x^{i+j+b} T_2(x))$. But $g(x)$ is not divisible by x because $g(x)$ divides $x^n - 1$. This concludes that x^i and x^{i+j+b} both are relatively prime to $g(x)$. Hence $g(x)$ must divide both $T_1(x)$ *and* $T_2(x)$. But it is impossible because

$$\deg g(x) = n - k, \tag{3.4}$$

and

$$\deg T_1(x) = \deg T_2(x) = b - 1 < n - k = \deg g(x). \tag{3.5}$$

Therefore, $T(x)$ cannot be a codeword and it will be detected.

This proves the theorem.

Theorem 3.2: *The fraction of 2-repeated solid bursts of length b> n-k that can go undetected by any (n, k) cyclic codes is*

$$\begin{cases} \dfrac{2q^{-2(n-k-1)}}{(n-2b+1)(n-2b+2)(q-1)^2} & \text{if } b = n-k+1 \\ \dfrac{2}{(n-2b+1)(n-2b+2)} q^{-2(n-k)} & \text{if } b > n-k+1. \end{cases} \tag{3.6}$$

Proof: Let us consider 2-repeated solid bursts of length $b \geq 1$. Each such burst will be of the form

$$T(x) = (x^i T_1(x) + x^{i+j+b} T_2(x)), \tag{3.7}$$

where: $T_1(x)\, and\, T_2(x)$ have degree $(b-1)$ each. There are $(q-1)$ choices for the each component of both sets. Thus there are $(q-1)^{2b}$ distinct polynomials $T_1(x)$ *and* $T_2(x)$.

The error will go undetected if and only if $T_1(X)$ *and* $T_2(X)$ have $g(X)$ as a factor, that is

$$T_1(x) = g(x)R_1(x),$$

$$T_2(x) = g(x)R_2(x).$$

Since $g(x)$ has degree $(n-k)$, $R_1(x)$ and $R_2(x)$ both must have degree $b-1-(n-k)$. If $b-1=(n-k)$, then $R_1(x)$ and $R_2(x)$ are nonzero constants and there are $(q-1)^2$ values they may take. The ratio of undetected 2-repeated solid bursts to the total number of 2-repeated solid bursts is, refer [6],

$$\frac{(q-1)^2}{\frac{(n-2b+1)(n-2b+2)}{2}(q-1)^{2b}} = \frac{q^{-2(b-1)}}{\frac{(n-2b+1)(n-2b+2)}{2}} = \frac{2q^{-2(n-k)}}{(n-2b+1)(n-2b+2)}.$$

Now if $b-1 > (n-k)$, $T_1(x)$ may have any of the $(q-1)$ non- zero field element as its each coefficient. There are therefore, $(q-1)^b$ choices of $T_1(x)$ which give undetected error patterns. Similarly, there are $(q-1)^b$ choices of $T_2(x)$ which give undetected error patterns.

The ratio in this case is

$$\frac{(q-1)^{2b}}{\frac{(n-2b+1)(n-2b+2)}{2}(q-1)^{2b}} = \frac{2}{(n-2b+1)(n-2b+2)}.$$

This proves the result.

4. CONCLUDING REMARKS

The results, presented in this paper, are inspired by the study of cyclic codes detecting burst errors by Peterson and Weldon [3].Such type of errors generally occurs in channels like super computers, space communication and semiconductor devices. By developing such codes, the number of parity-check digits required can be economized and hence the efficiency of the transmission of the code through anoisy channel can be increased.

Moreover, results obtained in this paper are with the consideration of Hamming weight. One can do similar study in a more generalized manner by considering other types of weight such as Lee weight, Sharma- Kaushik weight, Euclidean weight, generalized Hamming weight etc.

Remark 1: Technique used here to prove the results is based on [2].

Remark 2: It is clear from results obtained here that if we consider bursts of length 1 then the error pattern is same as of random errors. This verifies that repeated solid burst errors are generalization of random errors.

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