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Inventory Model for Deteriorating Items under Trade Credit and Inflation

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ABSTRACT

In this paper, we put forward two warehouse inventory model considering trade credit and inflation. Demand rate is function of time. The inventory costs in rented warehouse are more than in owned warehouse. Shortages are allowed and backlogged. The purpose of this model is to calculate the time of order optimizing the total cost of the system. Numerical example explain the model. Sensitivity analysis is also carried to check the stability of the model.

***Keywords*:** Inventory, Trade Credit, Deteriorating Items, Partial Backlogging

1. INTRODUCTION

Today's business world is highly competitive wherein invariably all the supplier offers their retailers trade credit in settling their payments. In this regard, the retailers are forced to purchase more stock. So, the retailer has to purchase warehouse name as rented warehouse to store the extra stock.

The total inventory system cost can be lowered if the capacity of owned warehouse increases. Hartley (1976) first established two warehouses system. Yang (2004) proposed inventory model considering the rate of inflation and allowable shortages. Shah (2006) presented an inventory model with time value of money and permissible delay in payments. Yang and Zhou (2011) came up with model with constant deterioration and permissible delay

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in payments. Sett *et. al.* (2012) put forward inventory model with deterioration rate depending on time. Jaggi *et. al*. (2015) produced model for deteriorating items imperfect quality and shortages. Palanivel *et. al*. (2016) produced inventory model for non-instantaneous deteriorating items with stock dependent demand and shortages.

In today's business world, the buyers commonly allow some grace period to settle the account with the supplier. The buyer does not have to pay any interest during the fixed grace period; if the payment, however, is delayed beyond this period, an interest is charged. Chung and Liao (2004) came up with model with trade credit depending on ordered quantity. Chen and Kang (2010) developed the model introducing a varying pricing from the above literature that researchers have paid less interest in developing two- strategy to obtain cost savings for long term cooperative relationships. Chung *et. al.* (2013) came up with inventory model considering that the demand rate is constant and replenishment is instantaneous. Majumder *et. al.* (2016) developed production model with an alternative approach of payment. In this model, inventory model is presented for deteriorating products with two warehouse and allowable shortages. The effect of permissible delay is considered. A numerical example is given to explain the model. Sensitivity analysis is also performed.

2. ASSUMPTIONS AND NOTATIONS

The assumptions and notations are as follows:

2. 1. ASSUMPTIONS

[1] Demand rate is function of time
[2] Shortages are allowed.
[3] A single item is taken over the prescribed period of time.
[4] The owned warehouse has a fixed capacity of W units, whereas the rented warehouse capacity is not fixed.
[5] The stock of the owned warehouse is used only after consuming the stock kept in rented warehouse.
[6] Shortages are allowed and the fractions of shortages are backordered

2. 2. NOTATIONS

r : Rate of Inflation.
K : Cost of ordering.
C_R : Holding cost rented warehouse per unit time.
C_O : Holding cost owned warehouse per unit time.
C_D : Cost of deterioration per unit time.
C_b : Cost of shortage per unit per unit time.
C_l : Cost of Lost sale per unit per unit time.
T : Total cycle time.
M : Period of Trade credit.

p : selling price.
c : purchase cost.
$D(t)$: Rate of Demand
I_e : Interest earned per year.
I_C : Interest charges per year by the supplier.
δ : Backlogging parameter.
W : Fixed capacity of owned warehouse

3. MATHEMATICAL FORMULATION

The model starts with shortages. During the interval t_1 and t_2; the inventory decreases due to both deterioration and demand in rented warehouse. During time t_1, the inventory in owned warehouse comes to decrease at t_2 due to deterioration. Also, during t_2 and T the inventory decreases in owned warehouse due to both deterioration and demand.
The equations of the system are given as follows:

$$\frac{dI_B(t)}{dt} = -D(t)e^{-\delta(t_1-t)}, \quad 0 \le t \le t_1 \tag{1}$$

$$\frac{dI_r(t)}{dt} + \theta\, I_r(t) = -D(t), \quad t_1 \le t \le t_2 \tag{2}$$

$$\frac{dI_{o_1}(t)}{dt} + \theta\, I_{o_1}(t) = 0, \quad t_1 \le t \le t_2 \tag{3}$$

$$\frac{dI_{o_2}(t)}{dt} + \theta\, I_{o_2}(t) = -D(t), \quad t_2 \le t \le T \tag{4}$$

with boundary conditions

$$I_B(0) = 0, I_r(t_2) = 0, I_{o_1}(t_1) = W, I_{O_2}(T) = 0 \tag{5}$$

The solutions of equations are given as follows:

$$I_B(t) = \frac{\mu e^{-\delta t_1}}{\lambda+\delta}\left(1 - e^{(\lambda+\delta)t}\right) \qquad 0 \le t \le t_1 \tag{6}$$

$$I_r(t) = \mu\, e^{-\alpha t^{\beta}}\left((t_2 - t) + \frac{\lambda}{2}\left(t_2^{\,2} - t^2\right) + \frac{\alpha}{(\beta+1)}\left(t_2^{\,\beta+1} - t^{\beta+1}\right)\right) \qquad t_1 \le t \le t_2 \tag{7}$$

$$I_{o_1}(t) = W\left(1 + \alpha\left(t_1^{\,\beta} - t^{\beta}\right)\right) \qquad t_1 \le t \le t_2 \tag{8}$$

$$I_{o_2}(t)=\mu e^{-\alpha t^{\beta}}\left(\left(T-t\right)+\frac{\beta}{2}\left(T^2-t^2\right)+\frac{\alpha}{(\beta+1)}\left(T^{\beta+1}-t^{\beta+1}\right)\right) \qquad t_2\le t\le T \qquad (9)$$

The cost components for the retailer in the given inventory cycle is as follows:

Present value ordering cost $= Ke^{-rt_1}$ (10)

Present value holding costs

Rented warehouse (11)

$$=c_R\int_{t_1}^{t_2} e^{-rt} I_r(t)dt$$

$$= c_R\mu\ (\frac{t_2^{\ 2}}{2}-\frac{\alpha\, t_2^{\ \beta+2}}{(\beta+1)(\beta+2)}-\frac{5}{6}r\,t_2^{\ 3}+\frac{\alpha\, r\, t_2^{\ \beta+3}}{(\beta+2)(\beta+3)}-t_2t_1+\frac{t_1^{\ 2}}{2}+\frac{\alpha\, t_2\, t_1^{\ \beta+1}}{\beta+1}-\frac{\alpha\, t_1^{\ \beta+2}}{\beta+2}$$

$$+\frac{r\,t_2t_1^{\ 2}}{2}+\frac{r\,t_1^{\ 3}}{3}-\frac{\alpha\, r\, t_2\, t_1^{\ \beta+2}}{(\beta+2)}+\frac{\alpha\, r\ t_1^{\ \beta+3}}{(\beta+3)})+\frac{\lambda}{2}(\frac{2\,t_2^{\ 3}}{3}-\frac{2\alpha\, t_2^{\ \beta+3}}{(\beta+1)(\beta+3)}-\frac{r\,t_2^{\ 4}}{4}+$$

$$+\frac{2\alpha\, r\, t_2^{\ \beta+4}}{(\beta+2)(\beta+4)}-t_2^{\ 2}t_1+\frac{t_1^{\ 3}}{3}+\frac{\alpha\, t_2^{\ 2}t_1^{\ \beta+1}}{(\beta+1)}-\frac{\alpha\, t_1^{\ \beta+3}}{\beta+3}+\frac{r\,t_2^{\ 2}t_1^{\ 2}}{2}-\frac{r\,t_1^{\ 4}}{4}-\frac{\alpha\, r\, t_2^{\ 2}t_1^{\ \beta+2}}{\beta+2}+\frac{\alpha\, r\, t_1^{\ \beta+4}}{\beta+4})$$

$$+\frac{\alpha}{(\beta+1)}(\frac{(\beta+1)t_2^{\ \beta+2}}{(\beta+2)}-\frac{\alpha\, t_2^{\ 2\beta+2}}{2(\beta+1)}-\frac{(\beta+1)r\,t_2^{\ \beta+3}}{2(\beta+3)}+\frac{(\beta+1)\alpha\, r\, t_2^{\ 2\beta+3}}{(\beta+2)(2\beta+3)}-t_2^{\ \beta+1}t_1+\frac{t_1^{\ \beta+2}}{(\beta+2)}$$

$$+\frac{\alpha\, t_2^{\ \beta+1}t_1^{\ \beta+1}}{(\beta+1)}-\frac{\alpha\, t_1^{\ 2\beta+2}}{(2\beta+2)}-\frac{r\,t_1^{\ 2}\ t_2^{\ \beta+1}}{2}-\frac{r\,t_1^{\ \beta+3}}{(\beta+3)}+\frac{\alpha\, r\, t_2^{\ \beta+1}t_1^{\ \beta+2}}{(\beta+2)}+\frac{\alpha\, r\, t_1^{\ 2\beta+3}}{(2\beta+3)}))$$

Owned warehouse

$$=c_O\left(\int_{t_1}^{t_2} e^{-rt}I_{o_1}(t)dt+\int_{t_2}^{T} e^{-rt}I_{o_2}(t)dt\right)$$

$$= c_o\ \ (W(t_2+\alpha\, t_1^{\ \beta}t_2-\frac{\alpha\, t_2^{\ \beta+1}}{(\beta+1)}-\frac{r\,t_2^{\ 2}}{2}-\frac{\alpha\, r\, t_1^{\ \beta}t_2^{\ 2}}{2}+\frac{\alpha\, r\, t_2^{\ \beta+2}}{\beta+2}-t_1+\frac{\alpha\ t_1^{\ \beta+1}(\beta+2)}{(\beta+1)}$$

$$+\frac{r\,t_1^{\ 2}}{2}-\frac{\alpha\, r\ t_1^{\ \beta+2}(\beta+4)}{2(\beta+2)})+\mu(\frac{T^2}{2}-\frac{\alpha\, T^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{r\,T^3}{6}+\frac{r\alpha\, T^{\beta+2}}{(\beta+2)}-\frac{r\alpha\, T^{\beta+3}}{(\beta+3)}$$

$$-T\,t_2+\frac{t_2^{\,2}}{2}+\frac{\alpha\,T\,t_2^{\,\beta+1}}{\beta+1}-\frac{\alpha\,t_2^{\,\beta+2}}{\beta+2}+\frac{r\,T\,t_2^{\,2}}{2}-\frac{r\,t_2^{\,3}}{3}-\frac{r\alpha\,t_2^{\,\beta+2}}{\beta+2}+\frac{r\alpha\,t_2^{\,\beta+3}}{\beta+3})$$

$$+\frac{\mu\,\beta}{2}(\frac{2T^3}{3}-\frac{2\alpha\,T^{\beta+3}(\beta+2)}{(\beta+1)(\beta+3)}-\frac{r\,T^4}{4}+\frac{2\alpha\,r\,T^{\beta+4}}{(\beta+2)(\beta+4)}-T^2t_2+\frac{t_2^{\,3}}{3}+\frac{\alpha\,T^2t_2^{\,\beta+1}}{(\beta+1)}+$$

$$+\frac{\alpha\,t_2^{\,\beta+3}}{(\beta+3)}+\frac{r\,T^2\,t_2^{\,2}}{2}-\frac{r\,t_2^{\,4}}{4}-\frac{\alpha\,r\,T^2t_2^{\,\beta+2}}{(\beta+2)}+\frac{\alpha\,r\,t_2^{\,\beta+4}}{(\beta+4)})$$

$$+\frac{\mu\alpha}{\beta+1}(\frac{T^{\beta+2}(\beta+1)}{(\beta+2)}-\frac{(4\beta+3)T^{2\beta+2}}{2(\beta+1)^2}-\frac{(\beta+1)\,r\,T^{\beta+3}}{2(\beta+3)}+\frac{(\beta+1)\,r\alpha\,T^{2\beta+3}}{2(\beta+2)(\beta+3)}$$

$$-T^{\beta+1}t_2+\frac{t_2^{\,\beta+2}}{(\beta+2)}+\frac{\alpha\,T^{\beta+1}\,t_2^{\,\beta+1}}{(\beta+1)}+\frac{2\;t_2^{\,2\beta+2}}{(2\beta+2)}-\frac{r\,T^{\beta+1}t_2^{\,2}}{2}-\frac{r\,t_2^{\,\beta+3}}{\beta+3}-\frac{\alpha\,r\,T^{\beta+1}\,t_2^{\,\beta+2}}{(\beta+2)}+\frac{r\alpha\,t_2^{\,2\beta+3}}{2\beta+3}))$$

Present value deterioration cost

$$=c_D\left(\int_{t_1}^{t_2}\alpha\;\beta\;t^{\beta-1}e^{-rt}I_r\left(t\right)dt+\int_{t_1}^{t_2}\alpha\;\beta\;t^{\beta-1}e^{-rt}I_{o_1}\left(t\right)dt+\int_{t_2}^{T}\alpha\;\beta\;t^{\beta-1}e^{-rt}I_{o_2}\left(t\right)dt\right) \tag{12}$$

$$=c_D\;\alpha\;\beta\;\;(W(\frac{t_2^{\,\beta+1}}{\beta(\beta+1)}-\frac{\alpha\;t_2^{\,2\beta+1}}{(2\beta)(2\beta+1)}-\frac{r\;t_2^{\,\beta+2}}{(\beta+1)(\beta+2)}+\frac{\alpha\;r\;t_2^{\,2\beta+2}}{(2\beta+1)(2\beta+2)}-\frac{t_2\;t_1^{\,\beta}}{\beta}+\frac{t_1^{\,\beta+1}}{\beta+1}$$

$$+\frac{\alpha\;t_2\;t_1^{\,2\beta}}{2\beta}-\frac{\alpha\;\;t_1^{\,2\beta+1}}{(2\beta+1)}+\frac{r\;t_2t_1^{\,\beta+1}}{(\beta+1)}-\frac{r\;t_1^{\,\beta+2}}{(\beta+2)}-\frac{\alpha\;r\;t_2\;t_1^{\,2\beta+1}}{2\beta+1}+\frac{\alpha\;r\;\;t_1^{\,2\beta+2}}{2\beta+2}+$$

$$\left(\frac{t_2^{\,\beta}}{\beta}+\frac{\alpha\,t_1^{\,\beta}\;t_2^{\,\beta}}{\beta}-\frac{\alpha\,t_2^{\,2\beta}}{2\beta}-\frac{r\;\;t_2^{\,\beta+1}}{\beta+1}-\frac{r\alpha t_1^{\,\beta}\;t_2^{\,\beta+1}}{\beta+1}+\frac{r\alpha\,t_2^{\,2\beta+1}}{2\beta+1}-\frac{t_1^{\,\beta}}{\beta}-\frac{\alpha}{2\beta}t_1^{\,2\beta}+\frac{r}{\beta+1}t_1^{\,\beta+1}\right.$$

$$+\frac{\beta\alpha\,rt_1^{\,2\beta+1}}{(\beta+1)(2\beta+1)})+\mu(\frac{T^{\beta+1}}{(\beta+1)\beta}-\frac{\alpha\,T^{2\beta+1}}{(2\beta)(2\beta+1)}-\frac{r\,T^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{r\,\alpha\,T^{2\beta+2}}{(2\beta+1)(2\beta+2)}-\frac{T\,t_2^{\,\beta}}{(\beta)}+\frac{t_2^{\,\beta+1}}{(\beta+1)}$$

$$\frac{T\alpha\,t_2^{\,2\beta}}{2\beta}-\frac{\alpha\,t_2^{\,2\beta+1}}{2\beta+1}+\frac{r\,T\,t_2^{\,\beta+1}}{\beta+1}-\frac{r\;\;t_2^{\,\beta+2}}{\beta+2}-\frac{\alpha\,rT\,t_2^{\,2\beta+1}}{2\beta+1}+\frac{r\alpha\,t_2^{\,2\beta+2}}{2\beta+2})+\frac{\mu\,\beta}{2}(\frac{2\,T^{\beta+2}}{\beta(\beta+2)}$$

$$-\frac{\alpha\,T^{2\beta+2}}{\beta(2\beta+2)}+\frac{4\,r\,T^{\beta+3}}{(\beta+1)(\beta+3)}+\frac{2\,\alpha\,r\,T^{2\beta+3}}{(2\beta+1)(2\beta+3)}-\frac{T^2t_2^{\,\beta}}{\beta}+\frac{t_2^{\,\beta+2}}{\beta+2}+\frac{\alpha\,T^2\;t_2^{\,2\beta}}{2\beta}-\frac{\alpha\;\;t_2^{\,2\beta+2}}{(2\beta+2)}+\frac{rT^2\;t_2^{\,\beta+1}}{\beta+1}-$$

$$-\frac{r\,t_2^{\,\beta+3}}{(\beta+3)}-\frac{\alpha\,r\,T^2t_2^{\,2\beta+1}}{(2\beta+1)}+\frac{\alpha\,r\,t_2^{\,2\beta+3}}{(2\beta+3)})+\frac{\mu\alpha}{\beta+1}(\frac{(\beta+1)}{\beta(2\beta+1)}T^{2\beta+1}-\frac{(\beta+1)\alpha T^{3\beta+1}}{2\beta(3\beta+1)}-\frac{r\,T^{2\beta+2}}{2\beta+2}+$$

$$+\frac{(\beta+1)r\alpha T^{3\beta+2}}{(2\beta+1)(3\beta+2)}-\frac{T^{\beta+1}t_2^{\beta}}{\beta}+\frac{t_2^{2\beta+1}}{(2\beta+1)}+\frac{\alpha T^{\beta+1}t_2^{2\beta}}{2\beta}-\frac{\alpha t_2^{3\beta+1}}{3\beta+1}+\frac{r t_2^{2\beta+2}}{\beta+1}-\frac{r t_2^{2\beta+2}}{2\beta+2}-\frac{r\alpha T^{\beta+1}t_2^{2\beta+1}}{2\beta+1}$$
$$+\frac{r\alpha t_2^{3\beta+2}}{3\beta+2}))$$

Present value shortage cost

$$=-c_b\int_0^{t_1}e^{-rt}I_B(t)dt=\frac{\mu c_b e^{-\delta t_1}}{(\lambda+\delta)}\left(\frac{1}{r}\left(e^{-rt_1}-1\right)+\frac{1}{(\lambda+\delta+r)}\left(e^{(\lambda+\delta-r)t_1}-1\right)\right) \tag{13}$$

Present value Lost sale cost

$$=c_l\int_0^{t_1}e^{-rt}\left(1-e^{-\delta(t_1-t)}\right)D(t)dt=\mu c_l\,\delta\left(t_1\left(t_1+\frac{(\lambda-r)}{2}t_1^2\right)-\left(\frac{t_1^2}{2}+\frac{(\lambda-r)t_1^3}{3}\right)\right) \tag{14}$$

Based on the parameter values t_1, t_2, M, T, there are three cases to be explored.

Case 1: $t_1+M<t_2$,

Interest paid

$$=c_pI_c\left(\int_{t_1+M}^{t_2}e^{-rt}\left(I_r(t)+I_{o_1}(t)\right)dt+\int_{t_2}^{T}e^{-rt}I_{o_2}(t)dt\right) \tag{15}$$

$$=c_pI_c\mu(\frac{t_2^2}{2}-\frac{\alpha\, t_2^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{5}{6}r\,t_2^3+\frac{\alpha\, r\, t_2^{\beta+3}}{(\beta+2)(\beta+3)}-t_2(t_1+M)+\frac{(t_1+M)^2}{2}+\frac{\alpha\, t_2\,(t_1+M)^{\beta+1}}{\beta+1}$$

$$-\frac{\alpha\,(t_1+M)^{\beta+2}}{\beta+2}-\frac{r\,t_2(t_1+M)^2}{2}+\frac{r\,(t_1+M)^3}{3}-\frac{\alpha\, r\, t_2\,(t_1+M)^{\beta+2}}{(\beta+2)}+\frac{\alpha\, r\,\,(t_1+M)^{\beta+3}}{(\beta+3)})$$

$$+\frac{\lambda}{2}(\frac{2t_2^3}{3}-\frac{2\alpha\, t_2^{\beta+3}}{(\beta+1)(\beta+3)}-\frac{r\,t_2^4}{4}+\frac{2\alpha\, r\, t_2^{\beta+4}}{(\beta+2)(\beta+4)}-t_2^2(t_1+M)+\frac{(t_1+M)^3}{3}$$

$$+\frac{\alpha\, t_2^2(t_1+M)^{\beta+1}}{(\beta+1)}-\frac{\alpha\,(t_1+M)^{\beta+3}}{\beta+3}+\frac{r\,t_2^2(t_1+M)^2}{2}-\frac{r(t_1+M)^4}{4}-\frac{\alpha\, r\, t_2^2(t_1+M)^{\beta+2}}{\beta+2}+\frac{\alpha\, r(t_1+M)^{\beta+4}}{\beta+4})$$

$$+\frac{\alpha}{\beta+1}(\frac{t_2^{\beta+2}(\beta+1)}{(\beta+2)}-\frac{\alpha^{\;2\beta+2}}{2(\beta+1)}-\frac{(\beta+1)r\,t_2^{\beta+3}}{2(\beta+3)}+\frac{(\beta+1)r\alpha\, t_2^{2\beta+3}}{(\beta+2)(2\beta+3)}$$

$$-t_2^{\beta+1}(t_1+M)+\frac{(t_1+M)^{\beta+2}}{(\beta+2)}+\frac{\alpha(t_1+M)^{\beta+1}t_2^{\beta+1}}{\beta+1}-\frac{\alpha(t_1+M)^{2\beta+2}}{(2\beta+2)}+\frac{r\,t_2^{\beta+1}(t_1+M)^2}{2}-\frac{r\,(t_1+M)^{\beta+3}}{(2\beta+3)}$$

$$-\frac{\alpha\,r(t_1+M)^{\beta+2}t_2^{\beta+1}}{(\beta+2)}+\frac{\alpha\,r\,(t_1+M)^{2\beta+3}}{(2\beta+3)}))+$$

$$(c_pI_cW(t_2+\alpha\,t_1^{\beta}t_2-\frac{\alpha\,t_2^{\beta+1}}{(\beta+1)}-\frac{r\,t_2^{2}}{2}-\frac{\alpha\,r\,t_1^{\beta}t_2^{2}}{2}+\frac{\alpha\,r\,t_2^{\beta+2}}{\beta+2}-(t_1+M)+\alpha\,(t_1+M)^{\beta+1}+\frac{\alpha(t_1+M)^{\beta+1}}{\beta+1}$$

$$+\frac{r\,(t_1+M)^2}{2}-\frac{\alpha\,r\,(t_1+M)^{\beta+2}}{2}-\frac{\alpha\,r\,(t_1+M)^{\beta+2}}{(\beta+2)})+\mu(\frac{T^2}{2}-\frac{\alpha\,T^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{r\,T^3}{6}$$

$$+\frac{r\alpha\,T^{\beta+2}}{(\beta+2)}-\frac{r\alpha\,T^{\beta+3}}{(\beta+3)}-T\,t_2+\frac{t_2^{2}}{2}+\frac{\alpha\,T\,t_2^{\beta+1}}{\beta+1}-\frac{\alpha\,t_2^{\beta+2}}{\beta+2}+\frac{r\,T\,t_2^{2}}{2}-\frac{r\,t_2^{3}}{3}-\frac{r\alpha\,t_2^{\beta+2}}{\beta+2}+\frac{r\alpha\,t_2^{\beta+3}}{\beta+3})$$

$$+\frac{\mu\,\beta}{2}(\frac{2T^3}{3}-\frac{2\alpha\,T^{\beta+3}(\beta+2)}{(\beta+1)(\beta+3)}-\frac{r\,T^4}{4}+\frac{2\alpha\,r\,T^{\beta+4}}{(\beta+2)(\beta+4)}-T^2t_2+\frac{t_2^{3}}{3}+\frac{\alpha\,T^2t_2^{\beta+1}}{(\beta+1)}+$$

$$+\frac{\alpha\,t_2^{\beta+3}}{(\beta+3)}+\frac{r\,T^2\,t_2^{2}}{2}-\frac{r\,t_2^{4}}{4}-\frac{\alpha\,r\,T^2t_2^{\beta+2}}{(\beta+2)}+\frac{\alpha\,r\,t_2^{\beta+4}}{(\beta+4)})$$

$$+\frac{\mu\alpha}{\beta+1}(\frac{T^{\beta+2}(\beta+1)}{(\beta+2)}-\frac{(4\beta+3)T^{2\beta+2}}{2(\beta+1)^2}-\frac{(\beta+1)r\,T^{\beta+3}}{2(\beta+3)}+\frac{(\beta+1)r\alpha\,T^{2\beta+3}}{(\beta+2)(\beta+3)}$$

$$-T^{\beta+1}t_2+\frac{t_2^{\beta+2}}{(\beta+2)}+\frac{\alpha\,T^{\beta+1}\,t_2^{\beta+1}}{(\beta+1)}+\frac{2\,t_2^{2\beta+2}}{(2\beta+2)}+\frac{r\,T^{\beta+1}t_2^{2}}{2}-\frac{r\,t_2^{\beta+3}}{\beta+3}-\frac{\alpha\,r\,T^{\beta+1}\,t_2^{\beta+2}}{(\beta+2)}+\frac{r\alpha\,t_2^{2\beta+3}}{2\beta+3}))$$

Interest earned

$$=pI_e\left(\int_{t_1}^{t_1+M}e^{-rt}(t-t_1)D(t)\,dt+Me^{-rt_1}I_B(t_1)\right) \tag{16}$$

$$=pI_e\mu(\frac{(t_1+M)^2}{2}+t_1(t_1+M)+\frac{\lambda\,(t_1+M)^3}{(2\beta+2)}+\frac{\lambda\,(t_1+M)^2t_1}{2}-\frac{r\,(t_1+M)^3}{3}+\frac{r\,t_1\,(t_1+M)^2}{2}$$
$$-\frac{r\lambda\,t_1(t_1+M)^3}{3}+\frac{1}{2}t_1^{2}+\frac{\lambda}{6}t_1^{3}-\frac{r}{6}t_1^{3}-\frac{r\lambda t_1^{4}}{12})+pI_eMe^{-rt_1}\frac{\mu e^{-\delta t_1}}{(\lambda+\delta)}\left(1-e^{(\lambda+\delta)t_1}\right)$$

Case 2: $t_2\le t_1+M<T$,

Interest paid

$$= c_p I_c \left(\int_{t_1+M}^{t_2} e^{-rt} I_{o_1}(t) dt + \int_{t_2}^{T} e^{-rt} I_{o_2}(t) dt \right) \tag{17}$$

Interest earned is same as case1

Case 3: $T \le t_1 + M$

Interest paid = 0

Interest earned

$$= pI_e \left(\int_{t_1}^{T} e^{-rt} (t - t_1) D(t)\, dt + \int_{T}^{t_1+M} e^{-rt} (T - t_1) D(t)\, dt + M e^{-rt_1} I_B(t_1) \right) \tag{18}$$

The total cost for the retailer is given by

$TC(t_1, t_2, T)$ = Present value ordering cost + Present value holding cost in rented warehouse + Present value holding cost in owned warehouse + Cost of deterioration + Cost of Lost sale + Interest payable – Interest earned.

That is,

$$TC(t_1, t_2, T) = \begin{cases} TC_1, \text{ if } t_1 + M < t \\ TC_2, \text{ if } t_2 \le t_1 + M < T \\ TC_3, \text{ if } T \le t_1 + M \end{cases} \tag{19}$$

4. NUMERICAL EXAMPLE

The inventory model is explained through the numerical example for which the input parameter are as follows:

$A = 0.01$ units, $\lambda = 2$ units, $\mu = 0.2$ units, $\alpha = 0.05$ units, $\beta = 0.03$ units, $r = 0.01$ units, $W = 500$ units
$M = 0.25$, $C_O = 1.2$ Rs/ units, $C_R = 0.6$ Rs/ units, $C_b = 0.5$ Rs/ units, $c_D = 0.6$ Rs/ units, $c_l = 0.3$ Rs/ units
$C = 0.3$ Rs/units, $p = 0.2$ Rs/units, $I_e = 0.25$ Rs/units, $I_C = 0.15$ Rs/units

For above mentioned parametric values, the optimal solution has been found. Using software Mathematica, the results are as follows: The optimal values are

$t_1 = 0.2947$, $t_2 = 0.7651$, $T = 1.65132$, $TC = $ Rs 47.3725

5. SENSITIVITY ANALYSIS

Parameter	% change	T^*	TC^*
M	0.26	1.64362	47.5432
	0.27	1.63457	47.7542
	0.28	1.62456	48.2245
	0.29	1.61994	45.3321
α	0.06	1.64721	47.5162
	0.07	1.64322	47.2335
	0.08	1.64473	47.1332
	0.09	1.64305	46.6934
β	0.04	1.63235	47.5245
	0.05	1.63123	47.5412
	0.06	1.63247	47.6603
	0.07	1.63178	47.6659
A	0.02	1.65115	47.6537
	0.03	1.65320	47.6572
	0.04	1.65347	47.6634
	0.05	1.66118	47.6759
C_{OW}	1.4	1.322585	47.2581
	1.6	1.32653	47.5245
	1.8	1.34455	48.3457
	2.0	1.43252	49.7745
C_{RW}	0.7	1.64189	47.3611
	0.8	1.64134	47.4102
	0.9	1.65324	47.7136
	1.0	1.65244	48.5923

W	550	1.78842	47.2757
	600	1.76664	47.4883
	700	1.70121	48.2332
	750	1.66492	48.2566

6. CONCLUSION

The purpose of the inventory model is to calculate the time of order that optimize the total cost of the system. The inventory cost such as cost of holding stock in rented warehouse is higher than that of warehouse that is owned. Due to permissible delay, the quantity purchased by the retailer is greater than the capacity of warehouse that is owned. This paper helps to minimize the additional cost of holding material etc. For future scope, the model can be extended for two-level permissible delay in payment, learning effect, stochastic demand etc.

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