Inventory Model for Deteriorating Items under Trade Credit and Inflation

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ABSTRACT

In this paper, we put forward two warehouse inventory model considering trade credit and inflation. Demand rate is function of time. The inventory costs in rented warehouse are more than in owned warehouse. Shortages are allowed and backlogged. The purpose of this model is to calculate the time of order optimizing the total cost of the system. Numerical example explain the model. Sensitivity analysis is also carried to check the stability of the model.

\textit{Keywords:} Inventory, Trade Credit, Deteriorating Items, Partial Backlogging

1. INTRODUCTION

Today’s business world is highly competitive wherein invariably all the supplier offers their retailers trade credit in settling their payments. In this regard, the retailers are forced to purchase more stock. So, the retailer has to purchase warehouse name as rented warehouse to store the extra stock.


In today’s business world, the buyers commonly allow some grace period to settle the account with the supplier. The buyer does not have to pay any interest during the fixed grace period; if the payment, however, is delayed beyond this period, an interest is charged. Chung and Liao (2004) came up with model with trade credit depending on ordered quantity. Chen and Kang (2010) developed the model introducing a varying pricing from the above literature that researchers have paid less interest in developing two- strategy to obtain cost savings for long term cooperative relationships. Chung et al. (2013) came up with inventory model considering that the demand rate is constant and replenishment is instantaneous. Majumder et al. (2016) developed production model with an alternative approach of payment. In this model, inventory model is presented for deteriorating products with two warehouse and allowable shortages. The effect of permissible delay is considered. A numerical example is given to explain the model. Sensitivity analysis is also performed.

2. ASSUMPTIONS AND NOTATIONS

The assumptions and notations are as follows:

2.1. ASSUMPTIONS

[1] Demand rate is function of time
[2] Shortages are allowed.
[3] A single item is taken over the prescribed period of time.
[4] The owned warehouse has a fixed capacity of W units, whereas the rented warehouse capacity is not fixed.
[5] The stock of the owned warehouse is used only after consuming the stock kept in rented warehouse.
[6] Shortages are allowed and the fractions of shortages are backordered

2.2. NOTATIONS

\[ r \] : Rate of Inflation.
\[ K \] : Cost of ordering.
\[ C_R \] : Holding cost rented warehouse per unit time.
\[ C_O \] : Holding cost owned warehouse per unit time.
\[ C_D \] : Cost of deterioration per unit time.
\[ C_b \] : Cost of shortage per unit per unit time.
\[ C_l \] : Cost of Lost sale per unit per unit time.
\[ T \] : Total cycle time.
\[ M \] : Period of Trade credit.
mathematical formulation

The model starts with shortages. During the interval $t_1$ and $t_2$, the inventory decreases due to both deterioration and demand in rented warehouse. During time $t_1$, the inventory in owned warehouse comes to decrease at $t_2$ due to deterioration. Also, during $t_2$ and $T$ the inventory decreases in owned warehouse due to both deterioration and demand.

The equations of the system are given as follows:

$$\frac{dI_b(t)}{dt} = -D(t)e^{-\delta(t-1)}, \quad 0 \leq t \leq t_1$$

(1)

$$\frac{dI_r(t)}{dt} + \theta I_r(t) = -D(t), \quad t_1 \leq t \leq t_2$$

(2)

$$\frac{dI_o(t)}{dt} + \theta I_o(t) = 0, \quad t_1 \leq t \leq t_2$$

(3)

$$\frac{dI_o(t)}{dt} + \theta I_o(t) = -D(t), \quad t_2 \leq t \leq T$$

(4)

with boundary conditions

$$I_b(0) = 0, I_r(t_2) = 0, I_o(t_1) = W, I_o(T) = 0$$

(5)

The solutions of equations are given as follows:

$$I_b(t) = \frac{\mu e^{-\delta t_1}}{\lambda + \delta} \left(1 - e^{(\lambda+\delta)t} \right) \quad 0 \leq t \leq t_1$$

(6)

$$I_r(t) = \mu e^{-\alpha t} \left(t_2 - t + \frac{\lambda}{2} (t_2^2 - t_2) + \frac{\alpha}{(\beta+1)} (t_2^{\beta+1} - t^{\beta+1}) \right) \quad t_1 \leq t \leq t_2$$

(7)

$$I_o(t) = W \left(1 + \alpha t - t^\beta \right) \quad t_1 \leq t \leq t_2$$

(8)
\[ I_{o_i}(t) = \mu e^{-r t} \left[(T-t) + \frac{\beta}{2}(T^2-t^2)+ \frac{\alpha}{(\beta+1)}(T^{\beta+1}-t^{\beta+1})\right], \quad t_2 \leq t \leq T \tag{9} \]

The cost components for the retailer in the given inventory cycle is as follows:

Present value ordering cost = \( Ke^{-rt_1} \) \tag{10}

Present value holding costs

Rented warehouse

\[ = c_R \int_{t_1}^{t_2} e^{-r t} I_i(t) \, dt \]

\[ = c_R \mu \left( \frac{t_2^2}{2} - \frac{\alpha t_2^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{5}{6} t_2^3 + \frac{\alpha r t_2^{\beta+3}}{(\beta+2)(\beta+3)} - t_2t_1^2 + \frac{t_1^2}{2} + \frac{\alpha t_1 t_1^{\beta+1}}{\beta+1} - \frac{\alpha t_1^{\beta+2}}{\beta+2} \right) \]

\[ + \frac{rt_2^4}{(\beta+2)(\beta+4)} \left( \frac{t_1^3}{3} + \frac{t_1^2 t_1^{\beta+1}}{(\beta+1)} - \frac{t_1 t_1^{\beta+2}}{(\beta+2)} + \frac{t_1^{\beta+3}}{2} - \frac{r t_1^2 t_1^{\beta+2}}{4} - \frac{r t_1^{2\beta+3}}{4} \right) \]

\[ + \frac{\alpha (\beta+1)}{(\beta+2)} \left( t_2^{\beta+2} - \frac{\alpha r t_2^{\beta+3}}{(\beta+1)} + \frac{\alpha r t_1^{2\beta+3}}{2(\beta+3)} \right) + \frac{\alpha r t_1^{2\beta+3}}{2(\beta+3)} \]

\[ + \frac{\alpha}{(\beta+1)} \left( t_1^{\beta+1} - \frac{\alpha r t_1^{\beta+2}}{2(\beta+2)} - \frac{t_1^2 t_1^{\beta+1}}{(\beta+3)} \right) + \frac{\alpha r t_1^{2\beta+3}}{2(\beta+3))} \]

Owned warehouse

\[ = c_o \left( \int_{t_1}^{t_2} e^{-r t} I_{o_1}(t) \, dt + \int_{t_2}^{T} e^{-r t} I_{o_2}(t) \, dt \right) \]

\[ = c_o \left( W(t_2 + \alpha t_2^{\beta+1} + \frac{rt_2}{2} - \frac{\alpha r t_2^{\beta+2}}{\beta+2} - t_1 + \frac{\alpha t_1^{\beta+1} t_1}{(\beta+1)} \right) \]

\[ + \frac{rt_2}{2} - \frac{\alpha r t_2^{\beta+2}(\beta+4)}{2(\beta+2)} + \mu \left( T^2 \right) + \frac{\alpha T^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{r T^3}{6} + \frac{r \alpha T^{\beta+2}}{(\beta+2)} - \frac{r \alpha T^{\beta+3}}{(\beta+3)} \]
\[ -T t_2 + \frac{\alpha T t_2^{\beta+1}}{\beta+1} - \frac{\alpha t_2^{\beta+2}}{\beta+2} + \frac{r T t_2^2}{2} + \frac{r t_2^3}{3} - \frac{r \alpha t_2^{\beta+2}}{\beta+2} + \frac{r \alpha t_2^{\beta+3}}{\beta+3} \]

\[ + \frac{\mu \beta}{2} \left( \frac{2 T^3}{3} - \frac{2 \alpha T^{\beta+3}(\beta+2)}{\beta+3} \right) - \frac{r T^4}{4} + \frac{2 \alpha r T^{\beta+4}}{(\beta+2)(\beta+4)} - T^2 t_2 + \frac{t_2^3}{3} + \frac{\alpha T^2 t_2^{\beta+1}}{\beta+1} + \frac{\alpha t_2^{\beta+3}}{\beta+3} + \frac{r T^2 t_2^{\beta+2}}{4} - \frac{\alpha r T t_2^{\beta+2}}{(\beta+2)} + \frac{\alpha r t_2^{\beta+1}}{(\beta+2)} \]

Present value deterioration cost

\[ = c_D \left[ \int_t^1 \alpha \beta e^{-\gamma} t_2 (t) dt + \int_{t_2}^{t_1} \alpha \beta e^{-\gamma} t_2 (t) dt + \int_{t_2}^T \alpha \beta e^{-\gamma} t_2 (t) dt \right] \]

(12)

\[ = c_D \alpha \beta \left( W \left( \frac{t_2^{\beta+1}}{\beta(\beta+1)} - \frac{r t_2^{\beta+2}}{(2\beta)(2\beta+1)} + \frac{\alpha r t_2^{\beta+1}}{\beta(\beta+1)} - \frac{t_2 t_2^{\beta+1}}{2\beta+1} + \frac{t_1^{\beta+1}}{\beta+1} \right) \]

\[ + \frac{t_2^{\beta+1}}{\beta+1} + \frac{t_2^{\beta+2}}{\beta+2} \right) - \frac{\alpha r T^{\beta+1}}{(\beta+1)(2\beta+1)} \]

\[ + \frac{\beta \alpha r t_2^{(2\beta+1)}}{(\beta+1)(2\beta+1)} + \mu (T^{\beta+1}) - \frac{T^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{T t_2^{\beta+1}}{(\beta+1)} + \frac{t_2^{\beta+1}}{\beta} \]

\[ + \frac{T a t_2^{(2\beta+1)}}{2\beta + 1} - \frac{r t_2^{(2\beta+1)}}{\beta+1} - \frac{r t_2^{(2\beta+2)}}{\beta+2} - \frac{r a t_2^{(2\beta+1)}}{2\beta+1} + \frac{r t_2^{(2\beta+1)}}{2\beta+2} - \frac{\beta r t_2^{(2\beta+2)}}{2\beta+2} \]

\[ - \frac{\alpha T^{(2\beta+2)}}{\beta(2\beta+2)} + \frac{r T^{(2\beta+3)}}{(\beta+1)(\beta+3)} + \frac{2 r T^{(2\beta+3)}}{(\beta+1)(2\beta+3)} + \frac{T^{(2\beta+2)}}{\beta+1} + \frac{T^{(2\beta+1)}}{\beta} + \frac{t_2^{(2\beta+1)}}{\beta+2} + \frac{t_2^{(2\beta+2)}}{\beta+2} + \]

\[ - \frac{r t_2^{(2\beta+3)}}{(\beta+3)} + \frac{\alpha r T^{(2\beta+1)}}{(\beta+1)(2\beta+3)} + \frac{\mu a t_2^{(2\beta+3)}}{\beta(2\beta+1)} \cdot \frac{T^{(2\beta+1)}}{2\beta+2} - \frac{r T^{(2\beta+2)}}{\beta+2} \]
Present value shortage cost

\[
= -c_p \int_0^{t_1} e^{-rt} I_B(t) dt = \frac{\mu c_p e^{-\delta t_1}}{(\lambda + \delta)} \left( \frac{1}{r} (e^{-r t_1} - 1) + \frac{1}{(\lambda + \delta + r)} (e^{(\lambda + \delta + r) t_1} - 1) \right)
\]

(13)

Present value Lost sale cost

\[
= c_l \int_0^{t_1} e^{-rt} (1 - e^{-\beta(t-1)}) \phi(t) dt = \mu c_l \delta \left( \frac{t_1 + (\lambda - r) t_1^2}{2} - \left( t_1^2 + \frac{(\lambda - r) t_1^3}{3} \right) \right)
\]

(14)

Based on the parameter values \(t_1, t_2, M, T\), there are three cases to be explored.

**Case 1:** \(t_1 + M < t_2\),

Interest paid

\[
= c_p I \left( \int_{t_1+M}^{t_2} e^{-rt} (I_r(t) + I_{o_1}(t)) dt + \int_{t_2}^{T} e^{-rt} I_{o_2}(t) dt \right)
\]

(15)

\[
= c_p I \left( \frac{t_2^2}{2} - \frac{\alpha t_2^{\beta+2}}{6} + \frac{\alpha r (t_2)^{\beta+3}}{(\beta+2)(\beta+3)} - \frac{r t_2^2 (t_1 + M)}{2} - \frac{\alpha r t_2 (t_1 + M)^{\beta+2}}{3} + \frac{\alpha r (t_1 + M)^{\beta+3}}{\beta+3} \right)
\]

\[
+ \frac{\lambda}{2} \left( \frac{2 t_3^2}{3} - \frac{2 \alpha t_2^{\beta+3}}{\beta+3} + \frac{2 \alpha r t_2^{\beta+4}}{\beta+4} - t_2^2 (t_1 + M) + \frac{(t_1 + M)^3}{3} \right)
\]

\[
+ \frac{\alpha t_2^2 (t_1 + M)^{\beta+4}}{(\beta+1)} - \frac{\alpha t_2^2 (t_1 + M)^{\beta+3}}{\beta+3} + \frac{r t_2^2 (t_1 + M)^{\beta+2}}{4} - \frac{\alpha r t_2 (t_1 + M)^{\beta+3}}{\beta+2} + \frac{\alpha r (t_1 + M)^{\beta+4}}{\beta+4}
\]

\[
+ \frac{\alpha}{\beta+1} \left( \frac{t_2^{\beta+2}(\beta+1)}{2(\beta+2)} - \frac{(\beta+1) r t_2^{\beta+3}}{2(\beta+3)} + \frac{(\beta+1) r \alpha t_2^{2\beta+3}}{(\beta+2)(2\beta+3)} \right)
\]

-25-
\[-t_2^\beta (t_1 + M) + \frac{(t_1 + M)^{2\beta+2}}{(\beta + 2)} + \alpha (t_1 + M)^{\beta+1} t_2^{\beta+1} \frac{t_1 + M}{2} - \frac{r t_2^{\beta+1} (t_1 + M)^2}{2} - \frac{r (t_1 + M)^{\beta+3}}{(2\beta + 3)} \]
\[-\alpha r (t_1 + M)^{\beta+1} t_2^{\beta+1} + \frac{\alpha r (t_1 + M)^{2\beta+3}}{(2\beta + 3)} \)
\[[c_p t_2 + \alpha t_2^2 (t_1 + M) \frac{r t_2^{\beta+1}}{2} - \frac{\alpha r t_2^{\beta+2}}{\beta + 2} - \frac{\alpha r t_2^{\beta+2}}{\beta + 2} - (t_1 + M) + \alpha (t_1 + M)^{\beta+1} + \frac{\alpha (t_1 + M)^{\beta+1}}{\beta + 1} \]
\[+ \frac{r (t_1 + M)^2}{2} - \frac{\alpha r (t_1 + M)^{\beta+2}}{2} - \frac{\alpha r (t_1 + M)^{\beta+2}}{(\beta + 2)} + \mu (\frac{T^2}{2} - \frac{\alpha T^{\beta+2}}{(\beta + 1)(\beta + 2)} - \frac{r T^3}{6} \]
\[+ \frac{r a T^{\beta+2}}{\beta + 2} - \frac{r a T^{\beta+3}}{(\beta + 3)} - T^2, r^2 \frac{r t_2^{\beta+1}}{\beta + 2} + \frac{\alpha r t_2^{\beta+2}}{\beta + 2} + \frac{r T^2 t_2^{\beta+2}}{\beta + 2} - \frac{3 r t_2^{\beta+3}}{\beta + 3} + \frac{\alpha r t_2^{\beta+3}}{\beta + 3} \]
\[+ \frac{\mu \alpha (T^{\beta+2} (\beta + 1) - 2 a T^{\beta+3} (\beta + 2) - T^4)}{2(\beta + 1)(\beta + 2)} - \frac{2 a r T^{\beta+4}}{2(\beta + 2)(\beta + 4)} - T^2 \frac{t_2^{\beta+3}}{3} + \frac{\alpha T^2 t_2^{\beta+1}}{\beta + 3} \]
\[+ \frac{\mu a}{\beta + 1} \frac{T^{\beta+2} (\beta + 1)}{\beta + 2} - \frac{4(\beta + 3)^2 T^{\beta+2}}{2(\beta + 1)^2} - \frac{(\beta + 1) r T^{\beta+3}}{2(\beta + 3)} + (\beta + 1) r T^2 \frac{T^{\beta+3}}{2(\beta + 3)} \]
\[-T^{\beta+1} t_2 + \frac{\alpha T^{\beta+1} t_2^{\beta+1}}{(\beta + 1)} + \frac{2 t_2 \beta+1}{(2\beta + 2)} - \frac{r T^{\beta+2} t_2^{\beta+2}}{2} + \frac{r T^{\beta+3} t_2^{\beta+2}}{\beta + 3} - \frac{\alpha r T^{\beta+1} t_2^{\beta+2}}{(2\beta + 3)} + \frac{r a t_2^{\beta+3}}{2(2\beta + 3)} \]

Interest earned

\[= p I_e \left( \int_{t_1}^{t_i + M} e^{-r t} (t - t_1) D(t) dt + M e^{-r t_i} I_B(t_1) \right) \]

(16)

\[= p I_e \left( \frac{(t_1 + M)^2}{2} + t_1 (t_1 + M) + \frac{\lambda}{(2\beta + 2)} (t_1 + M)^2 t_1 - \frac{r (t_1 + M)^2}{3} + r t_1 (t_1 + M)^2 \right) \]
\[+ \frac{r \lambda t_1 (t_1 + M)^3}{3} + \frac{1}{2} t_1^2 + \frac{\lambda}{6} t_1^3 - \frac{r \lambda t_1^4}{12} + p I_e M e^{-r t_1} \left( \frac{\mu e^{-\delta t}}{\lambda + \delta} \right) \left( 1 - e^{(\lambda + \delta) t} \right) \]

Case 2: \( t_2 \leq t_1 + M < T \),
Interest paid

\[ I_p(t) = c_p \int_{t_1}^{t_2} e^{-\gamma t} I_{o_1}(t) + \int_{t_2}^{T} e^{-\gamma t} I_{o_2}(t) \, dt \]  
(17)

Interest earned is same as case 1

**Case 3:** \( T \leq t_1 + M \)

Interest paid = 0

Interest earned

\[ = pI_e \left[ e^{-\gamma t} (t - t_1) D(t) \, dt + \int_{t_1}^{t_2} e^{-\gamma t} (T - t_1) D(t) \, dt + \int_{T}^{T} e^{-\gamma t} I_{o_1}(t) \right] \]  
(18)

The total cost for the retailer is given by

\[ TC(t_1, t_2, T) = \text{Present value ordering cost} + \text{Present value holding cost in rented warehouse} + \text{Present value holding cost in owned warehouse} + \text{Cost of deterioration} + \text{Cost of Lost sale} + \text{Interest payable} - \text{Interest earned}. \]

That is,

\[ TC(t_1, t_2, T) = \begin{cases} 
TC_1, & \text{if } t_1 + M < t \\
TC_2, & \text{if } t_2 \leq t_1 + M < T \\
TC_3, & \text{if } T \leq t_1 + M 
\end{cases} \]  
(19)

**4. NUMERICAL EXAMPLE**

The inventory model is explained through the numerical example for which the input parameter are as follows:

- \( A = 0.01 \) units, \( \lambda = 2 \) units, \( \mu = 0.2 \) units, \( \alpha = 0.05 \) units, \( \beta = 0.03 \) units, \( r = 0.01 \) units, \( W = 500 \) units
- \( M = 0.25 \), \( C_o = 1.2 \text{ Rs/units} \), \( C_R = 0.6 \text{ Rs/units} \), \( C_b = 0.5 \text{ Rs/units} \), \( c_D = 0.6 \text{ Rs/units} \), \( c_l = 0.3 \text{ Rs/units} \)
- \( C = 0.3 \text{ Rs/units} \), \( p = 0.2 \text{ Rs/units} \), \( I_e = 0.25 \text{ Rs/units} \), \( I_c = 0.15 \text{ Rs/units} \)

For above mentioned parametric values, the optimal solution has been found. Using software Mathematica, the results are as follows: The optimal values are

- \( t_1 = 0.2947 \), \( t_2 = 0.7651 \), \( T = 1.65132 \), \( TC = 47.3725 \)
5. SENSITIVITY ANALYSIS

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6. CONCLUSION

The purpose of the inventory model is to calculate the time of order that optimize the total cost of the system. The inventory cost such as cost of holding stock in rented warehouse is higher than that of warehouse that is owned. Due to permissible delay, the quantity purchased by the retailer is greater than the capacity of warehouse that is owned. This paper helps to minimize the additional cost of holding material etc. For future scope, the model can be extended for two-level permissible delay in payment, learning effect, stochastic demand etc.

References


