



World Scientific News

An International Scientific Journal

WSN 109 (2018) 211-234

EISSN 2392-2192

Performance probability distribution function for modeling visibility for free space optical link in Nigeria

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ABSTRACT

This paper attempt to assess the performance of the most appropriate distribution function for modeling atmospheric visibility over five selected locations (Akure, Enugu, Ikeja, Jos and Port-Harcourt) in Nigeria. The distribution functions are tested based on 5-year (2010-2014) visibility data obtained from Nigeria Meteorological Agency (NIMET). Five distribution functions are tested to determine most suitable one based on four different metric measures. The associated parameters of the most suitable fitted distribution are estimated and the trends in the characteristic of the visibility are deduced. The result shows that lognormal distribution present the best probability distribution function for modeling visibility over the selected locations. A typical result for one of the locations (Akure) show that the lognormal distribution has Root Mean Square Error (RMSE) of 0.0766 km, Mean Absolute Error (MAE) of 0.0095 km, Mean Absolute Percentage Error (MAPE) of 17.8 % and coefficient of determination (R^2) of 0.87. When compared with other distribution functions, the same trend could be seen in other locations although with different values of RMSE, MAE, MAPE and R^2 . The location and the scale parameters for the total distribution varies seasonal-wise and are location depended. The overall result will be useful for predicting future visibility over the locations. It will also be a good tool for free space optical link design in Nigeria.

Keywords: Free Space Optical, distribution functions, visibility, lognormal, Nigeria

1. INTRODUCTION

Atmospheric visibility is an important input for road and air transportation safety, as well as a good proxy to estimate the air quality; it is the most direct way to access the level of air pollution in any region of the world. Depending on the area of interest and application, visibility in meteorology is defined as maximum distance at which a dark object can be discerned against a light sky [1-2] on the other hand, it is defined in aviation as the greatest horizontal distance at which a large object can be seen and recognized against a bright sky [3]. The accurate prediction of visibility even over short term (0–6 h) forecasting periods is challenging. Since most numerical weather prediction models do not explicitly model visibility, forecasts of visibility must first be derived from other meteorological parameters such as cloud water content, relative humidity, and precipitation [4].

In the past few decades, researchers have developed several methods of forecasting visibility. For example, Jim *et al.*, [5] worked on prediction of visibility and aerosol within the operational met office unified model and validation of model performance using observational data. The modeling relies on parameterizations for the aerosol size distribution which account for the coagulation of aerosols into the accumulation mode, and for the hygroscopic uptake of water by aerosols. Richard and Adrian [4], worked on probabilistic visibility forecasting using Bayesian Model Averaging (BMA). In the study, BMA was applied to probabilistic visibility forecasting using a predictive PDF that is a mixture of discrete point mass and beta distribution components.

Vislocky and Fritsch [6] also compared the performance of observation-based, MOS-based, and persistence climatology models for short-term deterministic ceiling and visibility forecasts. Marzban *et al.* [7] applied neural networks to probabilistic visibility forecasting. Over the years visibility data has been related to weather forecasting for flight control and safety. However, recent studies revealed that the devices that operate at terahertz (THz) frequencies over free space optical (FSO) link are majorly affected by atmospheric fog [8]. It is then imperative to make use of visibility data to assess the level of the signal degradation cause by fog on FSO link. The assessment in this case is through probability distribution function.

Although, the visibility has been predicted using different empirical models, but, none has modeled the visibility using distribution function in Nigeria to the best knowledge of the author. Knowing the distribution function and the parameters of the distribution of visibility data of a location allows someone to be able to generate data that will have the same characteristic as the actual data of the location in the future.

This is important as it can serve as the starting point for design analysis of any communication links. In this study, the most appropriate probability distribution function that can best model visibility data over some selected locations in Nigeria is determined using statistical goodness of fit. The parameters of the distribution are determined and the variation in the visibility of the study location is presented.

The rest of the paper is structured as thus: section 2 presents information on the sites and how data are acquired while section 3 discusses the methodology adopted. Results and discussion are presented in section 4 while conclusions are drawn out in section 5.

2. SITE AND DATA ACQUISITION

The selected locations for the study are Akure (Ondo State), Enugu (Enugu State), Ikeja (Lagos State), Jos (Plateau State) and Port-Harcourt (River State) in Nigeria. The study areas cover some of the segmented airport in Nigeria where data are readily available. Nigeria lies between latitudes 4° and 14°N, and longitudes 2° and 15°E in West Africa. It has two different seasons: the dry season, influenced by the Northwest trade wind and runs normally from November to March; and the rainy season, which runs from April to October and is also influenced by the South-East trade winds. Most of the rainy season months are associated with heavy rainfall, which is sometimes accompanied by thunderstorms. The coastal region in Nigeria experiences rainfall throughout the year [9]. The visibility data required for the study are taken over the selected locations at 0900 hour of the day. Table 1 presents the characteristics of the locations, while Figure 1 shows the Map of Nigeria depicting the areas under study.

Table 1. Description of the study locations.

Location	Coordinate °N °E	Altitude above sea level (m)	Average annual accumulation (mm /year)
Akure	7.18 5.12	303.00	1485.57
Enugu	6.24 7.24	139.00	1876.30
Ikeja	6.46 3.38	41.00	1425.20
Jos	9.50 8.50	1217.00	1186.89
Port-Harcourt	4.42 7.02	18.00	2803.10

2. 1. Visibility and Fog

Table 2. International Visibility Code for different types of weather conditions [13].

Weather condition	Visibility (m)
Dense fog	50
Thick fog	200
Moderate fog	500
Light fog	770-1000
Very light fog	1000-2000
Light mist	2000-2800
Very light mist	4000-10000
Clear air	18000-20000
Very clear air	23000-50000

The distance to which human visual perception is limited by atmospheric conditions is called visibility. The physical mechanisms that influence visual perception during the night in distinguishing lights differ from those in the day time in distinguishing objects illuminated by daylight [10]. Basically, meteorologist describes visibility as the transparency of the air in the horizontal direction and represents the maximum distance that one can see in the atmosphere at any given time [11]. However the transparency of the atmosphere can be influenced by the presence of hydrometeors such as rain, snow, mist, fog or litho meteors such as dust, smoke among others. Fog is the visible cloud of small water droplets suspended in the atmosphere at or air near the earth’s surface, thereby scattering the incident light and hence reducing the visibility [12]. Different types of fog result in different levels of optical losses and this is mainly due to the distribution of the fog particles, size and the location. Table 2 shows the International Visibility Code for different types of weather conditions. Hence the amount of fog in the air/atmosphere determines the level of the visibility of the atmosphere.

3. METHODOLOGY ADOPTED

The understanding of existing visibility data of a location is a fundamental requirement before embarking on any installation and in attaining a reliable FSO wireless communication links in a given location. The atmospheric transmittance at visibility deterioration is usually measure using a transmissometer. The instrument is normally installed at the runways of airports in order to determine the visual range for the flight control safety services. The instrument measures the visibility at different synopsis hours of the day. For the sake of this study, only 9-hour of the day is considered due to the availability of the data. The availability of the equipment over the study period varies. For example the equipment is available for 96% of the year at Akure, while it is available for 93%, 97%, 95% and 94% at Enugu, Ikeja, Jos and Port-Harcourt respectively, the remaining % unavailable are mainly due to equipment maintenance. Four statistical distribution functions were used in this study to model the visibility data over the study locations. It is impractical to report all statistical distribution functions as they are numerous to be accommodated in the paper. It is worth mentioning that several probability distribution were tested, however, the best ones that are appropriate in terms of fitting were repeated in this paper. The models adopted for this study are: gamma, Rayleigh, normal and lognormal distributions.

3. 1. Gamma distribution

The probability density function and cumulative distribution function of a gamma distribution are given respectively as.

$$f_{GM} = \frac{x^{a-1}}{b^a \Gamma(a)} \exp \left[-\left(\frac{x}{b}\right) \right] \quad a, b > 0 \quad (1)$$

$$F_{GM} = \frac{1}{b^a \Gamma(a)} \int_0^x t^{a-1} \exp \left[-\left(\frac{t}{b}\right) \right] dt \quad (2)$$

where x is the measured visibility data, a is the scale parameter while b is the shape parameter of the distribution, $f, F_{(GM)}$ is the gamma function.

3. 2. Lognormal distribution

The probability density function and cumulative distribution function of lognormal distribution are also given respectively as:

$$f_{LN}(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp \frac{-(\ln x - \mu)^2}{2\sigma^2} \quad (3)$$

$$F_{LN}(x) = \frac{1}{x\sigma\sqrt{2\pi}} \int_0^x \frac{\exp \frac{-(\ln(t) - \mu)^2}{2\sigma^2}}{t} dt \quad (4)$$

where x is the measured visibility data, μ is the scale parameter while σ is the shape parameter of the distribution.

3. 3. Normal distribution

The probability density function and cumulative distribution function of normal distribution are given respectively as:

$$f_N(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp \frac{-(x - \mu)^2}{2\sigma^2} \quad (5)$$

$$F_N(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp \frac{-(t - \mu)^2}{2\sigma^2} dt \quad (6)$$

where x is the measured visibility data, μ is the scale parameter while is the shape parameter of the distribution.

3. 4. Rayleigh distribution

For the Rayleigh distributions, probability density function and cumulative distribution function are respectively given as [14]:

$$f_R(x) = \frac{x}{b^2} \exp \left(\frac{-x^2}{2b^2} \right) \quad (7)$$

$$F_R(x) = \int_0^x \frac{t}{b^2} \exp \left(\frac{-t^2}{2b^2} \right) dt \quad (8)$$

where, x is the measured visibility data, and b is the scale parameter of the distribution.

3. 5. Assessing the performance of the distribution functions

In order to check how accurately a theoretical distribution function fits with measured data, four different statistical goodness of fits were considered as benchmark. The metric measures are: the Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE), Root Mean Square Error (RMSE) and Coefficient of Determination (R^2).

The closer the value of RMSE, MAE, MAPE, to zero the better the goodness of fit. Similarly, R^2 refers to the square of correlation coefficient. It is used to determine to what

extent a prediction can be deduce from a model. The relationship between the variables is determined as $0 \leq R^2 \leq 1$ with 1 being the perfect fit. The closer the value of R^2 to 1, the better the fit to the actual variables.

The equations for the criteria of the fitness are given as [15]:

$$\begin{aligned}
 \text{RMSE} &= \sqrt{\frac{1}{n} \sum_{i=1}^n (x_a(i) - x_p(i))^2} \\
 \text{MAE} &= \frac{1}{n} \sum_{i=1}^n (|x_a(i) - x_p(i)|) \\
 \text{MAPE} &= \frac{1}{n} \sum_{i=1}^n \left| \frac{(x_a(i) - x_p(i))}{x_a(i)} \right| * 100\% \\
 R^2 &= \left(\frac{\sum_{i=1}^n ([x_a(i) - E(x_a(i))] \cdot [x_p(i) - E(x_p(i))])}{\sqrt{([\sum_{i=1}^n (x_a(i) - E(x_a(i))]^2) \cdot [\sum_{i=1}^n (x_p(i) - E(x_p(i))]^2)}} \right)^2
 \end{aligned} \tag{9}$$

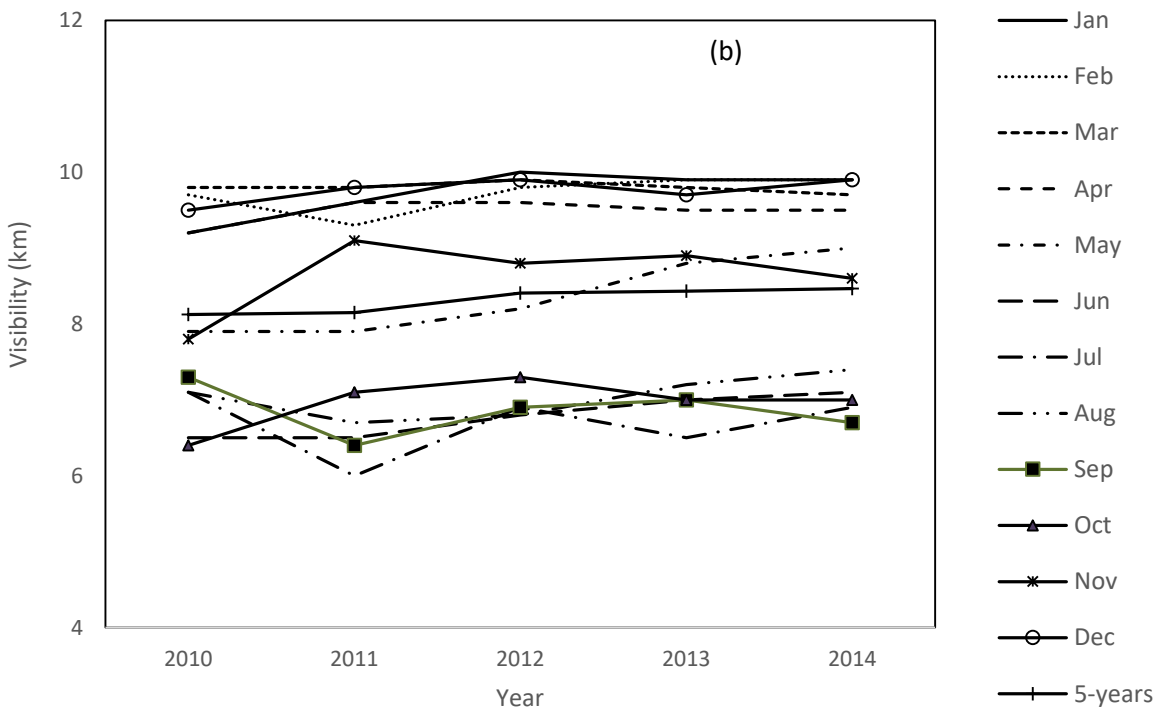
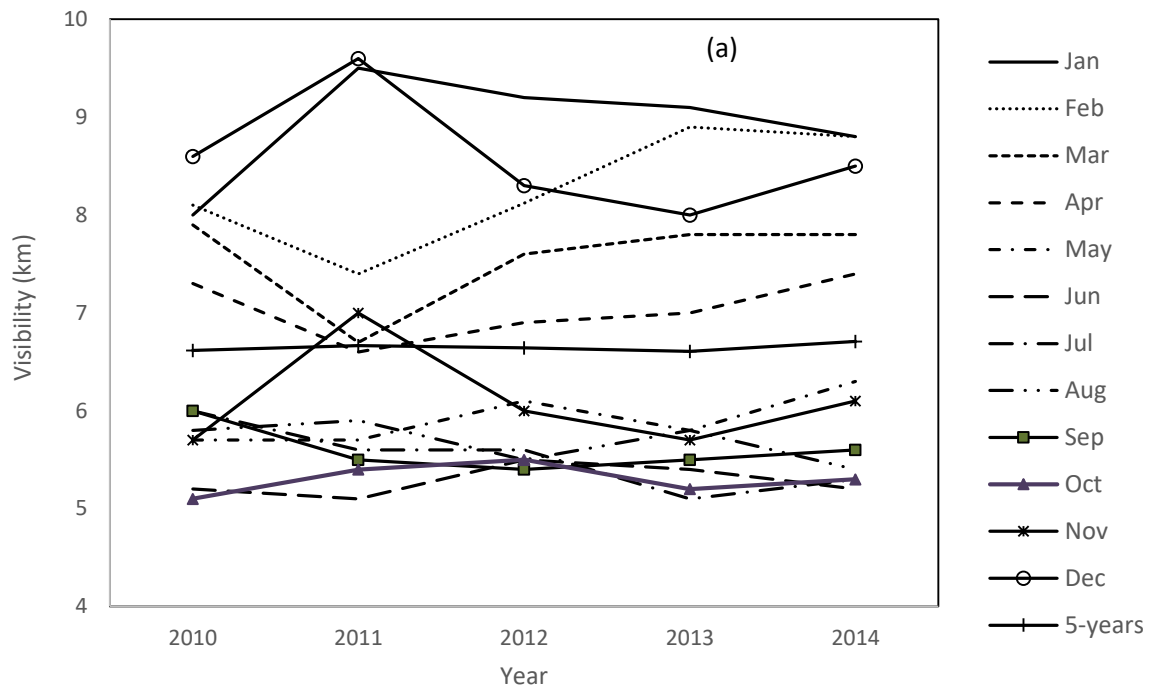
where $x_a(i)$ is the i^{th} measured visibility, $x_p(i)$ is the i^{th} predicted visibility, n is the number of observed visibility.

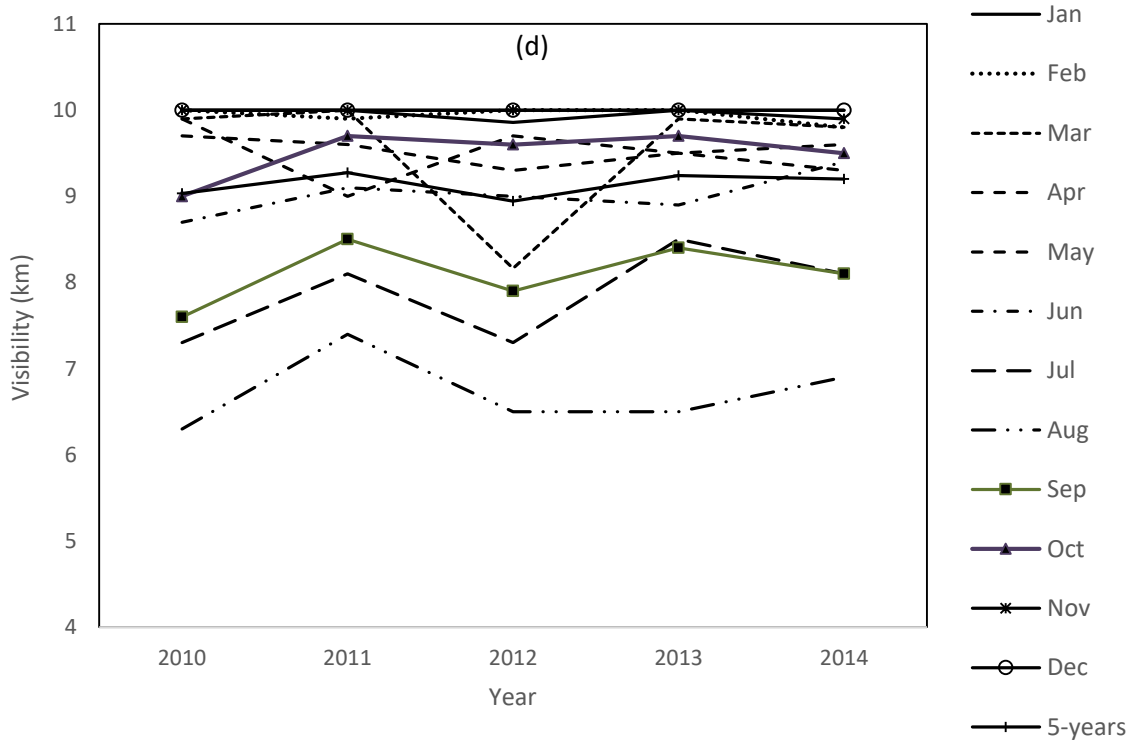
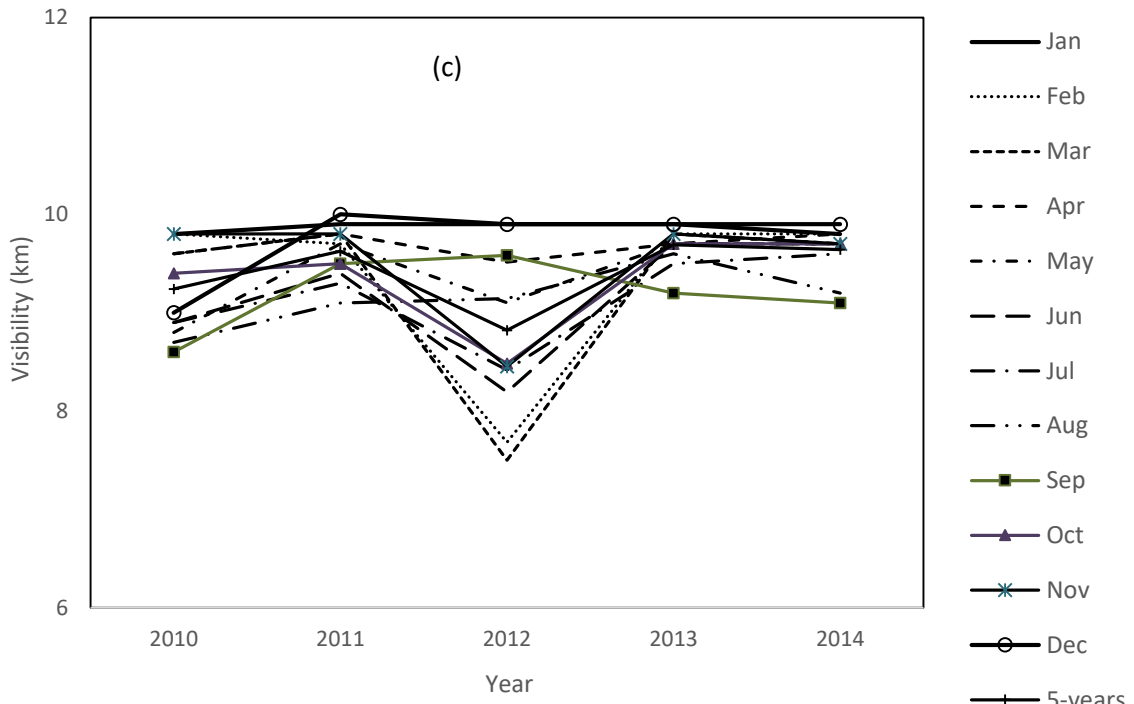
4. RESULTS AND DISCUSSION

Figure 1 (a-e) presents the comparison of the mean monthly visibility at Akure, Enugu, Ikeja, Jos and Port Harcourt respectively for each of the study years. Figure 1(a) shows the comparison of the visibility for Akure location where he month of June in most of the years under study followed by the month of October and July.

The average mean values of visibility for these months as presented in Table 3 are 5.28, 5.30 and 5.52 km, respectively with the standard deviation of 0.15, 0.15 and 0.32 km. Consequently, the month of January recorded the highest visibility value of about 8.92 km with standard deviation of 0.54 km follows by the month of December with the average value of about 8.60 km with standard deviation of 0.57 km.

This is expected since these months are within the dry season of the year when the cloud is dry and clear. The same trend could be seen in other locations as presented in Figures 1 (b-e) although with different values of the lowest and the highest visibility occurring at different months of the year. The summary of the average mean values of visibility for the years under study as well as the standard deviation are presented in Table 3.





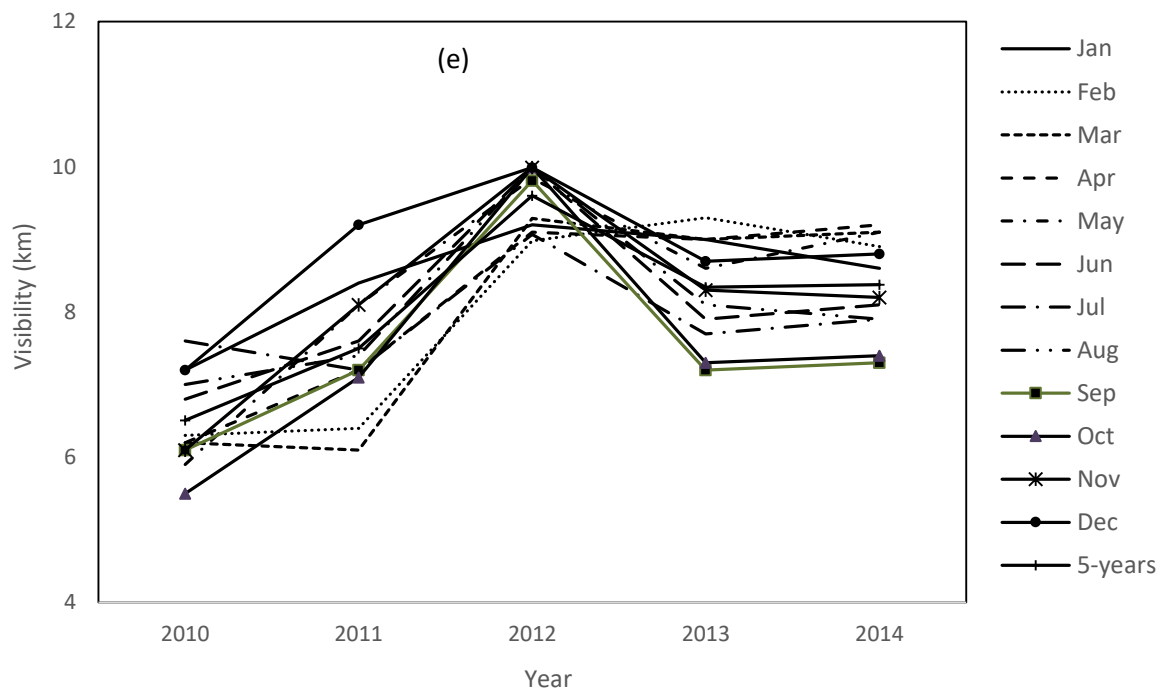


Figure 1. Comparison of the monthly visibility for each of the year at (a) Akure (b) Enugu (c) Ikeja (d) Jos and (e) Port-Harcourt.

Table 3. Average values of visibility over the study period.

Location		Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Akure	Mean	8.92	8.26	7.56	7.04	5.92	5.28	5.52	5.68	5.60	5.30	6.10	8.60
	STD	0.54	0.57	0.46	0.30	0.25	0.15	0.32	0.20	0.22	0.15	0.50	0.57
Enugu	Mean	9.72	9.72	9.80	9.48	8.36	6.78	6.68	7.04	6.86	6.96	8.64	9.76
	STD	0.31	0.23	0.07	0.15	0.48	0.26	0.41	0.27	0.32	0.32	0.47	0.16
Ikeja	Mean	9.86	9.36	9.28	9.68	9.40	9.18	9.14	9.15	9.20	9.36	9.51	9.74
	STD	0.05	0.88	0.94	0.12	0.40	0.60	0.46	0.30	0.37	0.48	0.56	0.39
Jos	Mean	9.97	9.94	9.55	9.54	9.48	9.02	7.86	6.72	8.10	9.50	9.98	10.00
	STD	0.06	0.08	0.73	0.32	0.17	0.24	0.51	0.41	0.35	0.27	0.04	0.00
Port-Harcourt	Mean	8.48	7.98	7.94	8.14	8.31	8.08	7.89	8.08	7.52	7.46	8.14	8.78
	STD	0.74	1.41	1.54	1.29	1.41	1.11	0.67	1.09	1.29	1.52	1.30	0.96

The statistical distribution functions (gamma, lognormal, normal and Rayleigh distributions) are fitted with the actual visibility. Figures 2 (a-e) present the probability function of visibility for Akure, Enugu, Ikeja, Jos and Port-Harcourt respectively. The actual value of visibility data are fitted alongside with PDF of Normal, Gamma, Lognormal and Rayleigh distribution, Figure 3 (a-e) also present the cumulative function of visibility for Akure, Enugu, Ikeja, Jos and Port-Harcourt respectively.

The actual value of visibility data are also fitted alongside with CDF of Normal, Gamma, Lognormal and Rayleigh distribution. It can be observed from the PDF plots that lognormal distribution closely matches with the measured data. The overall results show that most of the locations considered in this study can best be fitted by lognormal function as evident in Tables 3 (a-e) for each of the selected locations. It could also be observed that Cumulative Density Function (CDF) of statistical distribution function perform well than Probability Density Function (PDF). While the PDF graph mainly shows the shape of the data, the CDF graph actually determine how well the distributions fit to data. The location and scales parameters of lognormal distribution (most suitable distribution among the distribution considered) are determined for two distinct seasons observed in Nigeria for the study period, the results are presented in Table 4 (a-b).

The Table shows that the dry season has higher value of location parameters compared to the wet season months. This is because the measured values of visibility are higher during the dry season when compared with the wet season. About 30% of Nigeria’s total land area lies within Sahel belt of West Africa, so dust aerosol are regularly being transported towards Atlantic Ocean [11]. This could cause high concentration of aerosol in the atmosphere in addition to the moisture contents that could led to low visibility. The variation in the seasonal distribution for the five stations are also shown in Figures 4 (a-e) and 5 (a-e) for lognormal probability distribution and cumulative distribution functions respectively.

As earlier stated based on equation (9), the performance evaluation of statistical distribution for modeling diurnal variation of visibility in the five locations over the period of observation are presented in Tables 3 (a-e). It can be observed from the Tables 3, that lognormal distribution presents the best statistical goodness of fit in modeling the visibility data in Akure (For example) with RMSE of 0.0766 km, MAE of 0.0095 km, MAPE of 17.8010 % and R^2 of 0.8658. The same trend could be seen in other locations although with different values of RMSE, MAE, MAPE and R^2 .

Table 3. Performance evaluation of statistical distribution for modeling diurnal variation of visibility in (a) Akure (b) Enugu (c) Ikeja (d) Jos (e) Port-Harcourt.

(a)

Statistical distribution	RMSE (km)	MAE (km)	MAPE (%)	R^2	Order of good fit
Gamma	0.0951	0.0166	36.5519	0.7663	2 nd
Lognormal	0.0766	0.0095	17.8010	0.8658	1 st
Normal	0.0452	0.0126	38.6864	0.3515	3 rd
Rayleigh	0.0815	0.0203	65.1858	0.4527	4 th

(b)

Statistical distribution	RMSE (km)	MAE (km)	MAPE (%)	R ²	Order of good fit
Gamma	0.0203	0.0184	30.9051	0.5433	2 nd
Lognormal	0.0217	0.0163	27.3120	0.6129	1 st
Normal	0.0229	0.0210	42.4076	0.0553	4 th
Rayleigh	0.0298	0.0176	32.3532	0.3705	3 rd

(c)

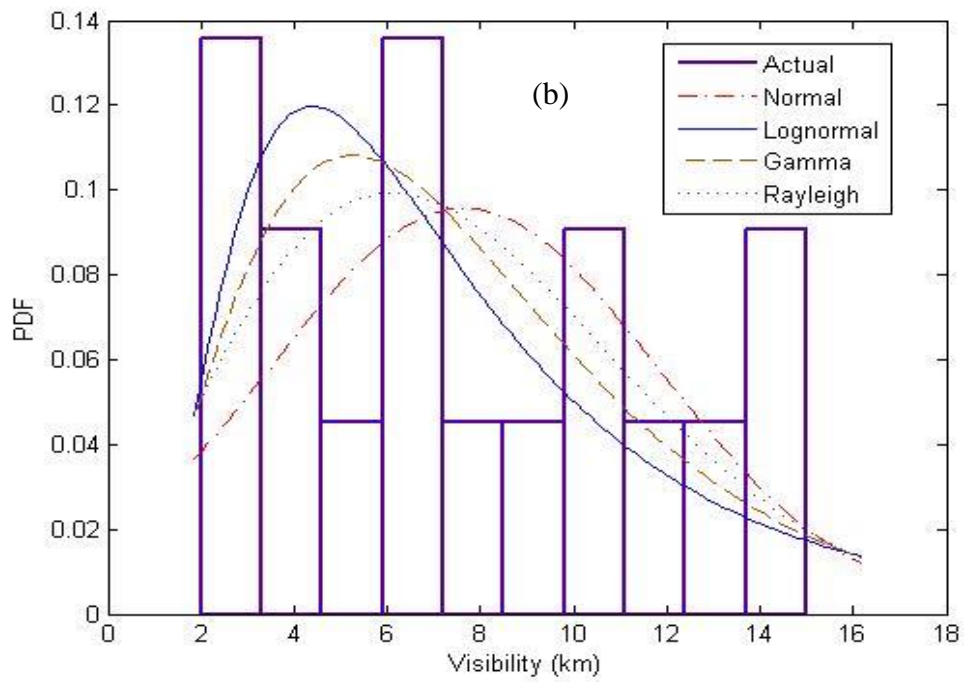
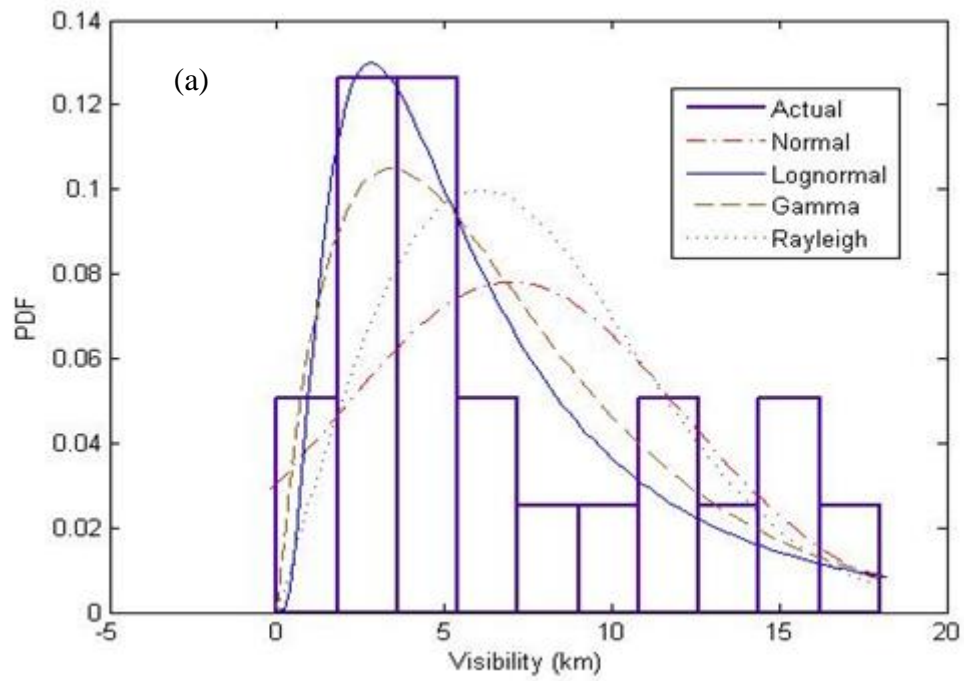
Statistical distribution	RMSE (km)	MAE (km)	MAPE (%)	R ²	Order of good fit
Gamma	0.0305	0.0246	40.0272	0.8020	2 nd
Lognormal	0.0199	0.0166	23.6361	0.8574	1 st
Normal	0.0307	0.0279	78.0102	0.2531	3 rd
Rayleigh	0.0430	0.0387	102.4567	0.3880	4 th

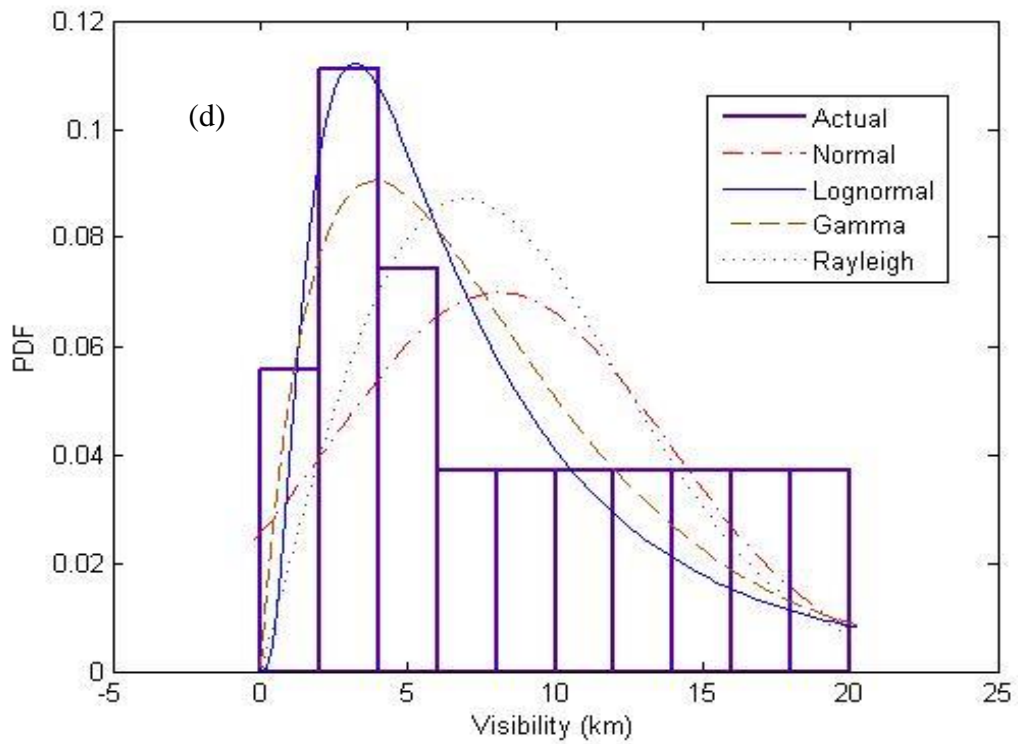
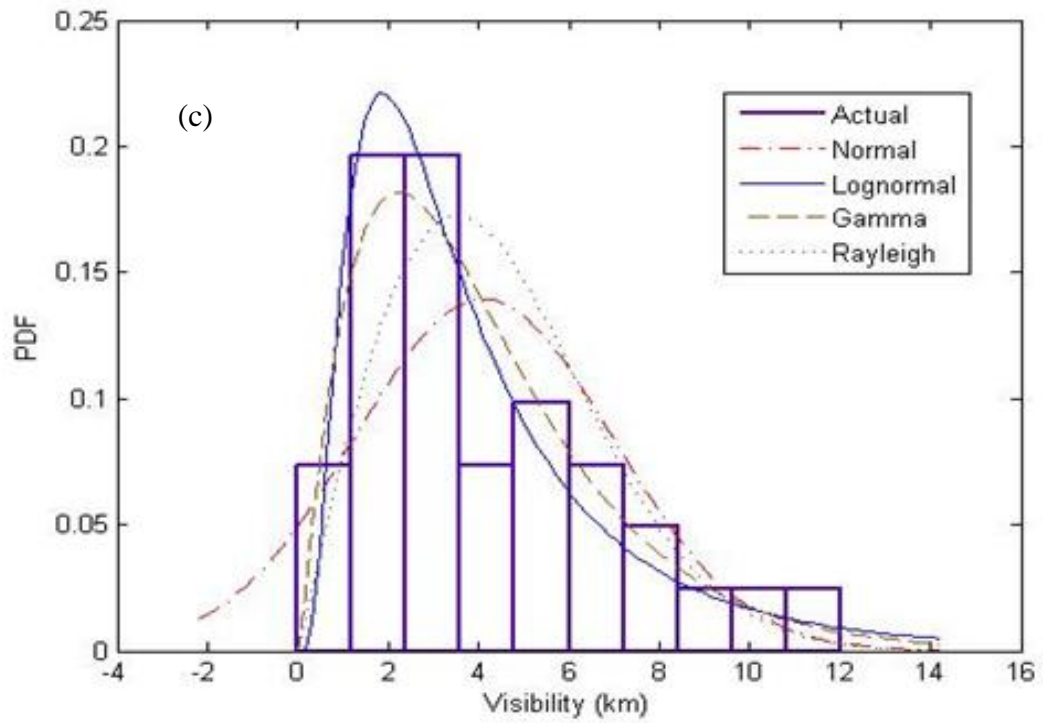
(d)

Statistical distribution	RMSE (km)	MAE (km)	MAPE (%)	R ²	Order of good fit
Gamma	0.0167	0.0133	31.0458	0.8054	2 nd
Lognormal	0.0120	0.0093	17.6906	0.8187	1 st
Normal	0.0144	0.0123	38.5646	0.2297	3 rd
Rayleigh	0.0221	0.0187	63.2272	0.3724	4 th

(e)

Statistical distribution	RMSE (km)	MAE (km)	MAPE (%)	R ²	Order of good fit
Gamma	0.0346	0.0258	42.5321	0.4197	2 nd
Lognormal	0.0268	0.0209	34.0792	0.4913	1 st
Normal	0.0380	0.0313	66.2115	0.1021	4 th
Rayleigh	0.0361	0.0263	45.0494	0.4413	3 rd





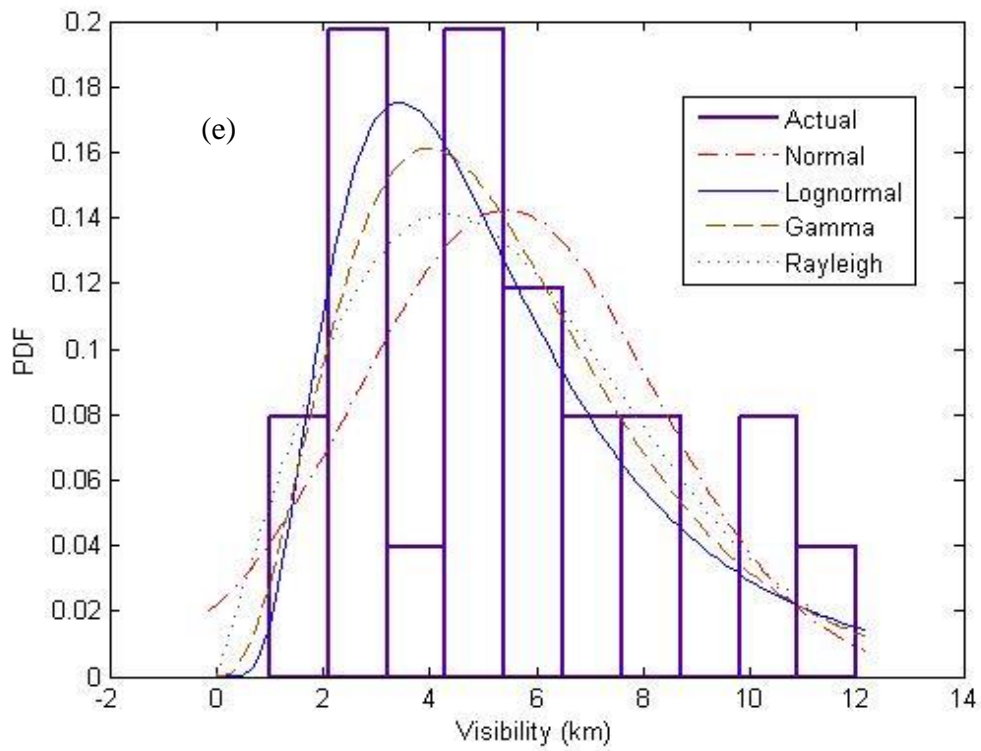
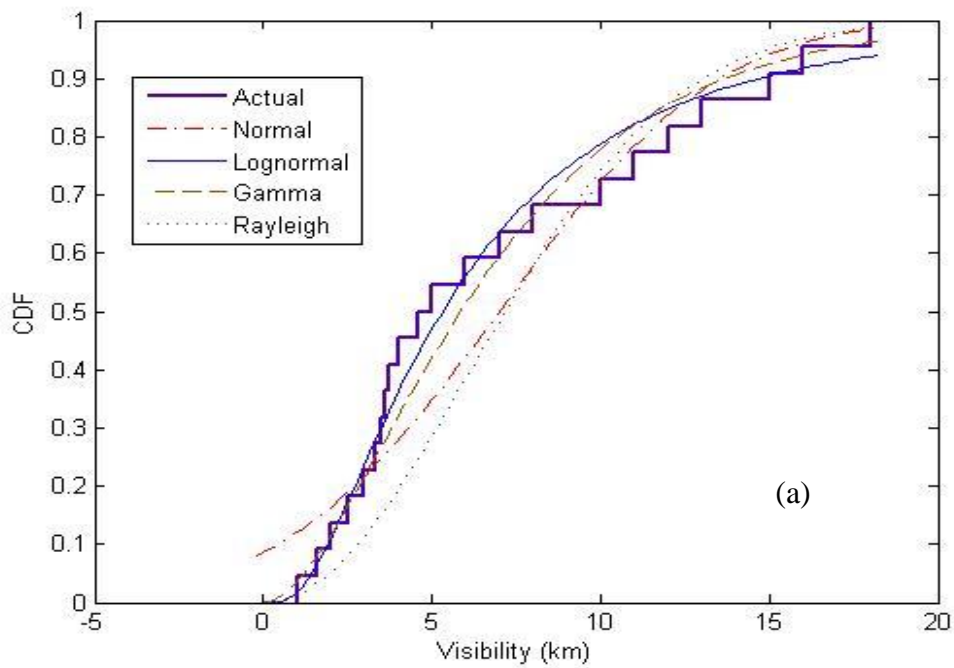
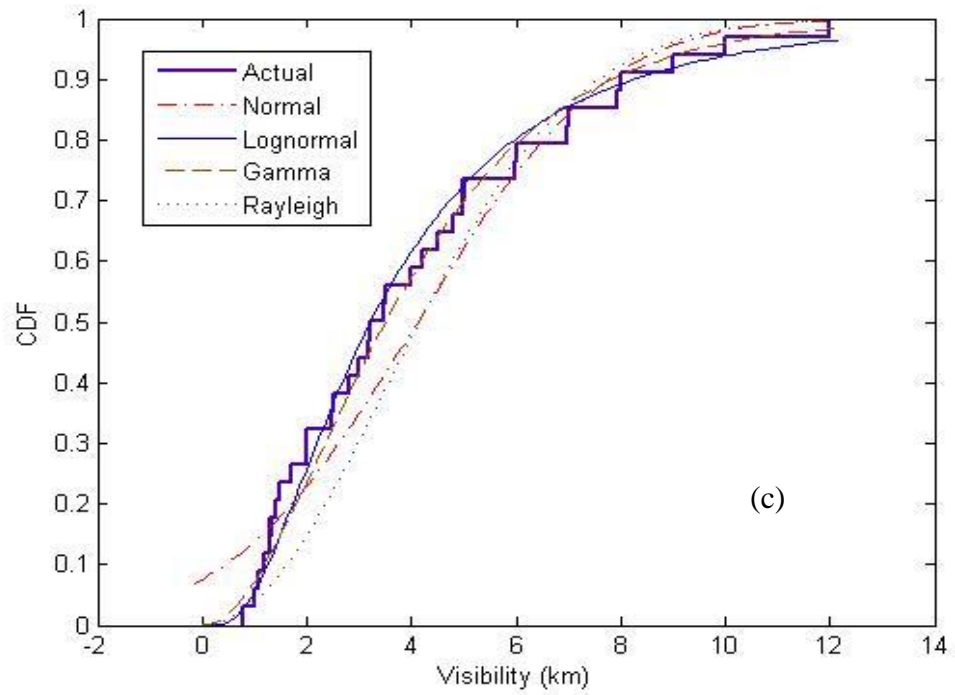
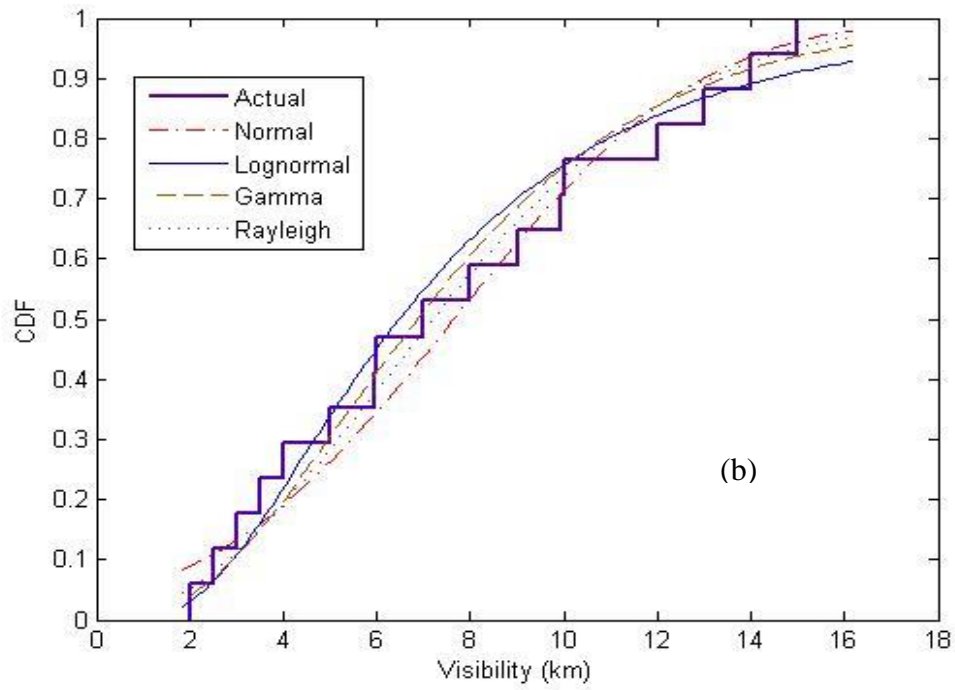


Figure 2. Probability distribution function for (a) Akure (b) Enugu (c) Ikeja (d) Jos and (e) Port-Harcourt.





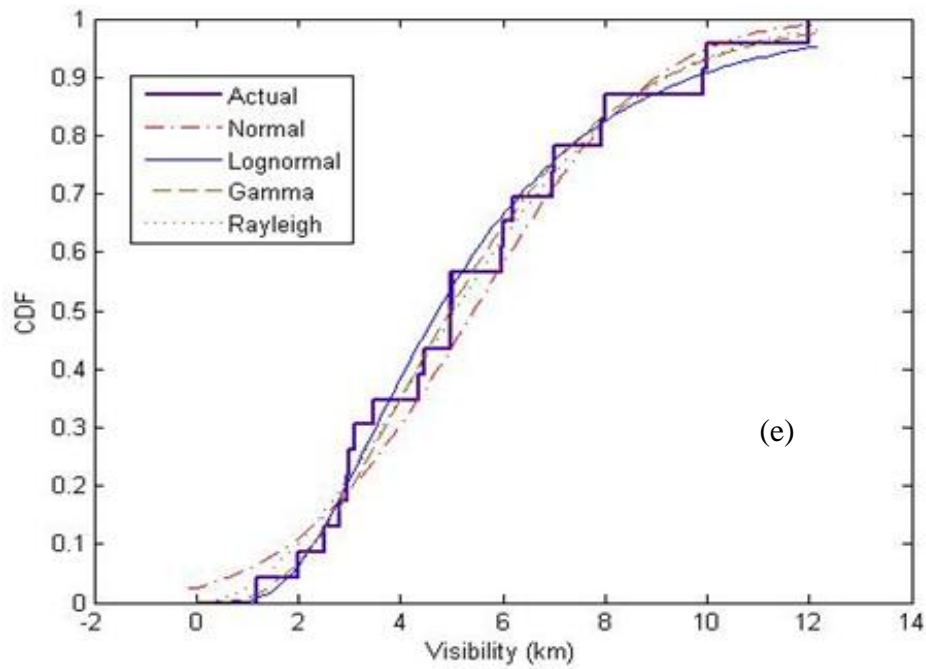
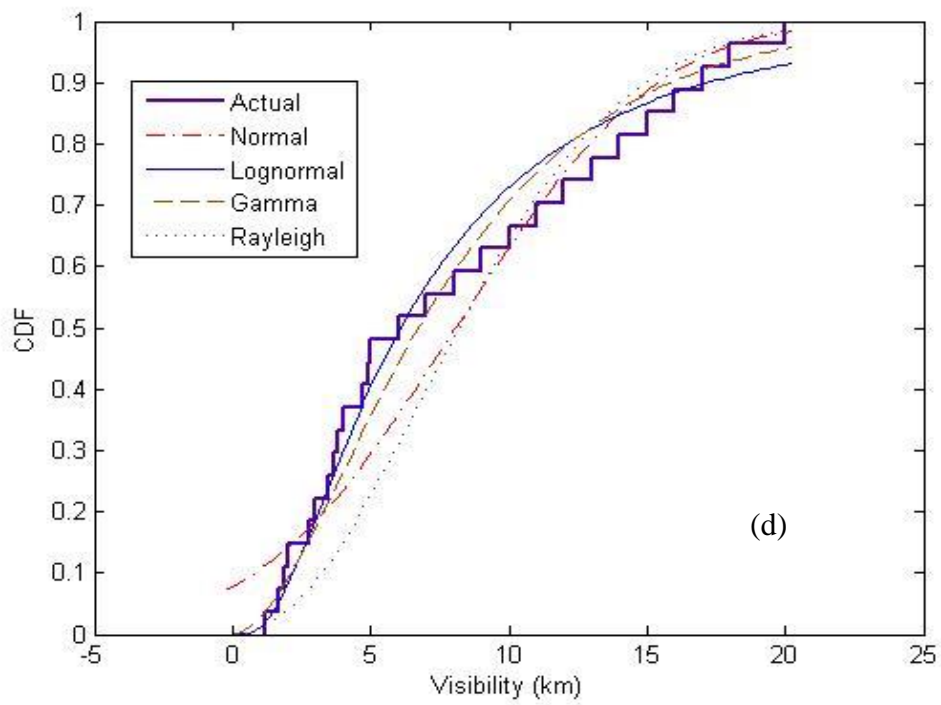


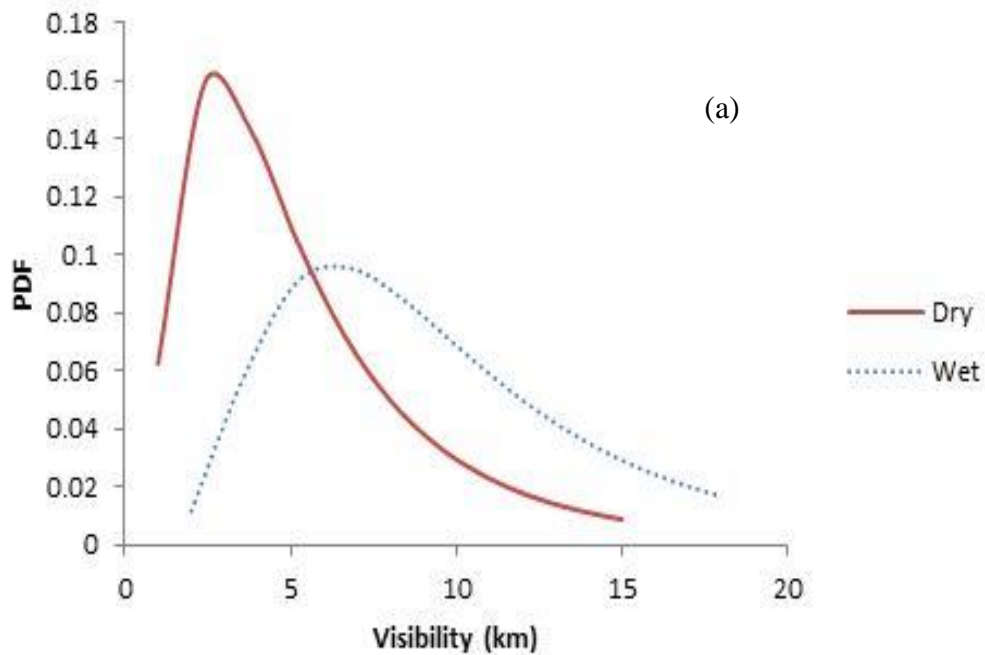
Figure 3. Cumulative distribution function for (a) Akure (b) Enugu (c) Ikeja (d) Jos and (e) Port-Harcourt.

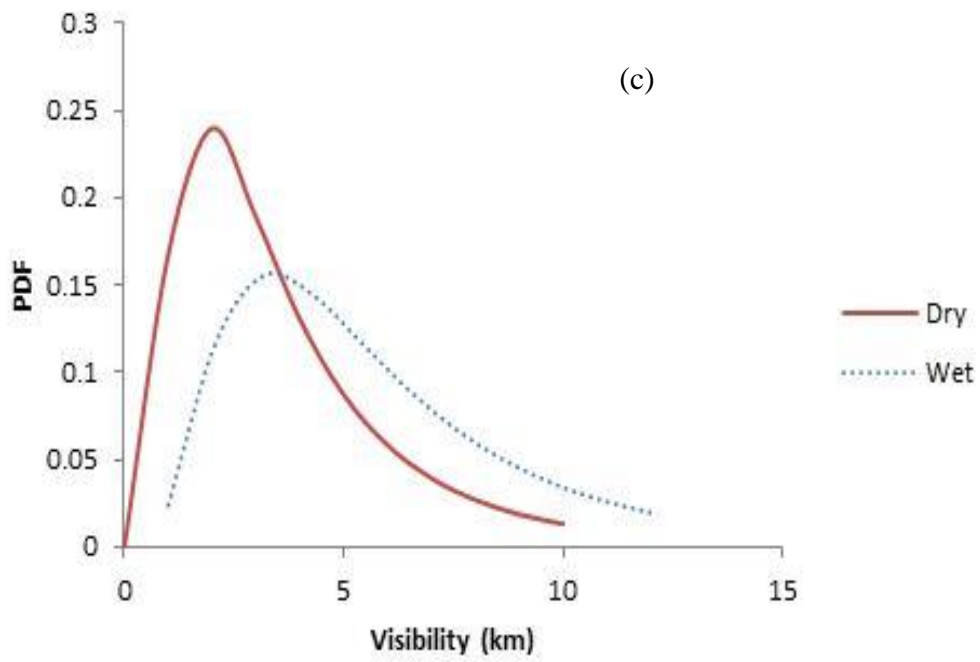
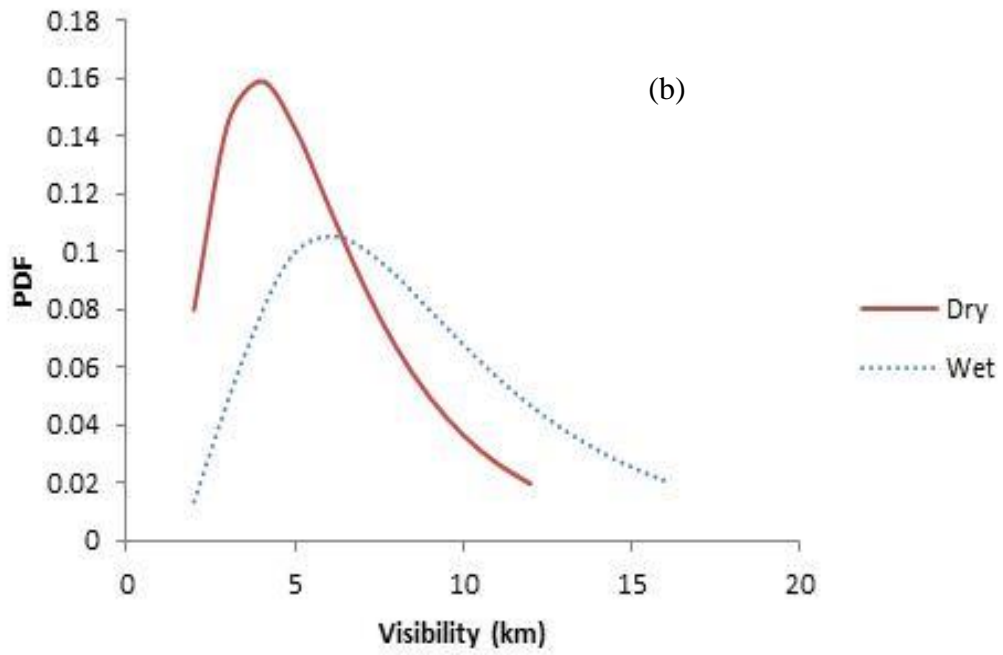
Table 4. Seasonal lognormal location and scale parameters for (a) dry (b) wet
(a)

Parameters	Akure	Enugu	Ikeja	Jos	Port-Harcourt
Location (μ)	0.6969	0.5348	0.6930	0.7548	0.6115
Scale (σ)	1.4972	1.6573	1.0997	1.7169	1.5000

(b)

Parameters	Akure	Enugu	Ikeja	Jos	Port-Harcourt
Location (μ)	0.5386	0.51792	0.6014	0.61701	0.5459
Scale (σ)	2.1564	2.0851	1.6014	2.0975	1.6803





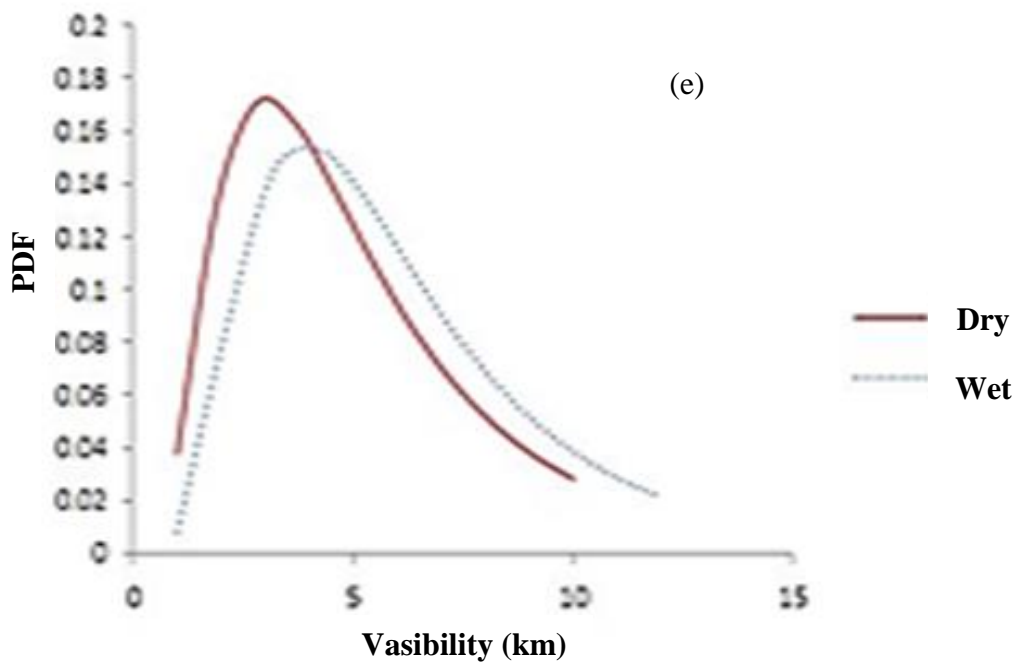
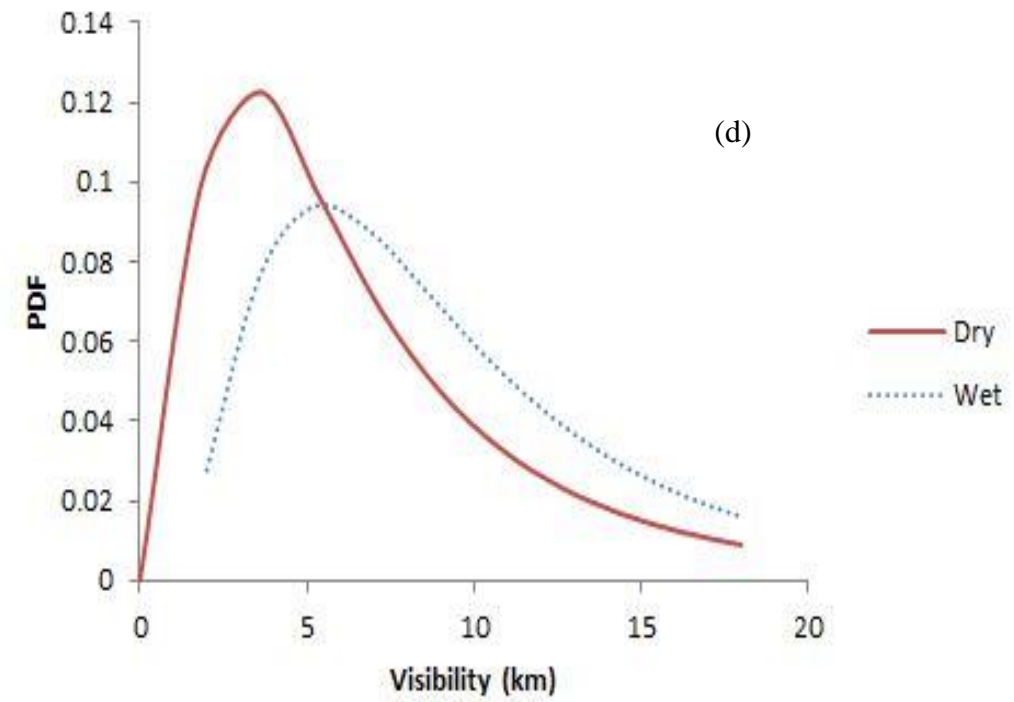
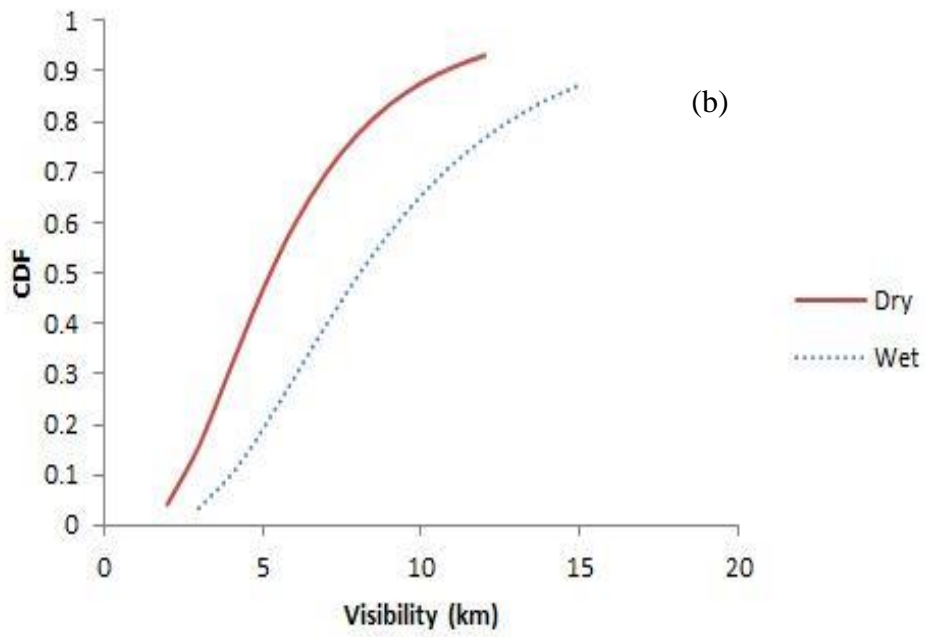
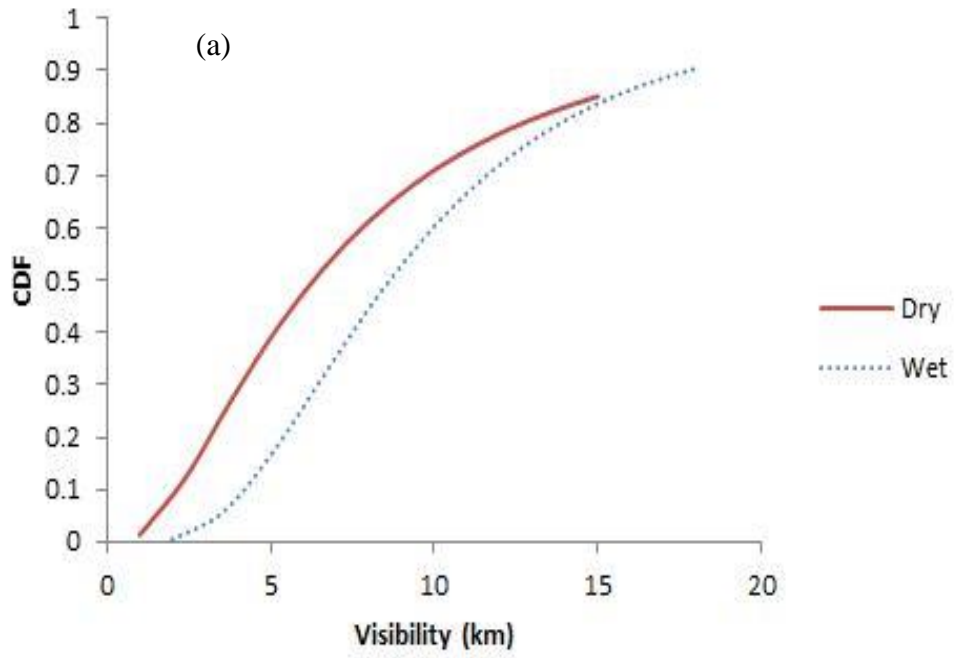
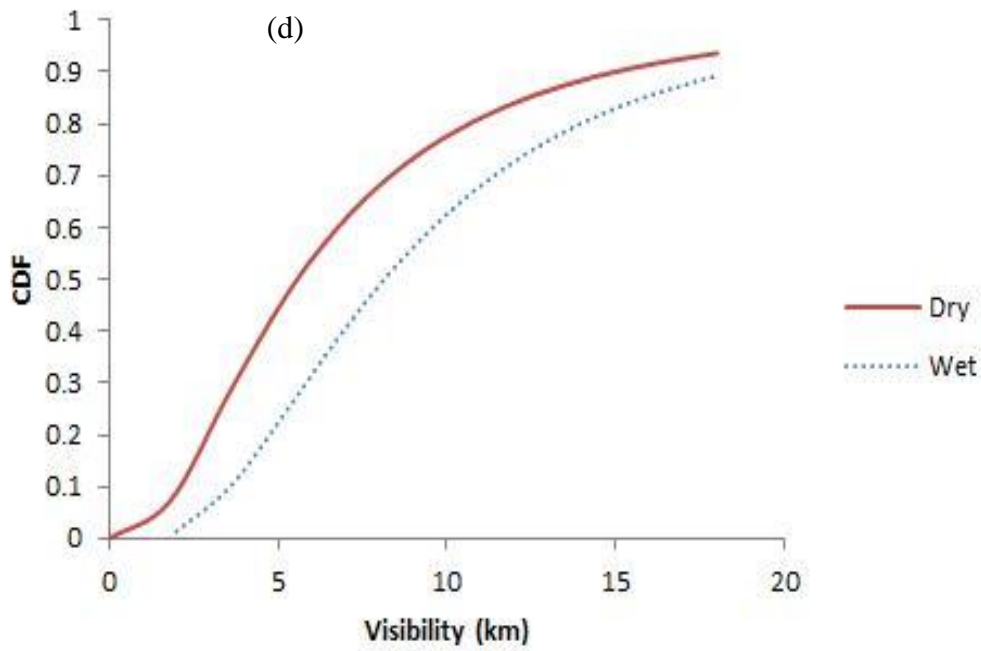
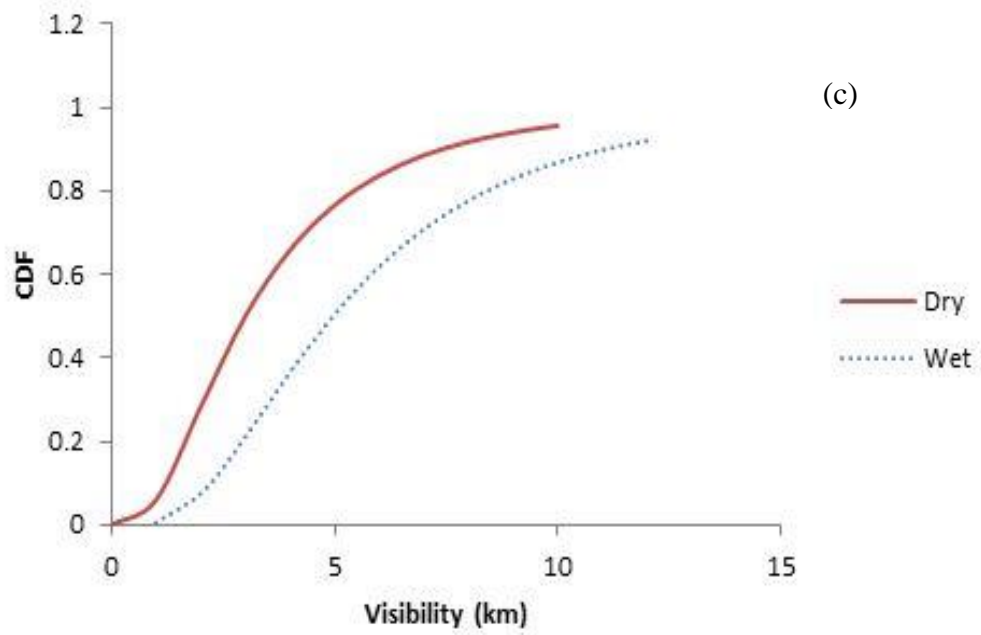


Figure 4. Seasonal variation in lognormal probability distribution function for (a) Akure (b) Enugu (c) Ikeja (d) Jos and (e) Port-Harcourt.





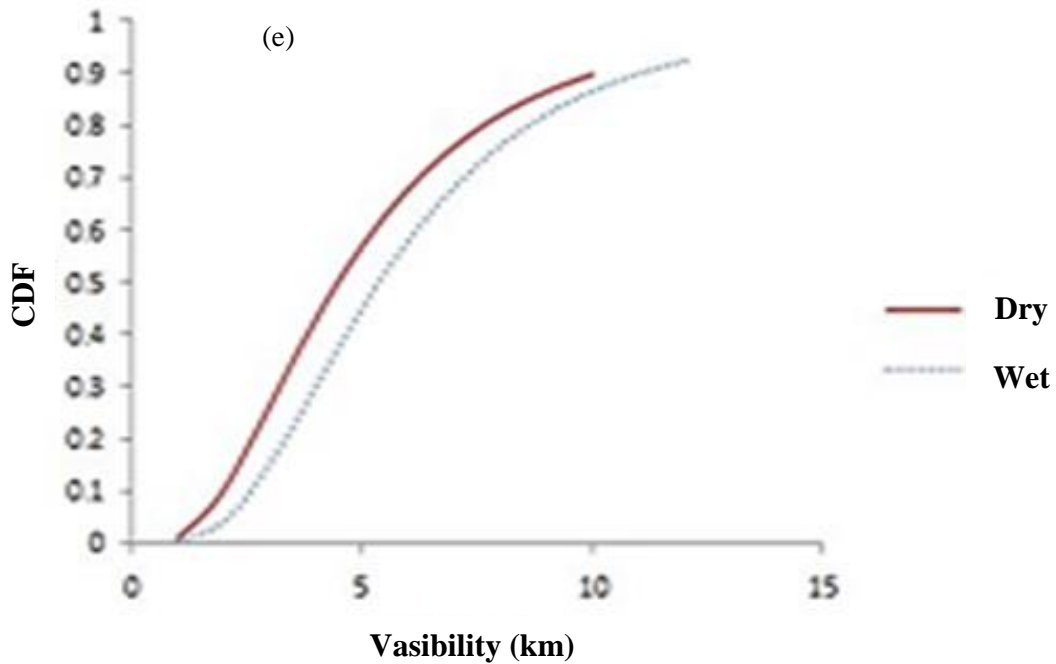


Figure 5. Seasonal variation in lognormal cumulative distribution function for (a) Akure (b) Enugu (c) Ikeja (d) Jos and (e) Port-Harcourt

5. CONCLUSIONS

In the present study an attempt has been made to investigate the most probable probability distribution function for modeling the visibility in the five cities in Nigeria. For this purpose, visibility data obtained at Nigeria Meteorological Agency (NIMET) were analyzed over a period of five years. The most probable probability distribution function for modeling the visibility in the five cities has been determined and the seasonal lognormal distribution parameters calculated. From the study, it is concluded that lognormal distribution is the most appropriate probability distribution function for modeling the visibility in all the selected stations considered based on some metric measures. The result when tested at Akure shows RMSE of 0.0766 km, MAE of 0.0095 km, MAPE of 17.8 % and R^2 of 0.87. The same was observed in other locations although with different values of RMSE, MAE, MAPE and R^2 . The results of this study is useful as a first-hand information to the system engineers who are interested in wireless communication links application in Nigeria. Future research emanating from this study will also assist in the predicting of future visibility of the study locations using lognormal distribution.

Acknowledgments

Authors will like to acknowledge the Nigerian Meteorological Agency (NIMET) for making data available for use.

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