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Markovian Epidemic Queueing Model with Exposed, Infection and Vaccination based on Treatment

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ABSTRACT

In this investigation, an epidemic Markov queueing model with alert, infection, vaccination and death has been considered. The main focus of this work is on the vaccination to provide the infection in a population. Healthy person may be affected from some disease that may cause death. The vaccination is the power tool for the prevention of the diseases spreading over the population size. Healthy person become alert when symptoms of the disease can be seen on them and the alert may be injected due to more infection. The provision of vaccination is provided in both alert and infected stages. The transition rates as followed by exponential distribution. A Markov model is developed by using inflow and outflow transition rates of the model. The transient state probabilities are evaluated by solving the transient state equations by using runge kutta method which are further used for calculating the model performances. A numerical illustration is also provided to validate the model.

Keywords: Epidemic Model, Infection, Vaccination, Markov model, Runge kutta method

1. INTRODUCTION

Epidemics are critical issues in the present world in the biological point of view that cause of a huge damage. Therefore the development of precise and improved epidemic models is highly recommended in order to study the complexity of these epidemics. Epidemic models play a critical role in disease dynamics for providing the internal assessment to current biological scenario. The provision of vaccination is proved helpful in the prevention of many epidemics with the recovering from the symptoms of these diseases. Markov modeling is very much useful for predicting the behavior of epidemic and disease models due their properties of uncertainty and randomness. Some remarkable work has been done by the research scholars in the field of epidemiology by considering markov and stochastic modeling. Longini *et al.*, (1989) did the statistical analysis of the stages of HIV infection using a Markov modeling. Gentleman *et al.*, (1994) considered a multi-state Markov models for analyzing incomplete disease history data with illustrations for HIV disease. Tan and Tang (1994) developed a general Markov model of the HIV epidemic in populations involving different types of sources. Debanne *et al.*, (2000) developed a multivariate Markov chain model to project tuberculosis (TB) progression among different races in the country.

Trapman and Bootsma (2009) established a relation between the spread of infectious diseases and the dynamics of M/G/1 queues with processor sharing. Bature *et al.* (2010) described a Markov chain model to use for tracking the movement of the virus from one generation to another in a period of 20 years. Sweeting *et al.*, (2010) considered multistate Markov modelling to explore the rate at which the Hepatitis C disease progresses by taking the experimental data. Lee *et al.*, (2014) used a Markov chain analysis to model the progression of the disease among vulnerable people, infective people and AIDS cases for the two races separately.

Oyewole (2014) discussed a discrete-time Markov process for HIV/AIDS epidemic modeling is to determine the behavior of the epidemic and to keep it under control. Bortolussi (2016) investigated models of continuous time markov chains in which some populations are approximated continuously while others are considered as discrete. Greenhalgh and Mao (2016) examined a SIRS model with the effect of environmental noise using a finite state markov chain. Giamberardino and Iacoviello (2017) considered SIR epidemic model with the dynamics of Susceptible, Infected & Removed subjects and an optimal vaccination strategy. Preeti (2017) has done Queueing modeling of a SAIS Model with Alert, Infection and Vaccination and obtained various system performance measures in terms of probabilities. In recent past (Shekhar *et al.*, 2017 and Shekhar *et al.*, 2016) had done extensive literature surveys on the queueing and retrial queueing modeling.

In the proposed work, an epidemic M/M/1 queueing model with alert, infection, vaccination and death has been taken into account. The provision of two types of vaccination is applied to alert and the infected persons for recovery. The remaining paper is structured as follows. In section 2, we describe the model by explaining the assumptions, notations and the various states of the system. Section 3 presents the governing equations of the model by using the inflow and outflow state transition rates.

The various system performances in terms of state probabilities are given in section 4. In section 5, a numerical illustration is facilitated to explore the validation of the model. Finally, conclusions are drawn in section 6.

2. MODEL DESCRIPTION

In this present investigation, we consider a epidemic queueing model with different states namely empty state, healthy state, alert state, vaccination state, infected state. We have consider a polpulation of size N which can be affected by an epiemic. Firstly, when the signs of this epidemic can be seeing in a healthy person, this stage at which he is not completely infected is known as alert. An alert person may be change into completely infected person by infected with this epidemic. The provision of the vaccination is also taken as a preventive tool to control on this epidemic. There are two types of vaccinations which are used in the model, the first one is used when the healty person is affected partialy from the epidemic mean at alert state and the second type of vaccination is exploitd for recovery of infected person from the complete infection. All the transition rates from one state to another state are taken as exponentially distributed. For the constraction of the model, the following assumptions are taken into consideration:

- The population size is taken as N i.e. there are total N living being in the system.
- Initially, the system is in empty state where there is no birth and no death and from this state after a birth system goes in the healty state.
- The birth and death of the persons are exponentially distributed with the rates λ and μ .
- The rate by which the healthy becomes alret is also exponentially distributed with the rate α_1 .
- Similary, The rate by which the infection take place is also exponentially distributed with the rate α_2 .
- The rate by which the alert changes into infected is also follow the exponential distribution with the mean β_1 .
- For both alert and infected persons, the vaccination of two types is carried out for the prevention and recovery of the epidemic.
- The vaccination of both types is provided to alert persons and infected persons with the exponentially distributed rates θ_1 and θ_2 , respectively.
- After the success of the both types of vaccination the alert persons and infected persons become healthy with the exponentially distributed rates γ_1 and γ_2 , respectively.
- The infected persons left the state after the recovery or the removal according to the exponential distribution with mean rate β_2 .

We develop the mathematical model by using some notations which are defined as below:

- N : The population size.
- λ : The birth rate.
- M : The death rate.
- α_1 : The partial infection rate (alert rate).
- α_2 : The complete infection rate.
- β_1 : The transition rate from alert state to infected state.
- β_2 : The transition rate from infection state to recovery or removal state.

- θ_1 : The first type vaccination rate from the alert state.
- θ_2 : The second type vaccination rate from the infected state.
- γ_1 : The transition rate from alert state to healthy state.
- γ_2 : The transition rate from infected state to healthy state.

Let $P_j(t)$ be the transient state probability that the system being in j^{th} [$j=0, (H,i), (A,i), (V_1,i), (I,i), (V_2,i), (R,i)$, where $i=1,2,3,\dots,N$] state as shown in Fig. 1 and j denotes the state of the system as follows:

- 0 The empty state at which there in no birth and no death takes place.
- H, i The healthy state where there are i ($i=1, 2, 3,\dots, N$) healthy persons in system.
- A, i The alert state where there are i ($i=1, 2, 3,\dots, N$) alert persons in system.
- V_1, i The first type vaccination state for alerts where there are i ($i=1, 2, 3,\dots, N$) alert persons in system.
- I, i The infection state where there are i ($i=1, 2, 3,\dots, N$) infected persons in system.
- V_2, i The second type vaccination state for infected persons where there are i ($i=1, 2, 3,\dots, N$) infected persons in system.
- R, i The recovery and removal state where there are i ($i=1, 2, 3,\dots, N$) persons left the system.

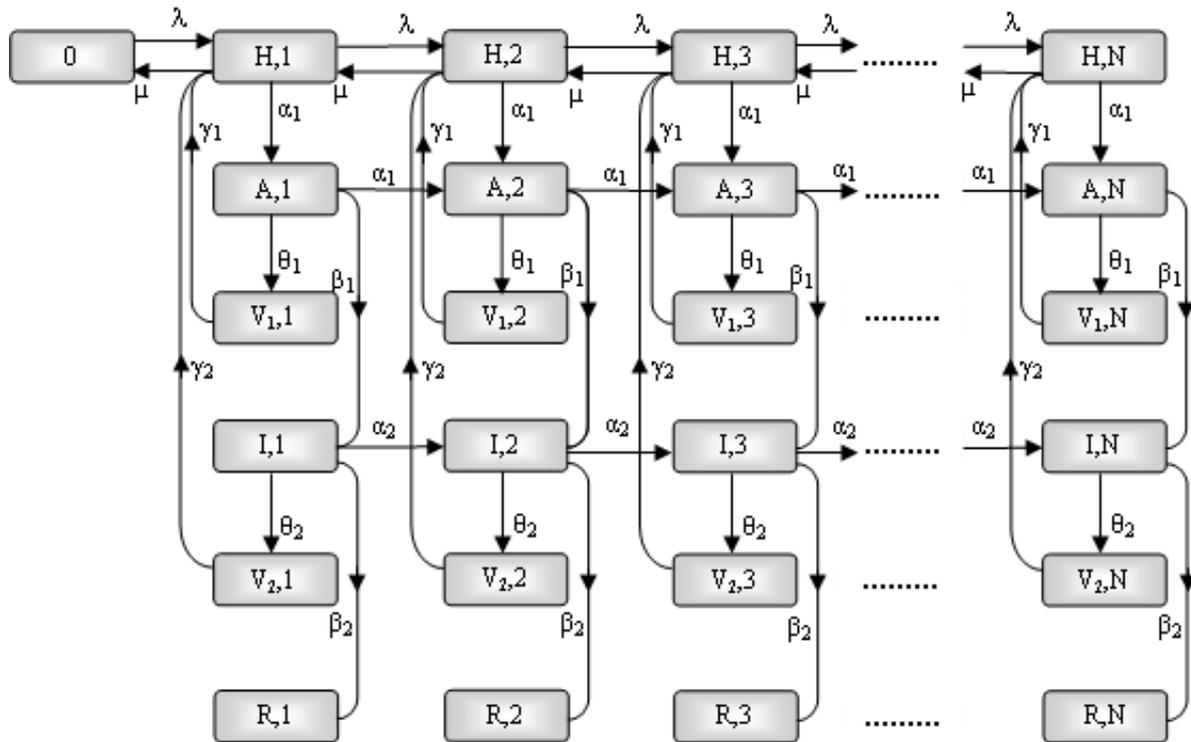


Fig. 1. State Transition Diagram

3. THE GOVERNING EQUATIONS

The steady state equations governing the epidemic model are constructed by equating the in-flow and out-flow (see fig. 1) as follows:

$$\frac{dP_0(t)}{dt} = -\lambda P_0(t) + \mu P_{H,1}(t) \tag{1}$$

$$\frac{dP_{H,1}(t)}{dt} = -(\lambda + \alpha_1 + \mu)P_{H,1}(t) + \lambda P_0(t) + \gamma_1 P_{V,1}(t) + \gamma_2 P_{V,2}(t) + \mu P_{H,2}(t) \tag{2}$$

$$\frac{dP_{H,i}(t)}{dt} = -(\lambda + \alpha_1 + \mu)P_{H,i}(t) + \lambda P_{H,i-1}(t) + \gamma_1 P_{V,i}(t) + \gamma_2 P_{V,i}(t) + \mu P_{H,i+1}(t), \quad i=2, 3, \dots, N-1 \tag{3}$$

$$\frac{dP_{H,N}(t)}{dt} = -(\alpha_1 + \mu)P_{H,N}(t) + \lambda P_{H,N-1}(t) + \gamma_1 P_{V,N}(t) + \gamma_2 P_{V,N}(t) \tag{4}$$

$$\frac{dP_{A,1}(t)}{dt} = -(\alpha_1 + \theta_1 + \beta_1)P_{A,1}(t) + \alpha_1 P_{H,1}(t) \tag{5}$$

$$\frac{dP_{A,i}(t)}{dt} = -(\alpha_1 + \theta_1 + \beta_1)P_{A,i}(t) + \alpha_1 P_{H,i}(t) + \alpha_1 P_{A,i-1}(t), \quad i=2, 3, \dots, N-1 \tag{6}$$

$$\frac{dP_{A,N}(t)}{dt} = -(\theta_1 + \beta_1)P_{A,N}(t) + \alpha_1 P_{H,N}(t) + \alpha_1 P_{A,N-1}(t) \tag{7}$$

$$\frac{dP_{V,i}(t)}{dt} = -\gamma_1 P_{V,i}(t) + \theta_1 P_{A,i}(t), \quad i=1, 2, 3, \dots, N \tag{8}$$

$$\frac{dP_{I,1}(t)}{dt} = -(\alpha_2 + \theta_2 + \beta_2)P_{I,1}(t) + \beta_1 P_{A,1}(t) \tag{9}$$

$$\frac{dP_{I,i}(t)}{dt} = -(\alpha_2 + \theta_2 + \beta_2)P_{I,i}(t) + \beta_1 P_{A,i}(t) + \alpha_2 P_{I,i-1}(t), \quad i=2, 3, \dots, N-1 \tag{10}$$

$$\frac{dP_{I,N}(t)}{dt} = -(\theta_2 + \beta_2)P_{I,N}(t) + \beta_1 P_{A,N}(t) + \alpha_2 P_{I,N-1}(t) \tag{11}$$

$$\frac{dP_{V,2,i}(t)}{dt} = -\gamma_2 P_{V,2,i}(t) + \theta_2 P_{I,i}(t), \quad i=1, 2, 3, \dots, N \tag{12}$$

$$\frac{dP_{R,i}(t)}{dt} = \beta_2 P_{I,i}(t), i=1, 2, 3, \dots, N \tag{13}$$

The transient probabilities of different states have been obtained by solving equations (1)-(13) by fourth order Runge-Kutta technique.

4. PERFORMANCE MEASURES

In section, our main objective of our investigation is to predict various performance metrics in terms of the steady state probabilities which are obtained by using fourth order Runge-Kutta method in previous section.

Some indices to characterize the system performance are as follows:

- The expected number of healthy persons in the epidemic system is

$$E(H) = \sum_{i=1}^N iP_{H,i}(t) \tag{20}$$

- The expected number of alert persons in the epidemic system is

$$E(A) = \sum_{i=1}^N iP_{A,i}(t) \tag{21}$$

- The expected number of infected persons in the epidemic system is

$$E(I) = \sum_{i=1}^N iP_{I,i}(t) \tag{22}$$

- The expected number of persons under the vaccinations of both types is

$$E(V) = \sum_{i=1}^N i(P_{V_1,i}(t) + P_{V_2,i}(t)) \tag{23}$$

- The expected number of persons those are recovered or removed is

$$E(R) = \sum_{i=1}^N iP_{R,i}(t) \tag{24}$$

5. NUMERICAL ILLUSTRATION

In this section, we perform a numerical illustration for the transient analysis of the epidemic M/M/1 queueing model with alert, infection, vaccination developed in previous sections. Runge-Kutta technique (RKT) of fourth order is used for solving these equations, which is implemented by exploiting MATLAB’s ‘ode45’ function. We have considered a time span with equal intervals. The numerical results by varying different parameters for various performance measures are summarized in the graphical presentations are also provided. For the computation purpose, we fix the values of the some parameters as follows: $\lambda=0.1, \mu=0.2, \alpha_1=0.0004, \alpha_2=0.0005, \beta_1=0.3, \beta_2=0.2, \gamma_1=0.0002, \gamma_2=0.0001$ for figures 2-5.

In graph 2, the effect of first type vaccination rate θ_1 on expected number of alert persons $E(A)$ has been shown by varying time. It is observed that the expected number of alert persons $E(A)$ is decreases as the first type vaccination rate θ_1 increases with increasing time.

While Fig. 3 shows the effect of first type vaccination rate θ_1 on the expected number of infected persons $E(I)$ and it is easily seen that $E(I)$ is decreases as first type vaccination rate θ_1 increases.

Now on the other hand in Fig. 4, the relationship between the first type vaccination rate θ_1 and the expected number of recovered or removed persons $E(R)$ is displayed. It is realized that the expected number of recovered or removed persons $E(R)$ is decreases as the first type vaccination rate θ_1 increases. The effect of the second type vaccination rate θ_2 and the expected number of recovered or removed persons $E(R)$ is shown in fig. 5. It is quite obvious that by increasing the value of t , the expected number of recovered or removed persons $E(R)$ decreases while on increasing the second type vaccination rate θ_2 .

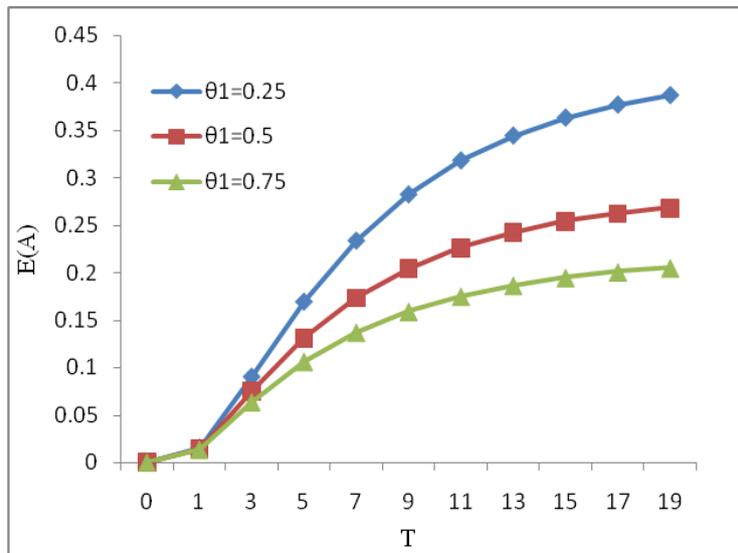


Fig. 2. Effect of θ_1 on $E(A)$

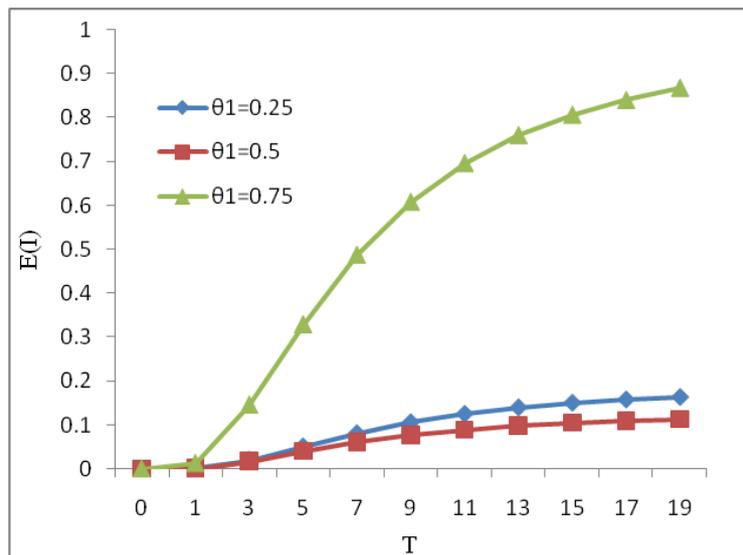


Fig. 3. Effect of θ_1 on $E(I)$

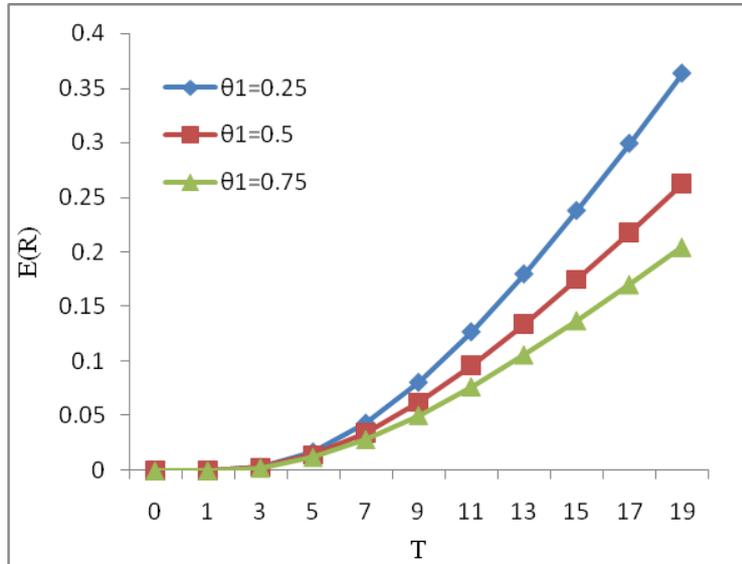


Fig. 4. Effect of θ_1 on $E(R)$

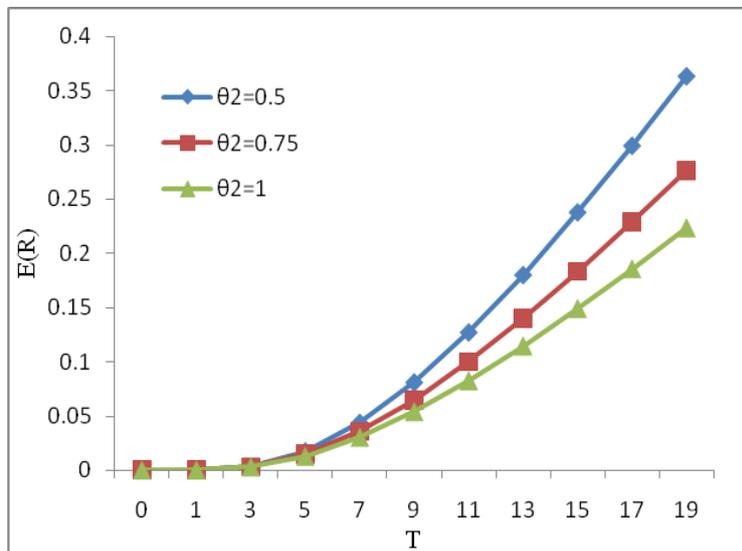


Fig. 5. Effect of θ_2 on $E(R)$

6. CONCLUSIONS

In this paper, a Markovian epidemic queueing model with exposed, infection and vaccination based on treatment is proposed and analyzed. The various concepts such as alerts, infection and vaccination have been incorporated to cope with the real-life issues. After constructing the model, the transient state equations are formed which are further used to find various system performances. We have also carried out a numerical example to find the numerical results for an epidemic model in the point of views of queueing theory and our

results are found quite satisfactory. The results obtained in simulation exhibit that in the presence of high dose of vaccination on the expected numbers of patients reduce the spread of diseases. We observed that, if a vaccination is well given, the number of infected person will be reduced for both types of vaccination rates. It is also observed that the both types of vaccination strategies improved the performance of the epidemic model. It is supposed that our investigation will be successful to provide a new direction to applications of queueing theory in the interdisciplinary field of epidemiology.

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