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Radio Number of Cycles and their Total Graphs

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ABSTRACT

A radio labeling f of G is an assignment of positive integers to the vertices of G satisfying, $|f(u) - f(v)| \geq diam(G) + 1 - d(u,v)$ $\forall$ u, v $\in$ V (G) where d (u ,v) is the distance between any two vertices in the graph. The radio number denoted by rn (G) is the minimum span of a radio labeling for G. In this paper, an alternate proof for radio number of cycles and exact radio number for their total graphs has been discussed.

***Keywords*:** Radio labeling, Radio number, Total graph

MSC 2010: 05C78

1. INTRODUCTION

A number of graph colorings have emerged from communication problem called as Channel Assignment Problem. In this problem, there can be interference between the transmitters due to the proximity in some geographical regions. The interference constraints between a pair of transmitters play a very important role in designing a radio network in a telecommunication system. To the geographical location of the radio stations, the level of interference is closely related. Lesser the distance between the stations more stronger is the interference between the pair of transmitters. The Channel Assignment Problem is the problem of assigning channels or frequencies to the transmitters in any optimal manner.

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This problem was first modeled by Hale [2] in 1980. Earlier, the minor and major level interference were considered by the designers of radio networks. If the interference level between a pair of transmitter was major then the pair was called very close transmitters and if it was minor then as close transmitters. Further, the interference graph was developed and the transmitters were represented as the vertices of the graph. If the transmitters were at a distance of one then the interference level was considered major and at a distance of two interference level was considered minor. If the transmitters were at a distance more than two then no interference was considered. Motivated by this, a new concept called distance two labeling was introduced by Roberts [5]. Later on, practically it was found that the interference among the transmitters might go beyond two levels. Motivated through the problem of channel assignment of FM radio stations, a concept of radio labeling of graph which is an extension of distance two labeling was introduced. In this labeling, the interference levels ranges from two to the largest possible level-the diameter of the graph.

Definition 1:

A radio labeling f of G is an assignment of positive integers to the vertices of G satisfying, $|f(u) - f(v)| \geq \text{diam}(G) + 1 - d(u, v)$ where $u, v \in V(G)$ where $d(u, v)$ is the distance between any two vertices in the graph. The radio number denoted by rn (G) is the minimum span of a radio labeling for G.

Definition 2:

The total graph of a graph G is the graph whose vertex set is $V(G) \cup E(G)$ and two vertices are adjacent whenever they are either adjacent or incident in G. The total graph of G is denoted by T(G).

From the definition of total graph, it is clear that $V(T(C_n)) = 2n$, $E(T(C_n)) = 4n$. Further, $\text{diam}(T(C_{2n})) = \text{diam}(C_{2n})$ and $\text{diam}(T(C_{2n+1}) = \text{diam}(C_{2n+1})+1$. It is always a challenging and tedious task to determine the radio number of graphs.

Literature Survey

Chatrand et. al. [1] proposed the bounds for radio number of cycles, Liu and Zhu [3] have determined the radio number of paths and cycles. Liu and Xie [6] and [7] have determined the radio number of square of even cycles and obtained lower bounds for the square of odd cycles. They have also obtained the radio number for square of any path. A. A Bhatti et. al.[8] have determined the radio number for Flower Wheel graph ($FW^k{}_n$), k-Wheel graph (k W), and Joint- wheel graph (WH_n). S. K Vaidya and P L Vihol [9] have obtained upper bounds of radio numbers for cycle with chords and n/2-Petal graph. They have also obtained radio number for the split graph and middle graph of cycle. S. K. Vaidya and D. D. Bantva [10] have determined the radio number of total graphs of paths. In this paper, we give an alternative proof for radio number of cycles and determine the radio number for their total graphs.

Throughout this paper we denote d as the diameter of the graph.

2. MAIN RESULTS

Theorem 1. For any integer k ≥ 3,

Then,

$$\phi(n) = \begin{cases} k+1, & if\ n = 4k \\ k+2, & if\ n = 4k + r\ for\ some\ r = 0, 2, 3. \end{cases}$$

$$rn(C_n) = \begin{cases} \frac{n-2}{2}\phi(n) + 1, & if\ n = 4k\ or\ 4k+2 \\ \frac{n-2}{2}\phi(n), & if\ n = 4k+1\ or\ 4k+3 \end{cases}$$

In this section, we determine the radio number rn of cycles and their total graphs. In the following results, we assign distinct labels to the vertices beginning with 0 and proceed in the order 0, 1, 2, … . We label a vertex in such a way that radio labeling requirements are met with already labeled vertices at every stage. We always move from current labeled vertex to the next unlabeled vertex at suitable distance so that label of least value can be assigned to it. We assign the label say x to a vertex only after all the possibilities of assigning labels of value less than x are ruled out for all the unlabeled vertices.

Theorem 2. *rn* (C_{2n}) = $\left[\left(\frac{n}{2} + 2 - \left\lceil\frac{n}{4}\right\rceil\right) * \left(\frac{n-2}{2}\right)\right] + 1$ *where* C_{2n} *is an even cycle with odd diameter d and n ≥ 2.*

Proof Consider C_{2n} with odd diameter d .

- Choose a vertex and assign it the label 0.
- Choose an unlabeled vertex at a distance of diameter i.e., *d* from the previously assigned vertex and assign it the label $d + 1 - d = 1$.
- Choose an unlabelled vertex at a distance $\left\lceil\frac{d}{2}\right\rceil$ from the previously assigned vertex and assign the label $d + 1 + 1 - \left\lceil\frac{d}{2}\right\rceil = d + 2 - \left\lceil\frac{d}{2}\right\rceil$ to the vertex. This is the least label that can be assigned to any vertex that maintains the requirements of radio labeling.
- Now, choose a vertex at a distance *d* from the last labeled vertex and assign the label $(d + 1 - d) + d + 2 - \left\lceil\frac{d}{2}\right\rceil = d + 3 - \left\lceil\frac{d}{2}\right\rceil$ to the vertex.
- Further, choose an unlabeled vertex at a distance $\left\lceil\frac{d}{2}\right\rceil$ from the current vertex, and assign it the label $(d + 1 - \left\lceil\frac{d}{2}\right\rceil) + d + 3 - \left\lceil\frac{d}{2}\right\rceil = d + 4 - \left\lceil\frac{d}{2}\right\rceil$.
- Continuing in this manner we see that the labels are $0, 1, d + 2 - \left\lceil\frac{d}{2}\right\rceil, d + 2 - \left\lceil\frac{d}{2}\right\rceil + 1, 2\left(d + 2 - \left\lceil\frac{d}{2}\right\rceil\right), 2\left(d + 2 - \left\lceil\frac{d}{2}\right\rceil\right) + 1, \ldots$.
- Continuing in this way, the label assigned to the second last vertex is $\left(d + 2 - \left\lceil\frac{d}{2}\right\rceil\right) * (d - 1)$. And the label assigned to the last vertex is $\left[\left(d + 2 - \left\lceil\frac{d}{2}\right\rceil\right) * (d - 1)\right] + 1$.

Hence rn $(C_{2n}) = \left[\left(d + 2 - \left\lceil\frac{d}{2}\right\rceil\right) * (d-1)\right] + 1$

$$= \left[\left(\frac{n}{2} + 2 - \left\lceil\frac{n}{4}\right\rceil\right) * \left(\frac{n-2}{2}\right)\right] + 1$$

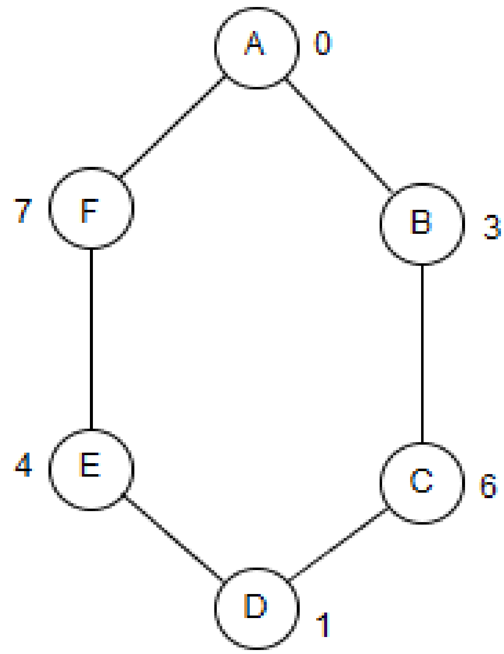

Fig. 1. Radio labeling of C_6

In Fig. 1, it can be observed that rn (C_6) is 7.

Theorem 3. *rn* (C_{2n}) = $\left[\left(\frac{n}{2} + 3 - \left\lceil\frac{n+1}{4}\right\rceil\right) * \left(\frac{n-2}{2}\right)\right] + 1$ *where* C_{2n} *is an even cycle with even diameter d and n* ≥ 2.

Proof: Consider C_{2n} with even diameter *d*.

- Choose a vertex and assign it the label 0.
- Choose an unlabeled vertex at a distance of diameter i.e., *d* from the previously assigned vertex and assign it the label $d + 1 - d = 1$.
- Choose an unlabelled vertex at a distance $\left\lceil\frac{d+1}{2}\right\rceil$ from the previously assigned vertex and assign the label $d + 1 + 1 - \left\lceil\frac{d+1}{2}\right\rceil + 1 = d + 3 - \left\lceil\frac{d+1}{2}\right\rceil$ to the vertex maintaining the label requirements of the vertex labeled 0.
- Now, choose a vertex at a distance *d* from the last labeled vertex and assign the label $(d + 1 - d)$+ $d + 3 - \left\lceil\frac{d+1}{2}\right\rceil = d + 4 - \left\lceil\frac{d+1}{2}\right\rceil$ to the vertex.

- Further, choose an unlabeled vertex at a distance $\lceil \frac{d+1}{2} \rceil$ from the current vertex, and assign it the label $(d+1-\lceil \frac{d+1}{2} \rceil) + d + 4 - \lceil \frac{d+1}{2} \rceil + 1 = 2d + 6 - 2\lceil \frac{d+1}{2} \rceil$ satisfying the label requirements of the vertex labeled $d + 3 - \lceil \frac{d+1}{2} \rceil$.
- Continuing in this manner we see that the labels are $0, 1, d+3-\lceil \frac{d+1}{2} \rceil, d+3-\lceil \frac{d+1}{2} \rceil + 1, 2\left(d+3-\lceil \frac{d+1}{2} \rceil\right), 2\left(d+3-\lceil \frac{d+1}{2} \rceil\right) + 1, \ldots$.
- Continuing in this way, the label assigned to the second last vertex is $(d+3-\lceil \frac{d+1}{2} \rceil) * (d-1)$. And the label assigned to the last vertex is, $\left[\left(d+3-\lceil \frac{d+1}{2} \rceil\right) * (d-1)\right] + 1$.

Hence rn (C_{2n}) $= \left[\left(d+3-\lceil \frac{d+1}{2} \rceil\right) * (d-1)\right] + 1$

$$= \left[\left(\frac{n}{2} + 3 - \lceil \frac{n+1}{4} \rceil\right) * \left(\frac{n-2}{2}\right)\right] + 1$$

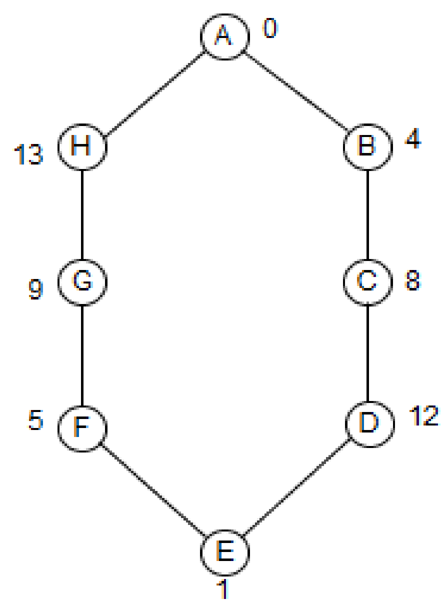

Fig. 2. Radio labeling of C_8

In Fig. 2, it can be observed that rn (C_8) is 13.

Theorem 4. $rn\ (C_{2n+1}) = \left(\left\lfloor \frac{n}{2} \right\rfloor + 2 - \lceil \frac{n}{4} \rceil\right) * \lceil \frac{n-2}{2} \rceil$ where C_{2n+1} is an odd cycle with odd diameter d and $n \geq 1$.

Proof : Consider C_{2n+1} with odd diameter d .

- Choose a vertex and assign it the label 0.
- Choose an unlabeled vertex at a distance of diameter i.e. d from the previously assigned vertex and assign it the label $d + 1 - d = 1$.

- Choose an unlabelled vertex at a distance $\lceil\frac{d}{2}\rceil$ from the previously assigned vertex and assign the label $(d+1-\lceil\frac{d}{2}\rceil)+1 = d+2-\lceil\frac{d}{2}\rceil$ to the vertex.
- Now, choose a vertex at a distance *d* from the last labeled vertex and assign the label $(d+1-d)$+ $d+2-\lceil\frac{d}{2}\rceil$ = $d+3-\lceil\frac{d}{2}\rceil$ to the vertex.
- Further, choose an unlabeled vertex at a distance $\lceil\frac{d}{2}\rceil$ from the current vertex, and assign it the label $(d+1-\lceil\frac{d}{2}\rceil$)+ $d+3-\lceil\frac{d}{2}\rceil = d+4-\lceil\frac{d}{2}\rceil$
- Continuing in this manner we see that the labels are $0, 1, d+2-\lceil\frac{d}{2}\rceil, d+2-\lceil\frac{d}{2}\rceil+1, 2\left(d+2-\lceil\frac{d}{2}\rceil\right), 2\left(d+2-\lceil\frac{d}{2}\rceil\right)+1, \ldots$
- Continuing in this way, the label assigned to the last vertex is $\left[\left(d+2-\lceil\frac{d}{2}\rceil\right)*(d-1)\right]$.

Hence rn (C_{2n+1}) $= \left[\left(d+2-\lceil\frac{d}{2}\rceil\right)*(d-1)\right]$

$$= \left(\left\lceil\frac{n}{2}\right\rceil+2-\lceil\frac{n}{4}\rceil\right)*\lceil\frac{n-2}{2}\rceil$$

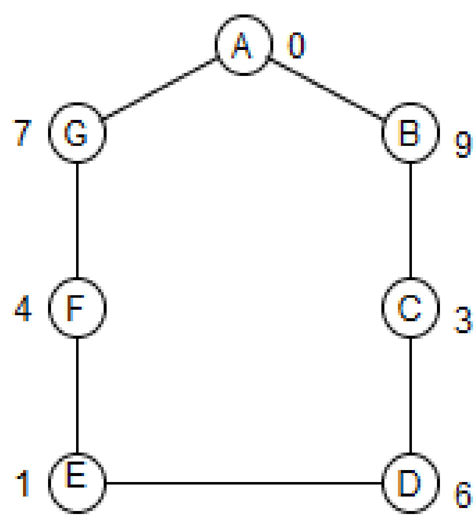

Fig. 3. Radio labeling of C_7

In Fig. 3, it can be observed that rn (C_7) is 9.

3. RADIO NUMBER OF TOTAL GRAPH OF CYCLES

Theorem 5. *$rn\left(T(C_{2n})\right) = \left[\left(\frac{n}{2}+2-\lceil\frac{n}{4}\rceil\right)*(n-1)\right]+1$ where $T(C_{2n})$ is the total graph of even cycle with odd diameter d and n ≥ 2.*

Proof: Consider a total graph of even cycle $T(C_{2n})$ with odd diameter d.

- Choose a vertex from $V\big(T(C_{2n})\big)$ and assign it the label 0.
- Choose a vertex at a distance d from the previously assigned vertex and assign the label $d + 1 - d = 1$.
- Choose an unlabelled vertex at a distance $\lceil \frac{d}{2} \rceil$ from the previously assigned vertex and assign the label $d + 1 + 1 - \lceil \frac{d}{2} \rceil = d + 2 - \lceil \frac{d}{2} \rceil$ to the vertex .
- Now, choose a vertex at a distance *d* from the last labeled vertex and assign the label $(d + 1 - d)$+ $d + 2 - \lceil \frac{d}{2} \rceil = d + 3 - \lceil \frac{d}{2} \rceil$ to the vertex .
- Further, choose an unlabeled vertex at a distance $\lceil \frac{d}{2} \rceil$ from the current vertex, and assign it the label $(d + 1 - \lceil \frac{d}{2} \rceil$)+ $d + 3 - \lceil \frac{d}{2} \rceil = d + 4 - \lceil \frac{d}{2} \rceil$.
- Continuing in this manner we see that there are *n* i.e. , *2d* pairs of vertices with the labels $0, 1,\ d + 2 - \lceil \frac{d}{2} \rceil,\ d + 2 - \lceil \frac{d}{2} \rceil + 1, 2\big(d + 2 - \lceil \frac{d}{2} \rceil\big), 2\big(d + 2 - \lceil \frac{d}{2} \rceil\big) + 1, \dots$.
- Continuing in this way, the label assigned to the second last vertex is $\big(d + 2 - \lceil \frac{d}{2} \rceil\big) * (2d - 1)$. And the label assigned to the last vertex is $\big[\big(d + 2 - \lceil \frac{d}{2} \rceil\big) * (2d - 1)\big] + 1$.

Hence rn $T(C_{2n}) = \big[\big(d + 2 - \lceil \frac{d}{2} \rceil\big) * (2d - 1)\big] + 1$

$$= \left[\left(\frac{n}{2} + 2 - \lceil \tfrac{n}{4} \rceil\right) * (n - 1)\right] + 1$$

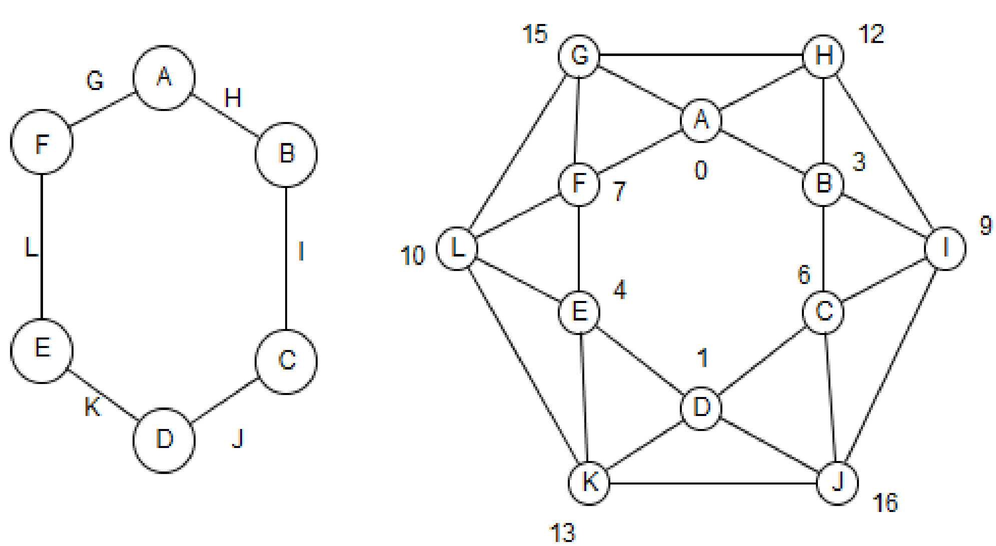

Fig. 4. C_6 and $T(C_6)$

In Fig. 4, it can be observed that rn $(T(C_6))$ is 16.

Theorem 6. *rn* $T(C_{2n}) = \left[\left(\frac{n}{2} + 2 - \left\lceil\frac{n+1}{4}\right\rceil\right) * (n-1)\right] + 1$ *where* $T(C_{2n})$ *is the total graph of an even cycle with even diameter d and n ≥ 2.*

Proof: Consider $T(C_{2n})$ with even diameter *d*.

- Choose a vertex and assign it the label 0.
- Choose an unlabeled vertex at a distance of diameter i.e., *d* from the previously assigned vertex and assign it the label $d + 1 - d = 1$.
- Choose an unlabelled vertex at a distance $\left\lceil\frac{d+1}{2}\right\rceil$ from the previously assigned vertex and assign the label $d + 1 + 1 - \left\lceil\frac{d+1}{2}\right\rceil = d + 2 - \left\lceil\frac{d+1}{2}\right\rceil$ to the vertex maintaining the label requirements of the vertex labeled 0.
- Now, choose a vertex at a distance *d* from the last labeled vertex and assign the label$(d + 1 - d)$+ $d + 2 - \left\lceil\frac{d+1}{2}\right\rceil = d + 3 - \left\lceil\frac{d+1}{2}\right\rceil$ to the vertex.
- Further, choose an unlabeled vertex at a distance $\left\lceil\frac{d+1}{2}\right\rceil$ from the current vertex, and assign it the label $(d + 1 - \left\lceil\frac{d+1}{2}\right\rceil$)+ $d + 3 - \left\lceil\frac{d+1}{2}\right\rceil = 2d + 4 - 2\left\lceil\frac{d+1}{2}\right\rceil$.
- Continuing in this manner we see that the labels are $0, 1, d + 2 - \left\lceil\frac{d+1}{2}\right\rceil, d + 2 - \left\lceil\frac{d+1}{2}\right\rceil + 1, 2\left(d + 2 - \left\lceil\frac{d+1}{2}\right\rceil\right), 2\left(d + 2 - \left\lceil\frac{d+1}{2}\right\rceil\right) + 1, \dots$.
- Continuing in this way, the label assigned to the second last vertex is $\left(d + 2 - \left\lceil\frac{d+1}{2}\right\rceil\right) * (2d - 1)$. And the label assigned to the last vertex is $\left[\left(d + 2 - \left\lceil\frac{d+1}{2}\right\rceil\right) * (2d - 1)\right] + 1$.

$$\text{Hence rn}(T(C_{2n}) = \left[\left(d + 2 - \left\lceil\frac{d+1}{2}\right\rceil\right) * (2d - 1)\right] + 1$$
$$= \left[\left(\frac{n}{2} + 2 - \left\lceil\frac{n+1}{4}\right\rceil\right) * (n - 1)\right] + 1$$

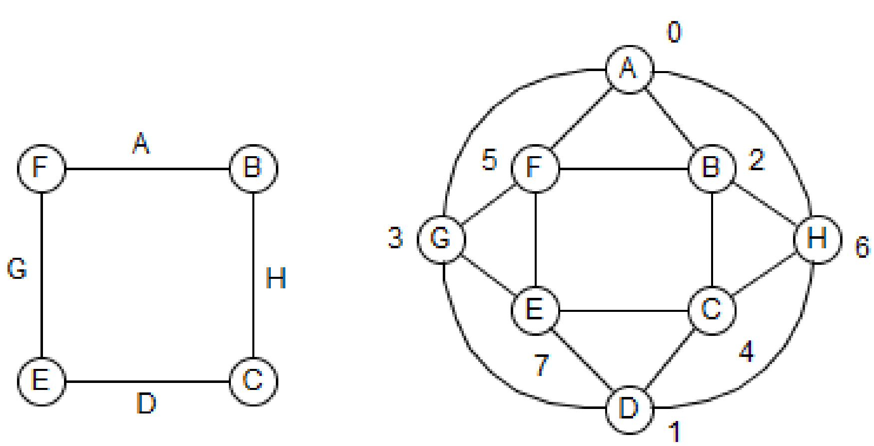

Fig. 5. C_4 and $T(C_4)$

In Fig. 5, it can be observed that rn (T(C_4)) is 7.

Theorem 7. *rn* $T(C_{2n+1}) = \left[\left(\left\lceil\frac{n}{2}\right\rceil + 2 - \left\lceil\frac{n}{4}\right\rceil\right) * (n-1)\right] + 1$ *where* $T(C_{2n+1})$ *is the total graph of an odd cycle and n ≥ 1.*

Proof: Consider the total graph of an odd cycle.

- Choose a vertex and assign it the label 0.
- Choose an unlabeled vertex at a distance of diameter i.e., *d* from the previously assigned vertex and assign it the label $d + 1 - d = 1$.
- Choose an unlabelled vertex at a distance $\left\lceil\frac{d}{2}\right\rceil$ from the previously assigned vertex and assign the label $\left(d + 1 - \left\lceil\frac{d}{2}\right\rceil\right) + 1 = d + 2 - \left\lceil\frac{d}{2}\right\rceil$ to the vertex.
- Now, choose a vertex at a distance *d* from the last labeled vertex and assign the label $(d + 1 - d) + d + 2 - \left\lceil\frac{d}{2}\right\rceil = d + 3 - \left\lceil\frac{d}{2}\right\rceil$ to the vertex.
- Further, choose an unlabeled vertex at a distance $\left\lceil\frac{d}{2}\right\rceil$ from the current vertex, and assign it the label $\left(d + 1 - \left\lceil\frac{d}{2}\right\rceil\right) + d + 3 - \left\lceil\frac{d}{2}\right\rceil = d + 4 - \left\lceil\frac{d}{2}\right\rceil$
- Continuing in this manner we see that the labels are $0, 1, d + 2 - \left\lceil\frac{d}{2}\right\rceil, d + 2 - \left\lceil\frac{d}{2}\right\rceil + 1, 2\left(d + 2 - \left\lceil\frac{d}{2}\right\rceil\right), 2\left(d + 2 - \left\lceil\frac{d}{2}\right\rceil\right) + 1, \ldots$.
- Continuing in this way, the label assigned to the last vertex is ,$\left[\left(d + 2 - \left\lceil\frac{d}{2}\right\rceil\right) * (2d - 1)\right]$

Hence rn (C_{2n+1}) $= \left[\left(d + 2 - \left\lceil\frac{d}{2}\right\rceil\right) * (2d - 1) + 1\right]$

$= \left[\left(\left\lceil\frac{n}{2}\right\rceil + 2 - \left\lceil\frac{n}{4}\right\rceil\right) * (n - 1) + 1\right]$

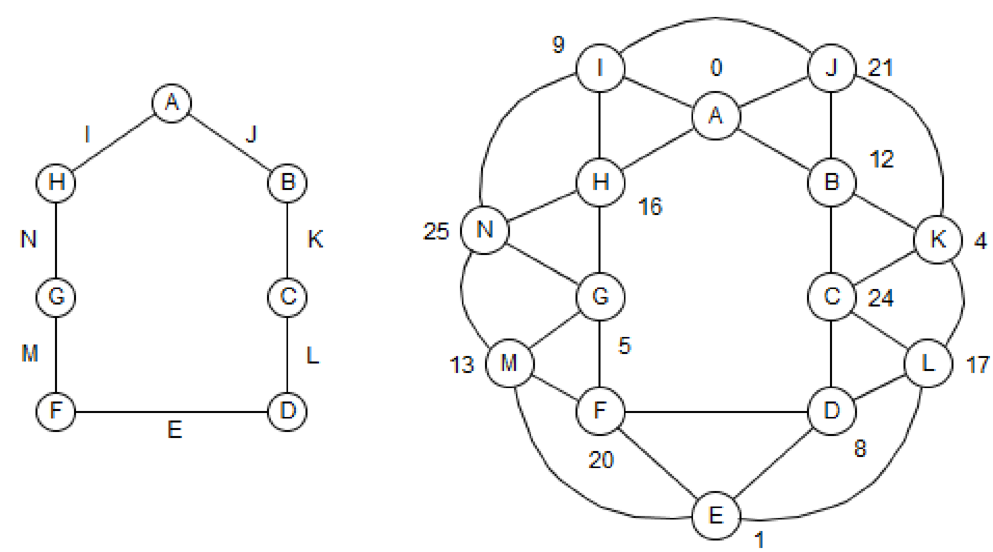

Fig. 6. C_7 and T (C_7)

In Fig. 6 it can be observed that rn $(T(C_7))$ is 25.

4. CONCLUSIONS

Designing a radio network with minimum interference involves a lot of challenges. We have taken up this problem in the context of total graph of cycles. In this paper, we have given an alternative proof for radio number of cycles and obtained the exact radio number for their total graphs. Further, our focus of research would be on the application of radio labeling in total graphs.

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