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Application of Cobb-Douglas Production Function to Manufacturing Industries in West Sumatra Indonesia

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ABSTRACT

In this paper, we discusses one of the production functions that shows the relationship between the level of output and the level of input combinations that called Cobb-Douglas production function. The estimation method used is the least square estimation with the settlement using Newton Raphson iteration. The Cobb-Douglas production function is applied to five selected manufacturing industries in West Sumatra. From the research result, the return to scale (RTS) of the rubber and plastic goods industry is 0.8424 and the return to scale of the food and beverage industry is 0.8496 in which the two industries produce RTS <1. Whereas return to scale of the publishing and printing industry is 1.0460, the return to scale of the textile industry is 1.0018, and the return to scale of the non-metallic mining industry is 1.3384. Of the three industries each produce RTS> 1.

***Keywords*:** Least Square Estimation, Cobb-Douglas Production Function, Newton Raphson Method, Manufacturing Industry

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1. INTRODUCTION

In general, the performance of the manufacturing industry sector is inseparable from the overall economic performance. The link between overall economic fluctuations and the performance of the manufacturing sector is clearly visible in Indonesia. Growth acceleration during the reign of the new order in the 1970s to the mid 1980s. The manufacturing sector grew to more than 14% per year on average. Conversely, the slowdown in national economic growth occurring in the mid-1980s resulting from the fall of international oil prices, made a slowdown also in the manufacturing sector [2].

The various policies of economic reform and liberalization proclaimed by the government at that time could boost the manufacturing sector to keep growing strongly until the mid-1990s, relying on exports and foreign investment. The growth took place until the economic crisis of July 1997. This crisis marked the beginning of a painful period for the Indonesian economy. Economic growth was recorded to fall by more than 14% during 1997-1998 [2].

Every industry tries to produce goods with optimal results. To obtain it, it is necessary to decide what to produce, how much to produce and how the production process. Thus, an industry needs to choose an optimal combination of inputs used, ie a combination of inputs that allows to produce the desired output level. The model used is Cobb-Douglas production function. The parameter estimation of the Cobb-Douglas production function was obtained by the least square estimation approach with the Newton-Raphson iteration procedure.

2. LITERATURE REVIEW

2. 1. Estimation

Statistical inference is a statistical method used to draw inferences or compartments or conclusions from a population with information from samples taken from that population [32]. In the classical method, inference is based entirely on information obtained through a random sample taken from the population [13]. Statistical inference can be divided into two, namely the assessment or estimation and testing the hypothesis [31].

In general, estimation is a conjecture of something that will happen in uncertain conditions. Estimation is the whole process using an estimator to generate an estimate of a parameter. The data used to estimate population parameters is the sample statistic as an estimator [8].

2. 2. Least Square Estimation

The observed thing as system output depends on two issues, the functional relationships that govern the system (pattern) and the random element (error) [7]. The general procedure for predicting a relationship pattern, either causally or periodically, is to match a functional form such that the error component can be minimized. One form of this estimate is the least squares [20]. This approach was first developed by Gauss (1980) [28]. The least squares term is based on the fact that this estimation procedure attempts to minimize the sum of the squares of its error [27]. Formulated as follows:

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \qquad (1)$$

In equation (1) it can be recognized as a formula that gives the mean value of n numbers and this formula gives a value that minimizes the sum of the error squares.

2. 3. Taylor Series

Taylor series is a representation of mathematical functions as an infinite sum of the tribes whose value is calculated from the derivative of the function at a point. Taylor series provides a formula for predicting a function price at x_{i+1} expressed in the price of that function and its derivative around point x_i [26].

The general formula of the Taylor series is as follows [29]:

$$f(x_{i+1}) = f(x_i) + f'(x_i)(x_{i+1} - x_i) + \cdots + \frac{f^{(n)}(x_i)}{n!}(x_{i+1} - x_i)^n + R_n \qquad (2)$$

In practice only the first few tribes are taken into account so that the results are not exactly as in the analytical settlement [21]. So there is an error called the truncation error (R_n), namely:

$$R_n = \frac{f^{(n+1)}(\xi)}{(n+1)!}(x_{i+1} - x_i)^{n+1} \qquad (3)$$

where: n is the residual of the n order approximation and ξ is an x value located just between x_i and x_{i+1}.

2. 4. Hessian Matrix

A matrix is a rectangular square arrangement of numbers. The numbers in the order are called entries in the matrix [15]. The hessian matrix of a function f (x) is the second partial derivative matrix of the function [23]. Expressed as:

$$H(f) = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_1 \partial x_n} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & \cdots & \frac{\partial^2 f}{\partial x_2 \partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 f}{\partial x_n \partial x_1} & \frac{\partial^2 f}{\partial x_n \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_n^2} \end{bmatrix} \qquad (4)$$

2. 5. Production Function

The production function is a function that shows the physical relationship between the output level and the level (combination) of inputs used [16]. The production function shows the maximum amount of output that can be obtained from a certain set of inputs [14]. The production function can be formulated in the form of [1]:

$$Q = f(K, L, M, \dots) \tag{5}$$

where Q denotes the output of a given good for a period, K denotes the capital input for a given period, L denotes the labor input, M denotes the raw material used, and the point notation indicates the possibility of other variables affecting the production process.

2. 6. Cobb-Douglas Production Function

The Cobb-Douglas Production Function was first investigated by CW. Cobb and PH. Douglas and published in the journal American Economic Review in 1928 [3]. The Cobb-Douglas function is a function or equation involving two or more variables, in which one variable is called a dependent variable and the other is called an independent variable [5,6].

Cobb-Douglas production function with multiplicative error term [18] is formulated by [19]:

$$Q_t = \beta_1 L_t^{\beta_2} K_t^{\beta_3} e_t \tag{6}$$

where: Q_t is output; L_t is the work input; K_t is the capital input; β_1 is a technology parameter; β_2 is the labor input elasticity; β_2 is the capital input elasticity; e_t is the random error term.

Cobb-Douglas function with additive error term [10] is formulated with:

$$Q_t = \beta_1 L_t^{\beta_2} K_t^{\beta_3} + e_t \tag{7}$$

where: Q_t is output; L_t is the work input; K_t is the capital input; β_1 is a technology parameter; β_2 is the labor input elasticity; β_2 is the capital input elasticity; e_t is the random error term.

2. 7. Production Elasticity

Production elasticity (E) defined percentage of output change divided by percentage of input change. The production elasticity shows the ratio of the relative change in output produced to the relative change in the number of inputs used [24].

The output elasticity of capital (E_K) is measured through:

$$E_K = \frac{\%\,\Delta Q}{\%\,\Delta K} = \beta_3 \tag{8}$$

it can be seen that the output elasticity of the capital can be measured directly through the β_3 coefficient of the Cobb-Douglas production function.

While the output elasticity of labor (E_L) can be measured through:

$$E_L = \frac{\%\,\Delta Q}{\%\,\Delta L} = \beta_2 \tag{9}$$

it can be seen that the output elasticity of the capital can be measured directly through the β_2 coefficient of the Cobb-Douglas production function.

2. 8. Return to Scale

The Cobb-Douglas production function can also be used to investigate returns to scale, by combining one or two inputs both capital and labor. Return to scale shows the relationship of input changes together (in percentage) to output changes [17].

Suppose that use of K_0 and L_0 will produce output Q_0, that is:

$$Q_0 = \beta_1 L_0^{\beta_2} K_0^{\beta_3} \tag{10}$$

If Q_1 is an output generated by a combination of capital inputs, then it is obtained:

$$Q_1 = \beta_1 L_0^{\beta_2} (2K_0)^{\beta_3} = 2^{\beta_3} Q_0 \tag{11}$$

If Q_2 is the output produced by a combination of labor input, then it is obtained:

$$Q_2 = \beta_1 (2L_0)^{\beta_2} K_0^{\beta_3} = 2^{\beta_2} Q_0 \tag{12}$$

If Q_3 is the output produced by a combination of capital and labor inputs, then it is obtained:

$$Q_3 = \beta_1 (2L_0)^{\beta_2} (2K_0)^{\beta_3} = 2^{\beta_2+\beta_3} Q_0 \tag{13}$$

So that can know the combination of production elasticity that is if $\beta_2 + \beta_3 = 1$ then the function will show constant return to scale, meaning that increase of input will be followed by increase of output proportionally [9]. If $\beta_2 + \beta_3 < 1$ then the function shows the decreasing returns to scale, which means the percentage increase in output is smaller than the percentage of input additions [25]. If $\beta_2 + \beta_3 > 1$ then it shows the scale with increasing return to scale, it means the percentage of output increase is greater than the percentage of input additions [11].

3. ESTIMATION PROCEDURE

In the production function model with multiplicative error termdapat directly transformed into linear form, so the settlement can be done by linear statistical estimation procedure. In this model, the regression coefficient shows the elasticity of production and the sum of its parameters shows return to scale.

Different from the production function model with the additive error term which this model can not be transformed into a linear form, so the estimation procedure must be solved by non-linear statistical technique [30].

The approach used in the estimation of non-linear parameters is to focus its objective function by minimizing the sum of squares or maximizing the likelihood function. Furthermore, in this paper is used estimation process with least square estimation approach using Newton Raphson iteration method.

The Newton-Raphson (NR) method is one of the most popular methods for approaching an iterative solution [4]. In this method $g(\beta)$ is approximated by the extension of the Taylor series of second order around the initial value $\beta^{(t)}$.

$$g\left(\beta^{(t+1)}\right) \approx g\left(\beta^{(t)}\right) + g'\left(\beta^{(t)}\right)\left(\beta^{(t+1)} - \beta^{(t)}\right) + \frac{1}{2} g''\left(\beta^{(t)}\right)\left(\beta^{(t+1)} - \beta^{(t)}\right)^2$$

$$g\left(\beta^{(t+1)}\right) \approx g(\beta^t) + \boldsymbol{G}(\beta^t)\left(\beta^{(t+1)} - \beta^t\right) + \frac{1}{2} \boldsymbol{H}(\beta^t)\left(\beta^{(t+1)} - \beta^{(t)}\right)^2$$

where: $\boldsymbol{G}(\beta^t) = \left[\frac{\partial g}{\partial \beta_i}\right]_{\beta^{(t)}}$ is the gradient vector and $\boldsymbol{H}(\beta^t) = \left[\frac{\partial^2 g}{\partial \beta_i \partial \beta_k}\right]_{\beta^{(t)}}$ is the Hessian matrix. This Hessian Matrix is positive definite[19].

The first derivative is obtained as follows:

$$\frac{\partial g(\beta)}{\partial(\beta)} = 0 + \boldsymbol{G}\left(\beta^{(t)}\right) + \boldsymbol{H}\left(\beta^{(t)}\right)\left(\beta^{(t+1)} - \beta^{(t)}\right) = 0$$

The next step is obtained to calculate $\beta^{(t+1)}$, to obtain the general equation of Newton Raphson iteration as follows:

$$\beta^{(t+1)} = \beta^{(t)} - \left[\boldsymbol{H}\left(\beta^{(t)}\right)\right]^{-\mathbf{1}} \boldsymbol{G}\left(\beta^{(t)}\right)$$

The iteration procedure continues until convergence is reached. If the iteration is convergent, ie $\beta^{(t+1)} = \beta^{(t)}$ then can be concluded that $\boldsymbol{G}(\beta^t) = 0$, which satisfies the requirement of first order (necessary condition). Convergence approaches maximum if:

$$\left|\beta_i^{(t+1)} - \hat{\beta}_i\right| \leq c \left|\beta_i^{(t)} - \hat{\beta}_i\right|^2, c \geq 0$$

when: $\beta_i^{(t)}$ approaches $\hat{\beta}_i$ for every i. So we get $\hat{\beta}_i$ estimation from Newton-Raphson method.

The parameter estimation of Cobb-Douglas production function with additive error term is done by minimizing the quantity of squaredrate, then adding the values (quadratic error) can be obtained:

$$S(\beta) = \sum_{t=1}^{n} {e_t}^2 = \sum_{t=1}^{n} \left(Q_t - \beta_1 L_t^{\beta_2} K_t^{\beta_3}\right)^2$$

By minimizing the sum of the squares of the error and substituted into the general equation of Newton Raphson obtained:

$$\beta^{(t+1)} = \beta^{(t)} - \left[\boldsymbol{H}\left(\beta^{(t)}\right)\right]^{-\mathbf{1}} \boldsymbol{G}\left(\beta^{(t)}\right)$$

$$\beta^{(t+1)} = \beta^{(t)} - \left[\left[\frac{\partial^2 g}{\partial\beta_i \partial\beta_k}\right]_{\beta^{(t)}}\right]^{-1} \left[\frac{\partial g}{\partial\beta_i}\right]_{\beta^{(t)}}$$

$$\beta^{(t+1)} = \beta^{(t)} - \begin{bmatrix} \frac{\partial^2 S(\beta)}{\partial\beta_1^2} & \frac{\partial^2 S(\beta)}{\partial\beta_1\partial\beta_2} & \frac{\partial^2 S(\beta)}{\partial\beta_1\partial\beta_3} \\ \frac{\partial^2 S(\beta)}{\partial\beta_1\partial\beta_2} & \frac{\partial^2 S(\beta)}{\partial\beta_2^2} & \frac{\partial^2 S(\beta)}{\partial\beta_2\partial\beta_3} \\ \frac{\partial^2 S(\beta)}{\partial\beta_1\partial\beta_3} & \frac{\partial^2 S(\beta)}{\partial\beta_2\partial\beta_3} & \frac{\partial^2 S(\beta)}{\partial\beta_3^2} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial S(\beta)}{\partial\beta_1} \\ \frac{\partial S(\beta)}{\partial\beta_2} \\ \frac{\partial S(\beta)}{\partial\beta_3} \end{bmatrix}$$

4. RESULT / EXPERIMENTAL

The data used in this research is secondary data taken from BadanPusatStatistik about manufacturing industry in West Sumatra. The industries studied are five industries, including: rubber and plastic goods industry, printing and publishing industry, textile industry, food and beverage industry, and non-metal mining industry. The data used are data from 2001 to 2010.

Table 1. Estimated results in rubber and plastic goods industries

Initial Value			Optimal Value			Number of Iterations
β_1	β_2	β_3	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	
0,55	0,55	0,55	4,0357	0,0609	0,7815	41
0,60	0,60	0,60	4,0357	0,0609	0,7815	12
0,65	0,65	0,65	4,0357	0,0609	0,7815	42
0,70	0,70	0,70	4,0357	0,0609	0,7815	39

Table 2. Estimated results in the publishing and printing industry

Initial Value			Optimal Value			Number of Iterations
β_1	β_2	β_3	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	
1	1	1	1,3186	0,0863	0,9597	50
0,9	0,9	0,9	1,3186	0,0863	0,9597	48

0,7	0,7	0,7	1,3186	0,0863	0,9597	33
0,5	0,5	0,5	1,3186	0,0863	0,9597	52

Table 3. Estimated results in the textile industry

Initial Value			Optimal Value			Number of Iterations
β_1	β_2	β_3	β_1	β_2	β_3	
1	1	1	1	1	1	1
0,6	0,6	0,6	0,6	0,6	0,6	0,6
0,4	0,4	0,4	0,4	0,4	0,4	0,4
0,2	0,2	0,2	0,2	0,2	0,2	0,2

Table 4. Estimated results in the food and beverage industry

Initial Value			Optimal Value			Number of Iterations
β_1	β_2	β_3	β_1	β_2	β_3	
1	1	1	1	1	1	1
0,8	0,8	0,8	0,8	0,8	0,8	0,8
0,5	0,5	0,5	0,5	0,5	0,5	0,5
0,3	0,3	0,3	0,3	0,3	0,3	0,3

Table 5. Estimated results in non-metal mining industry

Initial Value			Optimal Value			Number of Iterations
β_1	β_2	β_3	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	
1	1	1	0,1537	0,7430	0,5954	29
0,7	0,7	0,7	0,1537	0,7430	0,5954	16
0,5	0,5	0,5	0,1537	0,7430	0,5954	24
0,2	0,2	0,2	0,1537	0,7430	0,5954	13

Based on the estimation results in tables 1 to 5, with different initial values being obtained the same optimum value can be obtained. If a change is made to the initial value of the parameter, it changes the number of iterations.

4. 1. Elasticity

From the estimation results obtained, it can be explained about the elasticity that shows the ratio of the relative changes of output produced to the relative changes in the number of inputs used.

Table 6. Estimated results in the food and beverage industry

No	Industry	Labor Elasticity $(\hat{\beta}_2)$	Capital Elasticity $(\hat{\beta}_3)$
1	Rubber and plastic goods industries	0,0609	0,7815
2	Publishing and printing industry	0,0863	0,9597
3	Textile industry	0,0715	0,9303
4	Food and beverage industry	0,2752	0,5744
5	Non-metal mining industry	0,7430	0,5954

According to Table 6, in the rubber and plastic goods industry, the elasticity of labor input is 0.0609, which means that about 6% of the effect of changes in the amount of labor input on changes in industrial output results. While the elasticity of the capital input is 0.7815, which means that about 78% of the effect of changes in the amount of capital input on changes in industrial output results.

4. 2. Return to Scale

From the estimation result, the return to scale (RTS) from each industry shows the relationship of input change together to the output change.

In Table 7, the return to scale of the rubber and plastic goods industry is 0.8424 and the return to scale of the food and beverage industry is 0.8496, where the two industries produce RTS <1, it indicates decreasing returns to scale which means that the percentage increase in industrial output is less than the percentage of inputs of each industry.

While the return to scale of the publishing and printing industry is 1.0460, the return to scale of the textile industry is 1.0018, and the return to scale of the non-metallic mining industry is 1.3384. Of the three industries each produce RTS> 1, it shows increasingreturns to scale which means that the percentage increase in industrial output is greater than the percentage of input addition from each industry.

Table 7. Return to scale from each industry

No	Industry	Elastisitas Tenaga Kerja $(\hat{\beta}_2)$	Elastisitas Kapital $(\hat{\beta}_3)$	*Return to scale* $(\hat{\beta}_2)+(\hat{\beta}_3)$
1	Rubber and plastic goods industries	0,0609	0,7815	0,8424
2	Publishing and printing industry	0,0863	0,9597	1,0460
3	Textile industry	0,0715	0,9303	1,0018
4	Food and beverage industry	0,2752	0,5744	0,8496
5	Non-metal mining industry	0,7430	0,5954	1,3384

5. CONCLUSIONS

Model estimation of Cobb-Douglas production function parameters from each manufacturing industry is presented in the following table:

Table 8. Return to scale from each industry

No	Industry	Cobb-Douglas Estimation Model
1	Rubber and plastic goods industries	$\hat{Q} = 4{,}0357L^{0{,}0609}K^{0{,}7815}$
2	Publishing and printing industry	$\hat{Q} = 1{,}3185L^{0{,}0863}K^{0{,}9597}$
3	Textile industry	$\hat{Q} = 1{,}5132L^{0{,}0715}K^{0{,}9303}$
4	Food and beverage industry	$\hat{Q} = 2{,}8305L^{0{,}2752}K^{0{,}5744}$
5	Non-metal mining industry	$\hat{Q} = 0{,}1537L^{0{,}7430}K^{0{,}5954}$

The estimation model in Table 8 can be used by industry actors to select the optimal combination of inputs used, ie the allowable combination to generate the desired output level.

References

[1] A.D. Vîlcua, G.E. Vîlcu, A survey on the geometry of production models in economics. *Arab Journal of Mathematical Sciences* (2017) 18-31

[2] Bappenas, Directorate of Sectoral Development Performance Evaluation of Ministry of VAT, Changes in Productivity of Indonesia's Manufacturing Industry and Its Affecting Factors: Panel Data Analysis 2004-2009. *Bappenas Annual Report* (2010)

[3] CW Cobb, PH Douglas, A Theory of Production. *American Economic Review* (1928) 139-165

[4] Dan Anbar, A stochastic Newton-Raphson method. *Journal of Statistical Planning and Inference* (1978) 153-163

[5] E. Khater, A Cobb – Douglas Function Based Index for Human Development in Egypt. *International Journal Contemporer Math Science* (2012) 591-598

[6] Gabriel Eduard Vîlcu, A geometric perspective on the generalized Cobb–Douglas production functions. *Applied Mathematics Letters* 24 (2011) 777–783

[7] H.T. Banks, Michele L. Joyner. AIC under the framework of least squares estimation. *Applied Mathematics Letters* 74 (2017) 33-45

[8] J. Morio, M. Balesdent, Estimation of Rare Event Probabilities in Complex Aerospace and Other Systems: Introduction to rare event probability estimation. *Woodhead Publishing* (2015) 1-2

[9] J.H Tressler, C.Fmenezes, Constant returns to scale and competitive equilibrium under uncertainty. *Journal of Economic Theory* (1983) 383-391

[10] J.H. Proost, Combined proportional and additive residual error models in population pharmacokinetic modelling. *European Journal of Pharmaceutical Sciences* (2017) S78-S82

[11] James E.Rauch, Increasing returns to scale and the pattern of trade. *Journal of International Economics* 26 (1989) 359-369

[12] Judge, George G., et al, The Theory and Practice of Econometrics. *Wiley* (1980)

[13] K.J.Friston, et al, Classical and Bayesian Inference in Neuroimaging: Theory. *Neuro Image* 16 (2002) 465–483

[14] K.I. Kim, et al, Estimating Production Functions with Control Functions When Capital is Measured with Error. *Journal of Econometrics* (2016) 267-279

[15] Li Li, Yujin Hu, Xuelin Wang, Design sensitivity and Hessian Matrix of Generalized Eigenproblems. *Mechanical Systems and Signal Processing* 43 (2014) 272–294

[16] Lin B, Philip A, Inter-fuel substitution possibilities in South Africa: A translog production function approach. *Energy* (2017) 822-831

[17] LIU Anguo, GAO Ge, Yang Kaizhong, Returns to scale in the production of selected manufacturing sectors in China. *Energy Procedia* 5 (2011) 604–612

[18] M. Fosgerau, M. Bierlaire, Discrete choice models with multiplicative error terms. *Transportation Research Part B: Methodological* (2009) 494-505

[19] M. Hossain, A. Kumar Majumder, T. Basak, An Application of Non-Linear Cobb-Douglas Production Function to Selected Manufacturing Industries in Bangladesh. *Open Journal of Statistics* (2012) 460-468

[20] Madhumita Pal, M. Seetharama Bhat, Least Square Estimation of Spacecraft Attitude along with Star Camera Parameters. *IFAC Proceedings Volumes* (2014) 20-25

[21] N. Tsirivas, A generalization of universal Taylor series in simply connected domains. *Journal of Mathematical Analysis and Applications* 388 (2012) 361–369

[22] Nicholson W, Snyder C.M., Intermediate Microeconomics and Its Application. 11th Edition. Mason, Ohio: South-Western/Cengage Learning, c2010.

[23] Pingge Chen, Yuejian Peng , Suijie Wang, The Hessian matrix of Lagrange function. *Linear Algebra and its Applications* 531 (2017) 537–546

[24] R. Rogerson, J. Wallenius, Retirement, home production and labor supply elasticities. *Journal of Monetary Economics* 78 (2016) 23–34

[25] Rajiv D.Banker, R.M.Thrall, Estimation of returns to scale using data envelopment analysis. *European Journal of Operational Research* 62 (1992) 74-84

[26] Raymon P. Canale, Steven C. Chapra, Raymon P. Canale, Steven C. Chapra, Numerical Methods for Engineers. *McGraw-Hill Fourth Edition* (2002)

[27] Spyros G. Makridakis, Steven C. Wheelwright, Rob J. Hyndman, Forecasting: Methods and Applications, 3rd Edition (1998)

[28] Stephen M. Stigler, Gauss and the Invention of Least Squares. *The Annals of Statistics* (1981) 465-474

[29] S. Ghosh, et al, Taylor series approach for function approximation using 'estimated' higher derivatives. *Applied Mathematics and Computation* 284 (2016) 89–101

[30] V. Strijova, G.W. Weber, Nonlinear regression model generation using hyperparameter optimization. *Computers & Mathematics with Applications* (2010) 981-988

[31] Walpole E. Ronald, Myers H Raymond, Probability and Statistics for Engineers and Scientists. *Macmillan* (1972)

[32] Zhuang Yu, at al., Statistical inference-based research on sampling time of vehicle driving cycle experiments. *Transportation Research* (2017) 114–141