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Fuzzy multi-objective supplier selection problem in a supply chain

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ABSTRACT

The decision making of supplier selection and their allocation is one of the main concerns in supply chain management. In this paper, an attempt has been made to obtain an optimal allocation for supplier based on minimizing the net cost, minimizing the net rejections, and minimizing the net late deliveries subject to realistic constraints regarding buyer's demand, vendors' capacity, vendors' quota flexibility, purchase value of items, budget allocation to individual vendor, etc. We convert the problem into single objective fuzzy goal programming problem by using weighted root power mean the method of aggregation with linear, exponential and hyperbolic membership functions. The comparison has been made by assigning different weights to the objective functions. A numerical illustration is provided for the verification of applicability of the approach.

***Keywords*:** Multi-objective Optimization, Supplier Selection Problem, Fuzzy Goal Programming

1. INTRODUCTION

In a period of global sourcing, business's success often hinges on the most appropriate selection of its partners and suppliers. Specifically, suitable supplier's selection is one of the essential policies for improving the quality of output of any organization, which has a direct

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impact on the company's competitiveness and reputation. As business organizations become more dependent on suppliers, the direct and indirect concern of poor decision-making about supplier selection becomes more severe. As a result, an effective and efficient supplier selection and evaluation process become very important to the success of any manufacturing/Service organization. Suppliers' evaluation and selection process became energetic for the entire width of business sizes and activities. The selection process involves the determination of quantitative and qualitative factors so as to select the best possible suppliers, which ensure business competitiveness, sustainability and success.

Supplier selection is one of the most important functions of a company. The supplier selection is a multi-criterion problem, which includes both qualitative and quantitative factors. When a supplier selection decision needs to be made, the buyer generally establishes a set of evaluation criteria that can be used to compare potential sources. The relationship between a company and its supplier has always been critical and companies generally establish a set of evaluation criteria to be used to compare potential sources. The basic criteria typically utilized for this purpose are pricing structure, delivery product quality and service. Sometimes these evaluation criteria are in conflict with one another. Supplier selection is a key supply management decision.

The supplier selection problem (SSP) has been a focused area of research since the 1960s. Gaballa (1974) was the first author who has used a mixed integer programming (MIP) model to consider a real case study in which the objective is to minimise the total discount prices of the items allocation to the suppliers. After Gaballa, some other authors like Anthony and Buffa (1977) and Pan (1989) also developed a linear programming (LP) model for the SSP. Weber and Current (1993) developed a multi-objective MIP for supplier selection problem. Buffa and Jackson (1983) and Sharma et al. (1989), respectively, used linear and non-linear mixed-integer goal programming (GP) for price, service level, delivery and quality goals in supplier selection problem. A considerable number of decision models have been developed based on the multi-criteria decision making (MCDM) method presented by Brans and vinckle (1985), analytical hierarchy process (AHP) presented by Ghoudsypour and O'Brien (1998), total cost ownership (TCO) approach presented by Degraeve et al (2000), and data envelopment analysis (DEA) proposed by Weber et al (2000). Various research papers in literature deal with the imprecise information and uncertainty in supplier selection models. Narsimhan (1983) presented an analytical approach for supplier selection problem. Nydick and Hill (1992) used the analytic hierarchy process to structure the supplier selection procedure. Moreover, Holt (1998), Erol and Ferrel (2003) applied the fuzzy sets theory for the supplier selection with the best overall rating.

In past decade, several contributions have discussed fuzzy approaches with linear membership functions to the SSP problem in a supply chain. Amid et al. (2006) considered the decision maker to assign different weights to various criteria using an asymmetric fuzzy decision-making technique. This fuzzy model enables the purchasing managers to consider the imprecise information and also to consider the limitations of buyer and supplier for calculation of the order quantity assigned to each supplier. Kumar et al. (2004, 2006) proposed a multi-objective fuzzy linear programming supplier selection model with various input parameters treated as imprecise with a linear membership function. Özgen et al. (2008) developed a multi-objective possibilistic linear programming technique to define the optimum order quantities assigned to each of the supplier considering uncertainties in quality, cost and demand. Yang et al. (2008) presented fuzzy MCDM techniques interpretive structural modelling (ISM) and fuzzy analytical hierarchy process (AHP) for supplier selection problem. Faez et al. (2009) presented a case based reasoning approach for supplier selection problem. Madroñero et al. (2010) solved supplier selection problem using an interactive fuzzy multi-objective approach with modified S-curve membership functions.

In this paper, we incorporate the triangular (linear), exponential and hyperbolic membership functions with weighted root power mean the method of aggregation for fuzzy goal supplier selection problem. The results attained by the various membership functions are presented and compared. The advantage of the presented approach is that the decision maker can efficiently describe his strategies and preferences in terms of weight in order to get desired solutions. The remaining of the paper is organized as; Section 2 is the formulation of supplier selection problem, section 3 describes the fuzzy compromise programming and the various membership functions with weighted root power mean the method of aggregation. Section 4 contains the results analysis and section 5 contains the conclusion along with some future directions.

2. SUPPLIER SELECTION PROBLEM

Generally, the SSP is considered to be a complex multi-objective decision-making problem. Each objective involved in the problem may have an acceptable range of ideal value with a different type of achievement levels. Therefore, the SSP is formulated as a fuzzy goal programming problem. In the following formulation, it is assumed that only one item is purchased from one supplier and the quantity discounts are not taken into consideration. The shortages are also not allowed for any of the suppliers. It is considered that the demand and supply of the items are known with certainty. The objectives of minimizing the total net cost, late deliveries and number of rejected items are considered to be fuzzy in nature.

The following notations are used in the model formulation:

Nomenclature	
Indices	
i	Index for suppliers, $\forall\ i = 1,2,...,n$
j	Index for all items, $\forall\ j = 1,2,...,m$
k	Index for all items, $\forall\ k = 1,2,...,K$
Decision Variable	
x_i	Order quantity is given to the supplier i
Parameters	
p_i	Price of a unit item of the ordered quantity x_i to the supplier i
q_i	Percentage of the rejected units delivered by the supplier i

l_i	Percentage of the late delivered units by the supplier i
D	Aggregate demand of the item over a fixed planning period
P	Least total purchasing value that a supplier can have
F	Least value of flexibility in supply quota that a supplier should have
B_i	Budget constraint allocated to each supplier
U_i	Upper limit of the quantity available for supplier i
r_i	Supplier rating value for supplier i
f_i	Supplier quota flexibility for supplier i
Objective Functions	
Z_1	Total cost for ordering the aggregate demand
Z_2	Total rejected items
Z_3	Total late delivered items

The fuzzy goals of the SSP can be formulated as follows:

$$\text{Minimization } Z_1 \cong \sum_{i=1}^{n} p_i(x_i) \tag{1}$$

$$\text{Minimization } Z_2 \cong \sum_{i=1}^{n} q_i(x_i) \tag{2}$$

$$\text{Minimization } Z_3 \cong \sum_{i=1}^{n} l_i(x_i) \tag{3}$$

where: the symbol $\cong$ represents the fuzziness existing in the objective or the representation of fuzzy goal. The first fuzzy goal (1) is to minimize the net cost for ordering the aggregate demand. Second fuzzy goal (2) is to minimize the total rejected items of the suppliers and the third fuzzy goal (3) is to minimize the late deliveries of items from the suppliers.

Constraints

$$\sum_{i=1}^{n} x_i = D \tag{4}$$

The above constraint puts restrictions on the aggregate demand of the item.

$$x_i \le U_i \quad \textit{for all } i. \;\; i = 1,2,...,n \tag{5}$$

Above constraint puts restrictions on the maximum capacity of the suppliers.

$$\sum_{i=1}^{n} r_i(x_i) \ge P, \tag{6}$$

The total item purchasing value restriction is incorporated in above constraint.

$$\sum_{i=1}^{n} f_i(x_i) \le F, \tag{7}$$

Suppliers' quota flexibility is formulated in the above constraints.

$$P_i(x_i) \le B_i \tag{8}$$

The above constraint puts restrictions on the budget amount allocated to the suppliers for supplying the items.

$$x_i \ge 0 \text{ and integer.} \tag{9}$$

The above constraint puts a non-negativity restriction on the decision variable.

3. FUZZY COMPROMISE PROGRAMMING

The fuzzy compromise programming can provide preferred compromise solution which is also non-dominated one. Sometimes, the decision maker has to face conditions when he is worse off with respect to a goal and he wants to have a higher marginal rate of satisfaction with respect to that goal. Such behaviour is modelled using the convex shape of the membership function. The hyperbolic function is convex as well as concave over a different part of the objective function. In the proposed fuzzy compromise programming, we incorporate the linear, exponential and hyperbolic membership functions in the weighted root power mean the method of aggregation defined as follows:

For each particular objective $\tilde{Z}_i,\, i = 1,2,...,,K$ in the presented model, we obtain two values L_i & U_i that can be assumed as lower and upper bound of the objective function. One of the major assumptions in the fuzzy approach to solving mathematical programming problems in the literature involves the use of linear membership functions. A linear approximation is the most commonly used because of its simplicity and efficiency. It is denoted by obtaining two points, the upper and lower levels of acceptability.

Linear (or Triangular) Membership Function

Following triangular membership function can be employed to define the marginal evaluation mapping of the decision variable *X*.

$$\mu_i(x) = \begin{cases} 1 & If\ Z_i \le L_i \\ 1 - \dfrac{L_i - Z_i}{L_i - U_i} & If\ L_i < Z_i < U_i \\ 0 & If\ Z_i \ge U_i \end{cases}$$

$\mu_i : X \to [0,1]$ for the objectives $\tilde{Z}_i,\ i = 1,2,...,,K.$

Exponential Membership Function

A non-linear exponential membership function is defined as:

$$\mu_i(x) = \begin{cases} 1 & If\ Z_i \le L_k \\ \dfrac{e^{-S\theta_i(x)} - e^{-S}}{1 - e^{-S}} & If\ L_i < Z_i < U_i \\ 0 & If\ Z_i \ge U_i \end{cases}$$

where: $\theta_i(x) = \dfrac{L_i - Z_i}{L_i - U_i}$, $i = 1,2,...,K$, S is nonzero parameter which is prescribed by the decision maker.

Hyperbolic Membership Function

For each objective function a Hyperbolic Membership Function $\mu_i(x)$ is define as:

$$\mu_i(x) = \begin{cases} 1 & If\ Z_i \le L_i \\ \dfrac{1}{2}\tanh\left(\left(\dfrac{U_i + L_i}{2} - Z_i\right)\alpha_i\right) + \dfrac{1}{2} & If\ L_i < Z_i < U_i \\ 0 & If\ Z_i \ge U_i \end{cases}$$

where: $\alpha_i = \dfrac{6}{U_i - L_i}$

In most of the practical situation, these L_i & U_i can be viewed as an ideal solution and tolerance limit for the ideal solution. Ideal solutions are usually obtained by solving series of objective functions separately by ignoring all other objectives.

Li and Lai (2000) presented a fuzzy compromise programming approach using the weighted root power mean the method of aggregation for multi-objective transportation problem with the characteristic feature that all objectives are synthetically considered by marginally evaluating individual objectives and globally evaluating all objectives. Having defined marginal evaluation of X for all the objectives using these membership functions, the next step is to determine the global evaluation of X with respect to all objectives. Thus a mapping $\Psi_i : X \to [0,1]$ defines us that a solution X satisfies objective functions up to what degree. However, in this paper, we will consider the relative importance of all objective functions defined by preferences in terms of weights. The relative importance of objectives is usually given by a set of weights $w = (w_1, w_2, w_3, ..., w_k)$, for which $\sum_{i=1}^{K} w_i = 1$.

The aggregation operator employed here is the weighted root power mean operator $\Psi_w^{(\alpha)}$.

$$\Psi_w^{(\alpha)}(\mu_1, \mu_2, \mu_3, ..., \mu_K) = \left(\sum_{i=1}^{K} w_i \mu_i^{\alpha} \right)^{1/\alpha}, \qquad (0 < |\alpha| < \infty),$$

The above fuzzy compromise programming approach covers many approaches such as weighted sum method, quadratic programming method and Zimmermann's (1978) fuzzy programming approach for different values of α .

3. 1. Application of Fuzzy Compromise Programming

The mixed integer fuzzy multi-objective programming model presented above can be converted to mixed integer single objective programming model as follows:

$$\textit{Maximize } \Psi_w^{(\alpha)}(\mu_1, \mu_2, \mu_3, ..., \mu_K) = \left(\sum_{i=1}^{K} w_i \mu_i^{\alpha} \right)^{1/\alpha}, \qquad (0 < |\alpha| < \infty),$$

Subject to

$$\mu_i \le \mu_i^{'}$$

$$\sum_{i=1}^{n} x_i = D$$

$$x_i \le U_i \quad \textit{for all } i. \;\; i = 1,2,...,n$$

$$\sum_{i=1}^{n} r_i(x_i) \ge P,$$

$$\sum_{i=1}^{n} f_i(x_i) \le F,$$

$$P_i(x_i) \le B_i, \qquad x_i \ge 0 \text{ and integer.}$$

It is already proved in Li and Lai (2008) that this operator covers a wide range of aggregation operator used in multi-criteria decision making. Some of them are generated here by applying the supplier allocation problem formulated.

i) For $\alpha = 1$, the single objective of proposed model converges into the weighted arithmetic mean aggregating operator as follows.

$$\textit{Maximize} \ \ \Psi_w^{(\alpha)}(\mu_1, \mu_2, \mu_3) = w_1\mu_1 + w_2\mu_2 + w_3\mu_3$$

Subject to

$$\mu_i \le \mu_i^{'}$$
$$\sum_{i=1}^{n} x_i = D$$
$$x_i \le U_i \quad \forall i, \ \ i = 1,2,...,n$$
$$\sum_{i=1}^{n} r_i(x_i) \ge P,$$
$$\sum_{i=1}^{n} f_i(x_i) \le F,$$
$$P_i(x_i) \le B_i, \qquad x_i \ge 0 \text{ and integer.}$$

ii) For $\alpha = 2$, the single objective of proposed model converges into the weighted quadratic mean aggregating operator as follows.

$$\textit{Maximize} \ \ \Psi_w^{(\alpha)}(\mu_1, \mu_2, \mu_3) = \left(w_1\mu_1^{\ 3} + w_2\mu_2^{\ 3} + w_3\mu_3^{\ 3}\right)^{1/3},$$

Subject to

$$\mu_i \le \mu_i^{'}$$
$$\sum_{i=1}^{n} x_i = D$$
$$x_i \le U_i \quad \textit{for all i.} \ \ i = 1,2,...,n$$
$$\sum_{i=1}^{n} r_i(x_i) \ge P,$$
$$\sum_{i=1}^{n} f_i(x_i) \le F,$$
$$P_i(x_i) \le B_i, \qquad x_i \ge 0 \text{ and integer.}$$

iii) For $\alpha = 3$, the single objective of proposed model converges into the weighted quadratic mean aggregating operator as follows.

$$\textit{Maximize} \ \ \Psi_w^{(\alpha)}(\mu_1, \mu_2, \mu_3) = \left(w_1\mu_1^{\ 2} + w_2\mu_2^{\ 2} + w_3\mu_3^{\ 2}\right)^{1/3},$$

Subject to

$$\mu_i \le \mu_i^{'}$$
$$\sum_{i=1}^{n} x_i = D$$
$$x_i \le U_i \quad \text{for all } i. \; i = 1,2,\ldots,n$$
$$\sum_{i=1}^{n} r_i(x_i) \ge P,$$
$$\sum_{i=1}^{n} f_i(x_i) \le F,$$
$$P_i(x_i) \le B_i, \qquad x_i \ge 0 \text{ and integer.}$$

iv) For $\alpha = \infty$, the single objective of proposed model converges into the Zimmerman's *min* operator as follows.

$$\text{Maximize} \quad \lambda$$

Subject to

$$\lambda \le \mu_i$$
$$\sum_{i=1}^{n} x_i = D$$
$$x_i \le U_i \quad \text{for all } i. \; i = 1,2,\ldots,n$$
$$\sum_{i=1}^{n} r_i(x_i) \ge P,$$
$$\sum_{i=1}^{n} f_i(x_i) \le F,$$
$$P_i(x_i) \le B_i, \qquad x_i \ge 0 \text{ and integer.}$$

4. NUMERICAL ILLUSTRATION

To illustrate the proposed work we have used the case study provided in Kumar et al. (2006) for testing of the proposed models. Thus it is also possible for readers to compare the results generated from the proposed models.

The model was tested in a made-to-order firm which is part of a manufacturing multi-national group in the automobile sector. The external purchases of the firm annually accounted for more than 70% of the total costs. In order to improve the efficiency of the purchasing process management wants to reconsider the sourcing strategies. Management feels that it is essential to certify and evaluate their suppliers in order to ensure the inventory and supplier base reductions. Management appoints a special team which is responsible for recommending the suitable suppliers. The team consists of several managers from various departments of the firm such as production, engineering, purchasing, marketing and research & development. The team members organized several meetings to agree on the profiles of the contending suppliers and constructed an initial set of four suppliers and evaluated them. The supplier's profile is shown as follows:

Table 1. Supplier source data of the illustrative example

Supplier number	$p_i(\$)$	$q_i(\%)$	$d_i(\%)$	$U_i(units)$	f_i	r_i	$B_i(\$)$
1	5	0.05	0.04	5000	0.02	0.88	25000
2	7	0.03	0.02	15000	0.01	0.91	100000
3	6	0	0.08	6000	0.06	0.97	35000
4	2	0.02	0.01	3000	0.04	0.85	5500

The Supplier profiles are shown in Table 1. The least value of total purchase value of supplied items and flexibility in suppliers' quota described as $P = r \times D$ and $F = f \times D$ respectively, are policy decisions and depend on the demand. If overall flexibility (f) is 0.03 on the scale of 0–1, the aggregate demand (D) is 20 000 and the overall supplier rating (r) is 0.92 on the scale of 0–1, then the least value of flexibility in suppliers' quota (F) and the least total purchase value of supplied items (P) are 600 and 18400, respectively. The proposed approach for the multi-objective supplier selection problem illustrated as follows:

Linear Membership Function

$$\mu_1' = \frac{Z_1(x_i) - 63334}{60002 - 63334} \quad or \quad \mu_1' = \frac{63334 - Z_1(x_i)}{3332}$$

$$\mu_2' = \frac{Z_2(x_i) - 466.66}{433.34 - 466.66} \quad or \quad \mu_2' = \frac{466.66 - Z_2(x_i)}{33.32}$$

$$\mu_3' = \frac{Z_3(x_i) - 683.34}{641.69 - 683.34} \quad or \quad \mu_3' = \frac{683.34 - Z_3(x_i)}{41.65}$$

The final crisp formulation can be stated as:

$$Maximize\ \Psi_\alpha^w = \left(w_1\mu_1^\alpha + w_2\mu_2^\alpha + w_3\mu_3^\alpha\right)^{1/\alpha}$$

$$\mu_1 \le \frac{63334 - Z_1(x_i)}{3332}$$

$$\mu_2' \le \frac{466.66 - Z_2(x_i)}{33.32}$$

$$\mu_3' \le \frac{683.34 - Z_3(x_i)}{41.65}$$

$x_1 + x_{2+} x_3 + x_4 = 20000$
$x_1 \le 5000$
$x_2 \le 15000$
$x_3 \le 6000$
$x_4 \le 3000$
$0.88x_1 + 0.91x_2 + 0.97x_3 + 0.85x_4 \ge 18400$
$0.02x_1 + 0.01x_2 + 0.06x_3 + 0.04x_4 \le 600$
$3x_1 \le 25000$
$2x_2 \le 100000$
$7x_3 \le 35000$
$x_4 \le 5500$
$x_i \ge 0 \quad and \ i = 1,2,3,4$

Applying the integrated fuzzy goal programming technique with weighted root power mean Ψ_α^w as an aggregation operator with linear membership function, following results are generated.

Table 2. Solution for linear membership function with different values of α

α	w_i	μ_i	x_i	Z_i
$\alpha = 1$	$w_1 = 0.33333$ $w_2 = 0.33333$ $w_3 = 0.33333$	1 0 1	0 15000.00 4167.00	60002.00 466.66 641.69
	$w_1 = 0.6$ $w_2 = 0.2$ $w_3 = 0.2$	1 0 1	0 15000.00 4167.00	60002.00 466.66 641.69
$\alpha = 2$	$w_1 = 0.33333$ $w_2 = 0.33333$ $w_3 = 0.33333$	0 1 0	0 13334.00 5000.00	63334.0 433.34 683.34
	$w_1 = 0.6$ $w_2 = 0.2$ $w_3 = 0.2$	0 1 0	0 13334.00 5000.00	63334.0 433.34 683.34

$\alpha = 3$	$w_1 = 0.33333$ $w_2 = 0.33333$ $w_3 = 0.33333$	0 1 0	0 13334.00 5000.00	63334.0 433.34 683.34
	$w_1 = 0.6$ $w_2 = 0.2$ $w_3 = 0.2$	0 1 0	0 13334.0 5000.0	63334.0 433.34 683.34
$\alpha = \infty$	-----	0.4945	4.00	61686
			14174.00	450.18
			4584	662.74

Exponential Membership Function

$$\mu_1' = \frac{e^{-(Z_1 - 60002)/3332} - e^{-1}}{1 - e^{-1}}$$

$$\mu_2' = \frac{e^{-(Z_2 - 433.34)/33.32} - e^{-1}}{1 - e^{-1}}$$

$$\mu_3' = \frac{e^{-(Z_3 - 641.69)/41.65} - e^{-1}}{1 - e^{-1}}$$

The final crisp formulation can be stated as:

$$Maximize\ \Psi_\alpha^w = \left(w_1 \mu_1^\alpha + w_2 \mu_2^\alpha + w_3 \mu_3^\alpha\right)^{1/\alpha}$$

Subject to

$$\mu_1' <= \frac{e^{-(Z_1 - 60002)/3332} - e^{-1}}{1 - e^{-1}}$$

$$\mu_2' <= \frac{e^{-(Z_2 - 433.34)/33.32} - e^{-1}}{1 - e^{-1}}$$

$$\mu_3' <= \frac{e^{-(Z_3 - 641.69)/41.65} - e^{-1}}{1 - e^{-1}}$$

$$x_1 + x_2 + x_3 + x_4 = 20000$$

$x_1 \le 5000$
$x_2 \le 15000$
$x_3 \le 6000$
$x_4 \le 3000$
$0.88x_1 + 0.91x_2 + 0.97x_3 + 0.85x_4 \ge 18400$
$0.02x_1 + 0.01x_2 + 0.06x_3 + 0.04x_4 \le 600$
$3x_1 \le 25000$
$2x_2 \le 100000$
$7x_3 \le 35000$
$x_4 \le 5500$
$x_i \ge 0$ *and* $i = 1,2,3,4$

Table 3. Solution for exponential membership function with different values of α

α	w_i	μ_i	x_i	Z_i
$\alpha = 1$	$w_1 = 0.33333$ $w_2 = 0.33333$ $w_3 = 0.33333$	0.9981 0.0005 0.9981	0.0000 14998.0 4168.00	60006.00 466.62 641.74
	$w_1 = 0.6$ $w_2 = 0.2$ $w_3 = 0.2$	0.9981 0.0005 0.9981	0.0000 14998.0 4168.00	60006.00 466.62 641.74
$\alpha = 2$	$w_1 = 0.33333$ $w_2 = 0.33333$ $w_3 = 0.33333$	0.9981 0.0000 0.9981	0.0000 14998.0 4168.00	60006.00 466.62 641.74
	$w_1 = 0.6$ $w_2 = 0.2$ $w_3 = 0.2$	0.9981 0.0000 0.9981	0.0000 14998.0 4168.00	60006.00 466.62 641.74
$\alpha = 3$	$w_1 = 0.33333$ $w_2 = 0.33333$ $w_3 = 0.33333$	0.9981 0.0000 0.9981	0.0000 14998.0 4168.00	60006.00 466.62 641.74
	$w_1 = 0.6$ $w_2 = 0.2$ $w_3 = 0.2$	0.9981 0.0000 0.9981	0.0000 14998.0 4168.00	60006.00 466.62 641.74
$\alpha = \infty$	-----	0.3769	0.000	61670
			14168.00	449.98
			4584.00	662.54

Hyperbolic Membership Function

For each objective function a Hyperbolic Membership Function is defined as:

$$\mu_1 = \frac{1}{2}\tanh\left(\left(\frac{63334+60002}{2} - Z_1\right)\alpha_1\right) + \frac{1}{2} \qquad \& \qquad \alpha_1 = \frac{6}{63334-60002}$$

$$\mu_2 = \frac{1}{2}\tanh\left(\left(\frac{466.66+433.34}{2} - Z_2\right)\alpha_2\right) + \frac{1}{2} \qquad \& \qquad \alpha_2 = \frac{6}{466.66-433.34}$$

$$\mu_3 = \frac{1}{2}\tanh\left(\left(\frac{683.34+641.69}{2} - Z_3\right)\alpha_3\right) + \frac{1}{2} \qquad \& \qquad \alpha_3 = \frac{6}{683.34-641.69}$$

The final crisp formulation of can be stated as:

$$Maximize\ \Psi_\alpha^w = \left(w_1\mu_1^\alpha + w_2\mu_2^\alpha + w_3\mu_3^\alpha\right)^{1/\alpha}$$

Subject to

$$\mu_1 <= \frac{1}{2}\tanh\left(\left(\frac{63334+60002}{2} - Z_1\right)\alpha_1\right) + \frac{1}{2}$$

$$\mu_2 <= \frac{1}{2}\tanh\left(\left(\frac{466.66+433.34}{2} - Z_2\right)\alpha_2\right) + \frac{1}{2}$$

$$\mu_3 <= \frac{1}{2}\tanh\left(\left(\frac{683.34+641.69}{2} - Z_3\right)\alpha_3\right) + \frac{1}{2}$$

$$x_1 + x_{2+}x_3 + x_4 = 20000$$
$$x_1 \le 5000$$
$$x_2 \le 15000$$
$$x_3 \le 6000$$
$$x_4 \le 3000$$
$$0.88x_1 + 0.91x_2 + 0.97x_3 + 0.85x_4 \ge 18400$$
$$0.02x_1 + 0.01x_2 + 0.06x_3 + 0.04x_4 \le 600$$
$$3x_1 \le 25000$$
$$2x_2 \le 100000$$
$$7x_3 \le 35000$$
$$x_4 \le 5500$$
$$x_i \ge 0 \quad and \quad i = 1,2,3,4$$

Table 4. Solution for hyperbolic membership function with different values of α

α	w_i	μ_i	x_i	Z_i
$\alpha=1$	$w_1=0.33333$ $w_2=0.33333$ $w_3=0.33333$	0.9975 0.0024 0.9975	0.00000 15000.0 4167.00	60002.0 466.66 641.69
	$w_1=0.6$ $w_2=0.2$ $w_3=0.2$	0.9975 0.0024 0.9975	0.00000 15000.0 4167.00	60002.0 466.66 641.69
$\alpha=2$	$w_1=0.33333$ $w_2=0.33333$ $w_3=0.33333$	0.0024 0.9975 0.0024	0.0000 13334.0 5000.00	63334.0 433.34 683.34
	$w_1=0.6$ $w_2=0.2$ $w_3=0.2$	0.0024 0.9975 0.0024	0.0000 13334.0 5000.00	63334.0 433.34 683.34
$\alpha=3$	$w_1=0.33333$ $w_2=0.33333$ $w_3=0.33333$	0.0024 0.9975 0.0024	0.0000 13334.0 5000.00	63334.0 433.34 683.34
	$w_1=0.6$ $w_2=0.2$ $w_3=0.2$	0.0024 0.9975 0.0024	0.0000 13334.0 5000.00	63334.0 433.34 683.34
$\alpha=\infty$	-----	0.4981	0.0000	61666
			14168.00	450.02
			4583.00	662.49

It has been observed that the weights assigned to the objectives don't have any significant effect on the objective values.

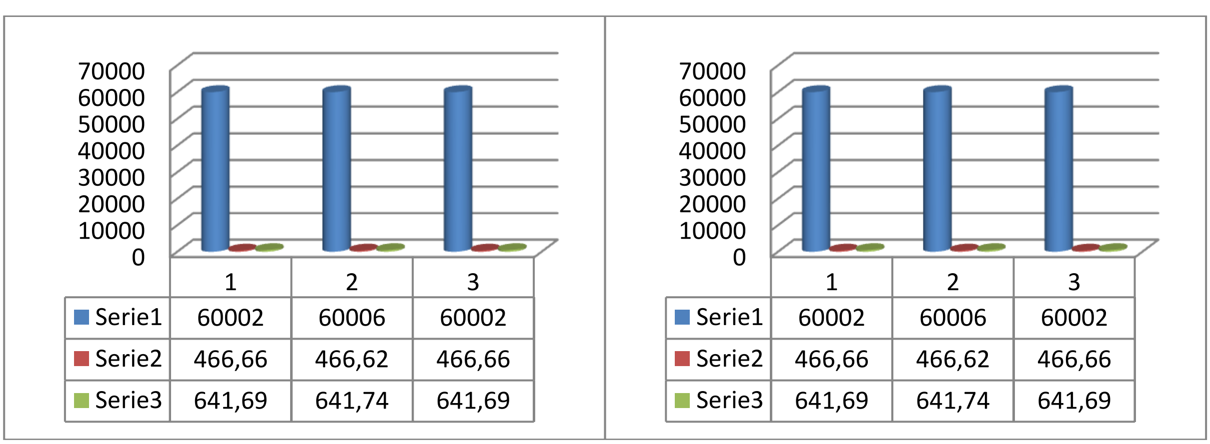

Figure 1. Comparison of objective values for the three membership function used for $\alpha=1$

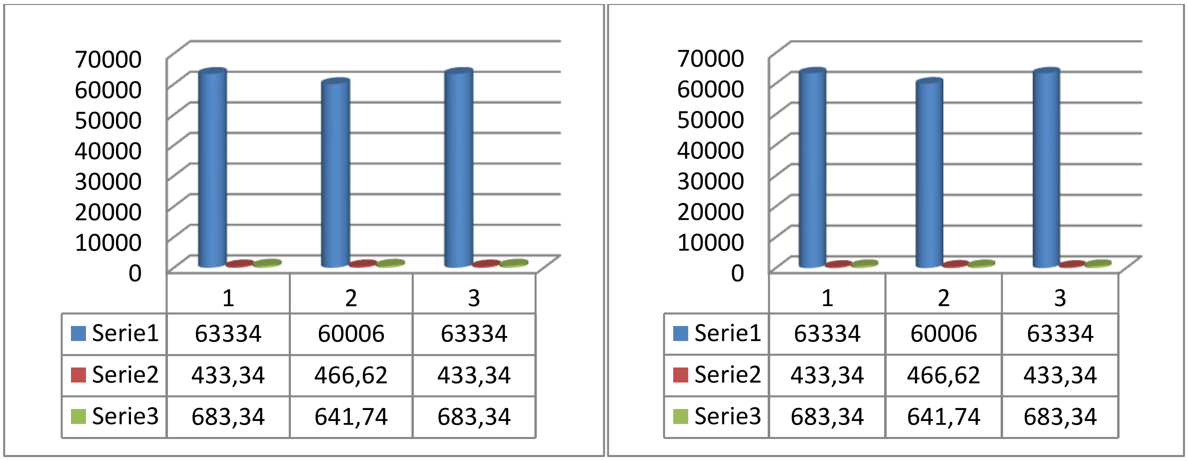

Figure 2. Comparison of objective values for the three membership function used for $\alpha = 2$

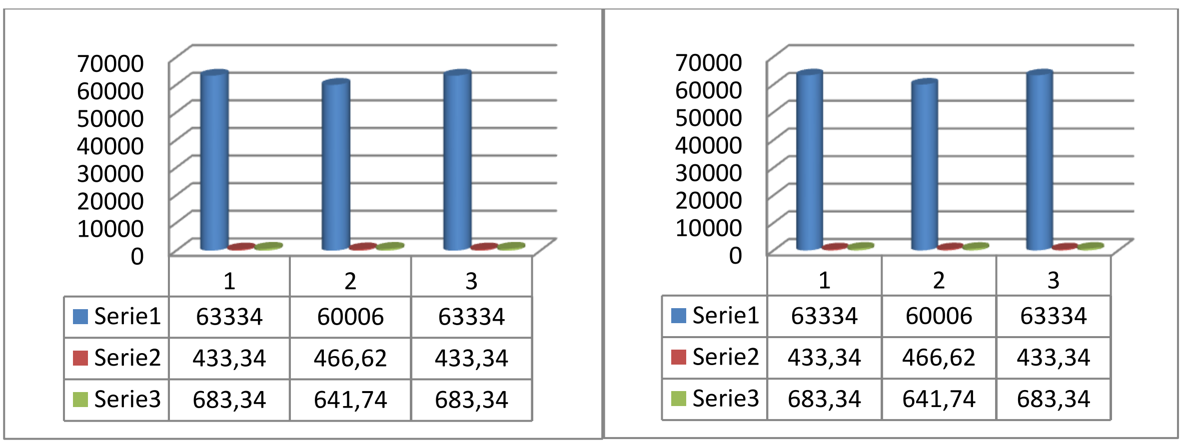

Figure 3. Comparison of objective values for the three membership function used for $\alpha = 3$

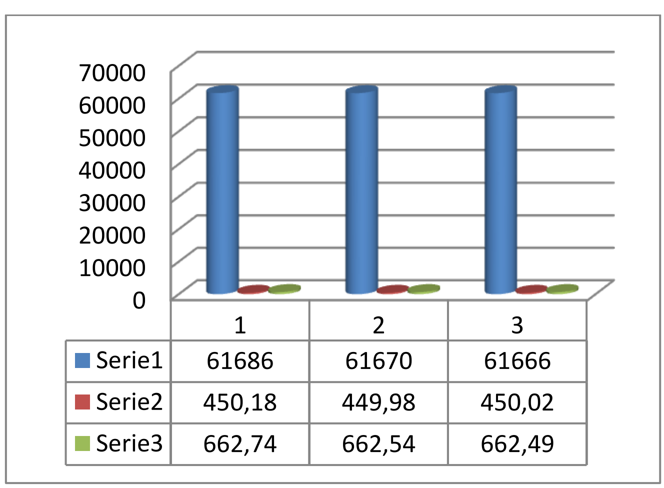

Figure 4. Comparison of objective values for the three membership function used for $\alpha = \infty$

In Figure (1-4), an overall comparison has been made in between the various objective values under different scenarios created on the basis of weights and value of the parameter α . In the above figures, the series 1, series 2 and series 3 represents the objective of total net cost, the total number of rejected items and the total late deliveries.

5. CONCLUSION

In this paper, supplier selection problem is considered with different fuzzy goals and different priorities. The weighted root power mean the method of aggregation is used to convert the multi-objective supplier selection optimization problem into a single supplier selection objective optimization problem. We incorporated the linear, exponential and hyperbolic membership functions in the weighted root power mean the method of aggregation. Different models have been generated for different parameter of alpha. A comparison has been made for the results obtained using various membership functions and weighted priority. It has been observed that the overall satisfaction is highest when the hyperbolic membership function is used. However, the proposed approach is very efficient and easy to apply. Also, the problem can be formulated and extended for the probabilistic cases.

References

[1] Anthony, T.F., Buffa, F.P., 1977. Strategic purchase scheduling. *Journal of Purchasing and Materials Management* 13 (3), 27–31.

[2] Bellman, R. E., & Zadeh, L. A. (1970). Decision making in a fuzzy environment. *Management Sciences* 17, 141–164.

[3] Buffa, F.P., Jackson, W.M., 1983. A goal programming model for purchase planning. *Journal of Purchasing and Materials Management* 19 (3), 27–34.

[4] C.A. Weber, J.R. Current, A. Desai, An optimization approach to determining the number of suppliers to employ, *Supply Chain Management: An International Journal* 5 (2) (2000) 90–98.

[5] Cachon, G. P., & Lariviere, M. A. (1999b). An equilibrium analysis of linear, proportional and uniform allocation. *IIE Transactions* 31(9), 835–849.

[6] Chen, L. H., & Tsai, F. C. (2001). Fuzzy goal programming with different importance and priorities. *European Journal of Operational Research* 133, 548–556.

[7] Conceição, S. V., Pedrosa, L. H. P., Neto, A. S. C., & Wolff, E. (2012). The facility location problem in the steel industry: a case study in Latin America. *Production Planning & Control: The Management of Operations* 23, 1, 26-46.

[8] Corstjens, M., & Doyle, P. (1979). Channel optimization in complex marketing systems. *Management Science* 25(10), 1014–1025.

[9] Dickson, G.W., 1966. An analysis of supplier selection systems and decisions. *Journal of Purchasing* 2 (1), 5–17.

[10] Gaballa, A.A., 1974. Minimum cost allocation of tenders. *Operational Research Quarterly* 25 (3), 398.

[11] J.P. Brans, P. Vincke, A preference ranking organization method. *Management Science* 31 (6) (1985) 647–656.

[12] Kumar, M., Vrat, P., & Shankar, R. (2004). A fuzzy goal programming approach for supplier selection problem in a supply chain. *Computers and Industrial Engineering* 46, 69–85.

[13] McAdam, R., & Brown, L. (2001). Strategic alignment and the supply chain for the steel stockholder sector. *Supply Chain Management* 6(2), 83–94.

[14] Moorthy, K. S. (1988). Strategic decentralization in channels. *Marketing Science* 7(4), 335–355.

[15] Narsimhan, R. (1980). Goal programming in a fuzzy environment. *Decision Sciences* 11, 325–336.

[16] Paksoy,T., & Pehlivan, N. Y. (2012). A fuzzy linear programming model for the optimization of multi-stage supply chain networks with triangular and trapezoidal membership functions. *Journal of the Franklin Institute* 349, 93-109.

[17] Potter, A., Mason, R., Naim, M., & Lalwani, C. (2004). The evolution towards an integrated steel supply chain: A case study from the UK. International Journal of Production Economics, 89, 207–216.

[18] Pan, A.C., 1989. Allocation of order quantity among suppliers. *Journal of Purchasing and Materials Management* 25 (3), 36–39.

[19] R. Verma, M.E. Pullman, An analysis of the supplier selection process, *Omega* 26 (6) (1998) 739–750.

[20] R.L. Nydick, R.P. Hill, Using the analytic hierarchy process to the supplier selection problem. *International Journal of Purchasing and Materials Management* 28 (2) (1992) 31–36.

[21] Rangan, V. K. (1987). The channel design decision: A model and an application. *Marketing Science* 6(2), 156–174.

[22] Rangan, V. K., & Jaikumar, R. (1991). Integrating distribution strategy and tactics: A model and an application. *Management Science* 37(11), 1377–1389.

[23] Rosenbloom, B. (2003). Marketing channels: A management view (3rd ed.). Dryden.

[24] S.H. Ghoudsypour, C.O. O'Brien, A decision support system for supplier selection using an integrated analytic hierarchy process and linear programming, *International Journal of Production Economics* 56–57 (1–3) (1998) 199–212.

[25] Sharma, D., Benton, W.C., Srivastava, R., 1989. Competitive strategy and purchasing decision. In: Proceedings of the 1989 Annual Conference of the Decision Sciences Institute, pp. 1088–1090.

[26] Tiwari, R. N., Dharmar, S., & Rao, J. R. (1986). Priority structure in fuzzy goal programming. *Fuzzy Sets and Systems* 19, 251–259.

[27] Tiwari, R. N., Dharmar, S.,&Rao, J. R. (1987). Fuzzy goal programming – an additive model. *Fuzzy Sets and Systems* 24, 27–34.

[28] Weber, C.A., Current, J.R., 1993. Theory and methodology: a multi-objective approach to supplier selection. *European Journal of Operational Research* 68, 173–184.

[29] Yang, T., Ignizio, J. P., & Kim, H. J. (1991). Fuzzy programming with nonlinear membership functions: Piecewise linear approximation. *Fuzzy Sets and Systems* 11, 39–53.

[30] Z. Degraeve, E. Labro, F. Roodhooft, An evaluation of supplier selection methods from a total cost ownership perspective. *European Journal of Operational Research* 125 (1) (2000) 34–59.

[31] Zadeh, L. A. (1965). Fuzzy sets. *Information and Control* 8, 338–353.

[32] Zimmermann, H. J. (1978). Fuzzy programming and linear programming with several objective functions. *Fuzzy Sets and Systems* 1, 45–56.