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SHORT COMMUNICATION

Analytical forms of deuteron wave function and density distribution

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ABSTRACT

Values of density distribution and transition density are calculated using the obtained coefficients of four deuteron wave function for analytical forms in coordinate representation for Argonne v18 potential. Calculations of these values can serve for an assessment of a correctness and accuracy of choice of an analytical form at concrete approximation of radial deuteron wave function. Also these calculations help to evaluate information on such characteristics of a deuteron as a charge form factor, tensor polarization and momentum distribution.

Keywords: deuteron, wave function, analytic form, approximation, density distribution, transition density

1. INTRODUCTION

The deuteron is the simplest kernel. It consists of two elementary particles - a proton and a neutron. Simplicity and presentation of a structure of a deuteron does it by convenient laboratory for studying and modeling a nucleon-nucleon forces. However, despite of good

experimental and theoretical study of a deuteron, there are some certain theoretical inconsistency and a problem. For example, in papers group the deuteron wave function (DWF) in coordinate representation has knots near the beginning of coordinates [1, 2]. Existence of such knots of the ground and only state of a deuteron testifies to not coherences and inaccuracies in realization of numerical algorithms in the solution of similar tasks or on features of potential models of a deuteron.

Also it is necessary to notice, that such potentials of NN interaction as Bonn, Paris, Moscow, Nijmegen group [3, 4], Argonne v18 [5], NLO, NNLO and N³LO, Idaho N³LO or Oxford potentials have rather a complicated structure and rather bulky record.

DWF can be presented as the table [6]: through the respective arrays of values of radial wave functions. Sometimes at numerical calculations to operate with such arrays of numbers very difficult and in general it is inconvenient, and the text of a program code for numerical calculations is bulky, overloaded and unreadable. Therefore receiving simpler analytical forms of representation of DWF is expedient. Further on them it is possible to calculate form factors and tensor polarization that characterize structure of a deuteron. DWF in a convenient form are necessary for used in calculations as polarization characteristics of a deuteron, and for an assessment of theoretical values of the spin observables in dp- scattering.

In this paper influence of a choice of analytical forms for approximation on calculation of sizes of density distribution and transition density in a deuteron is considered.

2. ANALYTICAL FORM OF THE DEUTERON WAVE FUNCTION

For the S- and D-wave components of the DWFs for local deep NN potential were obtained in the form of the gaussian expansions [7]

$$\begin{cases} u(r) = r \sum_{i=1}^N A_i e^{-a_i r^2}, \\ w(r) = r^3 \sum_{i=1}^N B_i e^{-b_i r^2}. \end{cases} \quad (1)$$

In 2000-x years the new analytical DWF forms were used. Except the mentioned parameterization, in scientific literature there is one more analytical form [8]

$$\begin{cases} u(r) = r^2 \sum_{i=1}^N A_i e^{-a_i r^3}, \\ w(r) = r^2 \sum_{i=1}^N B_i e^{-b_i r^3}. \end{cases} \quad (2)$$

Except forms (1) and (2), in paper [6] such analytical forms as are used

$$\begin{cases} u(r) = r^{3/2} \sum_{i=1}^N A_i e^{-a_i r^3}, \\ w(r) = r \sum_{i=1}^N B_i e^{-b_i r^3}. \end{cases} \quad (3)$$

The most popular, the quoted and used parameterization of DWF are the analytical forms, which were offered by the Paris group. Known numerical values of radial DWF in coordinate space for the Paris potential were approximated with the expansions [9] in such form

$$\begin{cases} u(r) = \sum_{j=1}^N C_j \exp(-m_j r), \\ w(r) = \sum_{j=1}^N D_j \exp(-m_j r) \left[1 + \frac{3}{m_j r} + \frac{3}{(m_j r)^2} \right], \end{cases} \quad (4)$$

where: $N = 13$; $m_j = \beta + (j-1)m_0$; $\beta = \sqrt{ME_d}$; $m_0 = 0.9 \text{ fm}^{-1}$; M is nucleon mass; E_d is binding energy of deuteron.

The accuracy of parameterization (1)-(4) is characterized by [6]:

$$\chi^2 = \frac{1}{n-p} \sum_{i=1}^N (y_i - f(x_i; a_1, a_2, \dots, a_p))^2, \quad (5)$$

where: n - the number of points of the array y_i of the numerical values of DWF; f - approximating function u (or w) according to formulas (1)-(4); a_1, a_2, \dots, a_p - parameters; p - the number of parameters (coefficients in the sums of formulas (1)-(4)). Hence, χ^2 is determined not only by the shape of the approximating function f , but also by the number of the selected parameters.

Based on the known DWFs (1)-(4) at $N = 30$ and their coefficients one can calculate the deuteron properties: the D- state probability P_D , deuteron radius r_m , the quadrupole moment Q_d , the magnetic moment μ_d and the ‘‘D/S- state ratio’’ η . They are in good agreement with the theoretical [5] and experimental [10] data’s. The deuteron properties for Argonne v18 potential have been numerically calculated (Table 1).

Table 1. Deuteron properties.

	P_D (%)	r_m (fm)	Q_d (fm ²)	μ_d	η
DWF (1)	5.7581	1.9628	0.26871	0.846996	0.025061
DWF (2)	5.7601	1.9574	0.26832	0.846985	0.024973
DWF (3)	5.7596	1.9587	0.26870	0.846988	0.025026

DWF (4)	5.7599	1.9674	0.26965	0.846986	0.025013
DWF [5]	5.76	1.967	0.270	0.847	0.025
Exp. [10]	-	1.9753	0.2859	0.85744	0.0256

In Figs. 1 and 2 is specified for asymptotic near the beginning of coordinate for deuteron wave function $u(r)$ and $w(r)$. Here 1, 2, 3, 4 - correspond for DWFs (1)-(4). The obtained wave functions do not contain any superfluous knots.

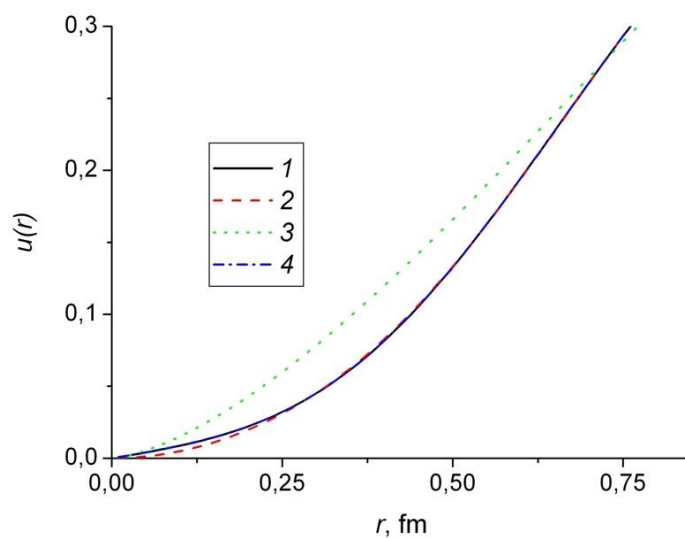


Figure 1. Asymptotic for deuteron wave function $u(r)$.

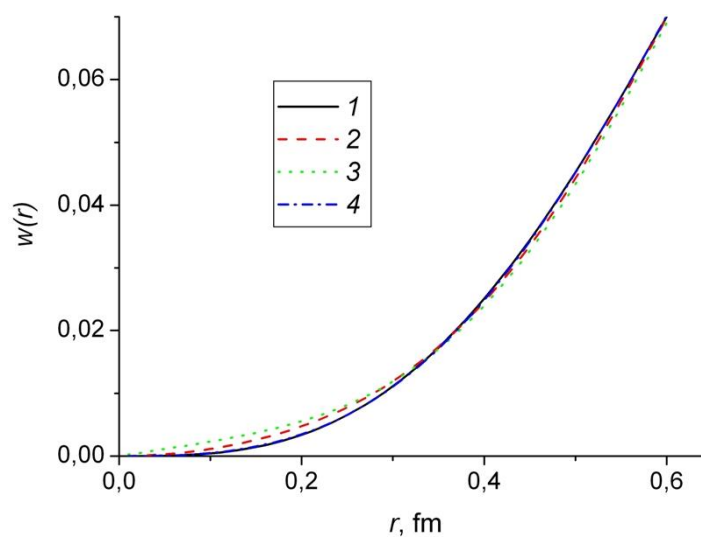


Figure 2. Asymptotic for deuteron wave function $w(r)$.

3. THE TWO-NUCLEON DENSITY DISTRIBUTIONS

DWF in coordinate representation is two-component [11]:

$$\Psi_d^{M_d}(r) = R_0(r)Y_{011}^{M_d} + R_2(r)Y_{211}^{M_d}, \quad (6)$$

where: $R_0 = u/r$; $R_2 = w/r$ - the radial functions for S- and D- states; $Y_{LSJ}^{M_d}$ - spin-angle functions.

In the short range the structure of a deuteron is visually described by means of density distribution [11] (or nucleon density distribution of substance in a deuteron [12]) $\rho_d^{M_d}(r', \theta)$, which depends on a projection M_d of full angular momentum, distance r' from the center of masses and a polar corner θ to r' . Standard rationing depends from between partial distance $r = 2r'$

$$\int_0^\infty r^2 [R_0^2(r) + R_2^2(r)] dr = 1;$$

$$\int \rho_d^{M_d}(r') d^3 r' = 2.$$

Using (6), for projections $M_d = 0; \pm 1$ receive density distribution $\rho_d^{M_d}$ in the form [11]

$$\begin{cases} \rho_d^0 = \frac{4}{\pi} [C_0(2r') - 2C_2(2r')P_2(\cos \theta)]; \\ \rho_d^{\pm 1} = \frac{4}{\pi} [C_0(2r') + C_2(2r')P_2(\cos \theta)]; \end{cases} \quad (7)$$

where: $C_0 = R_0^2 + R_2^2$;

$C_2 = \sqrt{2}R_0R_2 - \frac{1}{2}R_2^2$ - components of density distribution;

P_2 - Legendre's polynom.

At angles $\theta_1 = 0$ and $\theta_2 = \pi/2$ it is obvious that fair identity $\rho_d^0(r', \theta_2 = \pi/2) = \rho_d^{\pm 1}(r', \theta_1 = 0)$.

Except density distribution $\rho_d^{M_d}$, the internal structure of a deuteron is described as well by of transition density $\rho_{tr}^{\pm 1}$ [11]

$$\rho_{tr}^{\pm 1}(r') = \frac{2}{\pi} \left\{ R_0^2(2r') - \frac{1}{2}R_2^2(2r') - \frac{1}{2} \left[\sqrt{2}R_0(2r')R_2(2r') + R_2^2(2r') \right] P_2(\cos \theta) \right\}. \quad (8)$$

Values of density distribution and transition density (with components R_0, R_2, C_0, C_2) received on DWFs (1)-(4) for Argonne v18 potential are given in Figs. 3-6.

Depending on a choice of approximation for DWF the calculated values differ only in the area at 0-0.25 fm, and also in the peak for some quantities density distribution. In fact, this indicates which of the approximations applied is the «best» near the origin, despite the absence of an excessive node of the radial DWF.

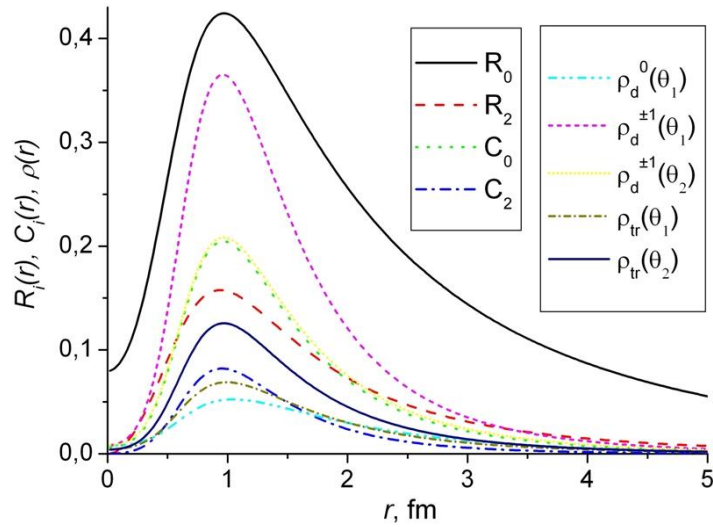


Figure 3. R_i , C_i and density distributions for deuteron wave function (1).

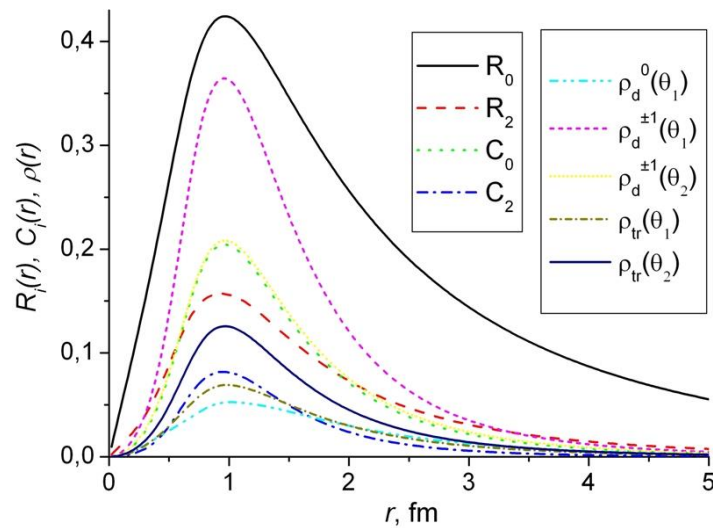


Figure 4. R_i , C_i and density distributions for deuteron wave function (2).

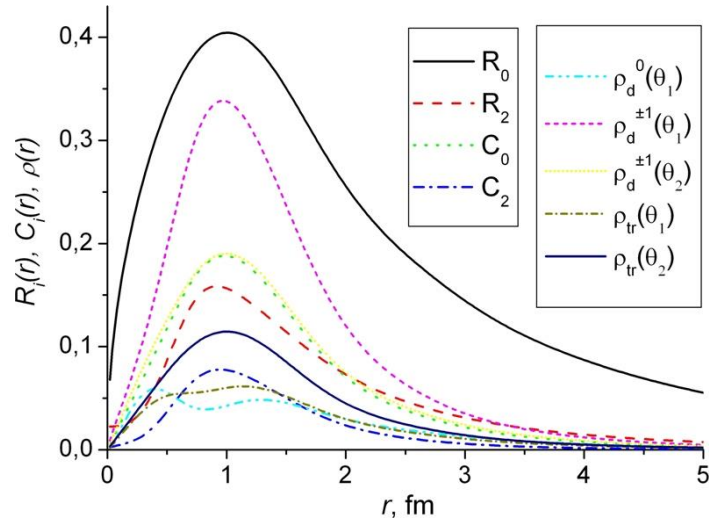


Figure 5. R_i , C_i and density distributions for deuteron wave function (3).

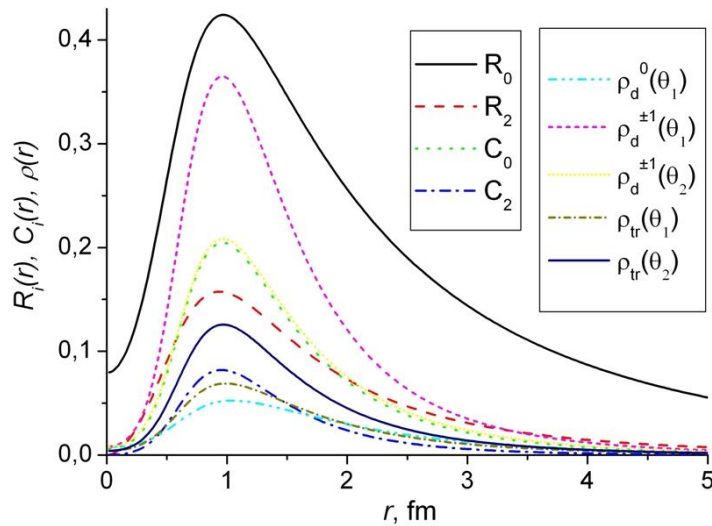


Figure 6. R_i , C_i and density distributions for deuteron wave function (4).

Results of similar calculations of density distribution and transition density for Reid93 [3, 4] and Moscow [13] potentials are quoted in paper [14].

4. CONCLUSIONS

Values of density distribution $\rho_d^{M_d}$ and transition density $\rho_{tr}^{\pm 1}$ are calculated using the obtained coefficients of four DWFs (1)-(4) of analytical forms in coordinate representation for a nucleon-nucleon Argonne v18 potential.

Calculations of values of density distribution in deuteron and transition density can serve for an assessment of a correctness of a choice of an analytical form at concrete approximation of DWFs.

Finally the knowledge of density distribution in a deuteron allows receiving information [11] on its charge form factor and tensor polarization. Spin-flip part of a magnetic form factor and momentum distribution can be received from transition density. It will also provide an independent assessment of the spatial sizes of toroidal structure in a deuteron and sections for $d(e,e'p)n$ - reaction in one-photon exchange approach.

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