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Oblong sum labeling of some special graphs

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ABSTRACT

Numbers of the form $n(n+1)$ are called oblong numbers. Let O_n be the n^{th} oblong number. An oblong sum labeling of a graph $\mathrm{G} = (V, E)$ with p vertices and q edges is a one to one function f : $\mathrm{V}(G) \to \{0,2,4,6,8,\dots\}$ that induces a bijection f^*: $\mathrm{E}(G) \to \{O_1, O_2, O_3, \dots, O_q\}$ of the edges of G defined by $f^*(uv) = f(u) + f(v)$ for all e = uv ∈ $\mathrm{E}(G)$. The graph that admits oblong sum labeling is called oblong sum graph. In this paper, oblong sum labeling of some special graphs is studied.

Keywords**:** Star, oblong numbers, oblong sum labeling

AMS classification: 05C78

1. INTRODUCTION

Graphs considered in this paper are finite, undirected and simple. For all graph theoretic terminologies and notations, Harary[3] is followed. Let $\mathrm{G} = (V, E)$ be a graph with p vertices and q edges. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions.

A vertex labeling of a graph G is an assignment f of labels to the vertices of G that induces for each edge xy a label depending on the vertex labels $f(x)$ and $f(y)$. Similarly, an edge labeling of a graph G is an assignment f of labels to the edges of G that induces for each

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vertex v a label depending on the edge label incident on it. Total labeling involves a function from the vertices and edges to some set of labels. There are several types of graph labeling and a detailed survey is found in [4].

Labeled graphs are becoming an increasing useful family of mathematical models for a broad range of applications like designing X-Ray crystallography, formulating a communication network addressing system, determining optimal circuit layouts, problems in additive number theory etc. A systematic presentation of diverse applications of graph labeling is given in [1] and [2].

Definition 1.1: The H-graph of a path P_n is the graph obtained from two copies of P_n with vertices $v_1, v_2, \dots, v_n$ and $u_1, u_2, \dots, u_n$ by joining the vertices $v_{\frac{n+1}{2}}$ and $u_{\frac{n+1}{2}}$ if n is odd and the vertices $v_{\frac{n}{2}+1}$ and $u_{\frac{n}{2}}$ if n is even.

Definition 1.2: Let O_n be the n^{th} oblong number. An oblong sum labeling of a graph $G = (V, E)$ with p vertices and q edges is a one to one function f: $V(G) \rightarrow \{0,2,4,6,8, \dots\}$ that induces a bijection f^*: $E(G) \rightarrow \{O_1, O_2, O_3, \dots, O_q\}$ of the edges of G defined by $f^*(uv) = f(u) + f(v)$ for all $e = uv \in E(G)$. The graph that admits oblong sum labeling is called oblong sum graph.

Result 1.3: H_n is oblong sum for all $n \geq 1$.

2. MAIN RESULTS

Definition 2.1: The Jelly Fish graph $J(m,n)$ is obtained from a 4- cycle u,v,s and t by joining s and t with an edge and appending m pendent edges to u and n pendent edges to v.

Theorem 2.2: For $m, n \geq 1$ Jelly Fish $J(m,n)$ is a oblong sum graph.

Proof: Let $J(m,n)$ be the Jelly fish graph.

Let $V(J(m,n)) = \{u, v, s, t, u_i, v_j ; 1 \leq i \leq m, 1 \leq j \leq n\}$ and
$E(J(m,n)) = \{us, sv, ut, tv, st, uu_i, vv_j \; ; \; 1 \leq i \leq m, 1 \leq j \leq n\}$

Then $J(m,n)$ has $(m+n+4)$ vertices and $(m+n+5)$ edges.

Let $f: V(J(m,n)) \rightarrow \{0,2,4,6, \dots\}$ be defined as follows

$f(u) = 0$
$f(s) = 12$
$f(t) = 30$
$f(v) = 60$
$f(u_i) = O_i \; ; \; 1 \leq i \leq 2$
$f(u_i) = O_4 \; ; \; i = 3$
$f(u_i) = O_7 \; ; \; i = 4$
$f(u_i) = O_{i+5} \; ; \; 5 \leq i \leq m$
$f(v_j) = O_{(m+5)+j} - f(v) \; ; \; 1 \leq j \leq n$

Let f^* be the induced edge labeling of *f*.

$f^*(us) = 12$
$f^*(ut) = 30$
$f^*(sv) = 72$
$f^*(tv) = 90$
$f(uu_i) = O_i\ ;\ 1 \le i \le 2$
$f(uu_i) = O_4\ ;\ i = 3$
$f(uu_i) = O_7\ ;\ i = 4$
$f(uu_i) = O_{i+5}\ ;\ 5 \le i \le m$
$f(vv_j) = O_{(m+5)+j}\ ;\ 1 \le j \le n$

The induced edge labels are distinct and are $O_1, O_2, O_3, \dots, O_{(m+n+5)}$.
Hence the Jelly Fish $J(m,n)$ is oblong sum.

Example 2.3: Oblong sum labeling of $J(8,5)$ is given in Fig. 1

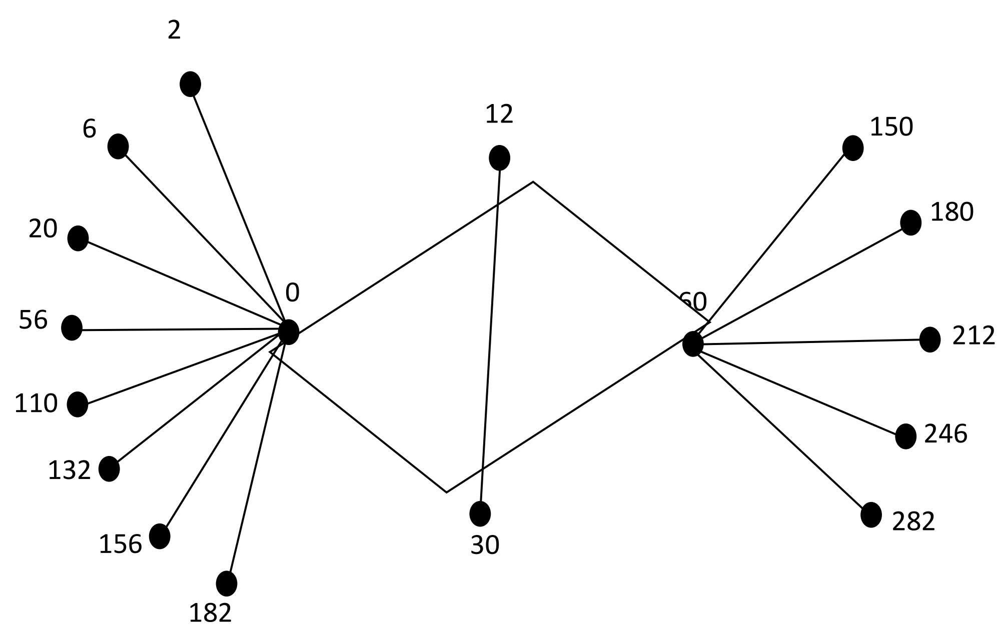


Fig. 1.

Definition 2.4: The Corona $G_1 \odot G_2$ of two graphs G_1 and G_2 is defined as the graph G by taking one copy of G_1(which has p_1 points) and $p_1$copies of G_2 and then joining the i^{th} point of G_1 to every point in the i^{th} copy of G_2.

Theorem 2.5: If H-graph G is a oblong sum graph then $G \odot S_1$ is also a oblong sum graph.

Proof: Let f be a oblong sum labeling of G with vertices $u_1, u_2, \dots, u_n$ and $v_1, v_2, \dots, v_n$ and let f^* be the induced edge labeling of G, as defined in Result 1.2.

Let $u_1', u_2', \ldots, u_n'$ and $v_1', v_2', \ldots, v_n'$ be the corresponding new vertices in $G \odot S_1$.
Define a labeling $g: (G \odot S_1) \rightarrow \{0,2,4,6, \ldots\}$ as follows:

Case(i): Let n be odd.

$$g\left(u_{\frac{n+1}{2}}\right) = f\left(u_{\frac{n+1}{2}}\right) = 0$$
$$g\left(u_{\frac{n+1}{2}-i}\right) = f\left(u_{\frac{n+1}{2}-i}\right) \;;1 \leq i < \frac{n+1}{2}$$
$$g\left(u_{\frac{n+1}{2}+i}\right) = f\left(u_{\frac{n+1}{2}+i}\right) \;;1 \leq i < \frac{n+1}{2}$$
$$g\left(v_{\frac{n+1}{2}}\right) = f\left(v_{\frac{n+1}{2}}\right) = 2$$
$$g\left(v_{\frac{n+1}{2}-i}\right) = f\left(v_{\frac{n+1}{2}-i}\right) \;;1 \leq i < \frac{n+1}{2}$$
$$g\left(v_{\frac{n+1}{2}+i}\right) = f\left(v_{\frac{n+1}{2}+i}\right) \;;1 \leq i < \frac{n+1}{2}$$
$$g\left(u'_{(2i+1)}\right) = (2n+2i)(2n+2i+1) - g\left(u_{(2i+1)}\right) \;;\; 0 \leq i < \frac{n+1}{2}$$
$$g\left(u'_{(2i+2)}\right) = (2n+2i+1)(2n+2i+2) - g\left(u_{(2i+2)}\right) \;;\; 0 \leq i < \frac{n-1}{2}$$
$$g\left(v'_{(2i+1)}\right) = (3n+2i)(3n+2i+1) - g\left(v_{(2i+1)}\right) \;;\; 0 \leq i < \frac{n+1}{2}$$
$$g\left(v'_{(2i+2)}\right) = (3n+2i+1)(3n+2i+2) - g\left(v_{(2i+2)}\right) \;;\; 0 \leq i < \frac{n-1}{2}$$

Let g^* be the induced edge labeling of g.
Then

$$g^*\left(u_{\frac{n+1}{2}+i}u_{\frac{n+1}{2}+(i+1)}\right) = f^*\left(u_{\frac{n+1}{2}+i}u_{\frac{n+1}{2}+(i+1)}\right) \;;0 \leq i < \frac{n-1}{2}$$
$$g^*\left(u_{\frac{n+1}{2}-i}u_{\frac{n+1}{2}-(i+1)}\right) = f^*\left(u_{\frac{n+1}{2}-i}u_{\frac{n+1}{2}-(i+1)}\right) \;;0 \leq i < \frac{n-1}{2}$$
$$g^*\left(u_{\frac{n+1}{2}}v_{\frac{n+1}{2}}\right) = f^*\left(u_{\frac{n+1}{2}}v_{\frac{n+1}{2}}\right) = 2$$
$$g^*\left(v_{\frac{n+1}{2}-i}v_{\frac{n+1}{2}-(i+1)}\right) = f^*\left(v_{\frac{n+1}{2}-i}v_{\frac{n+1}{2}-(i+1)}\right) \;;0 \leq i < \frac{n-1}{2}$$
$$g^*\left(v_{\frac{n+1}{2}+i}v_{\frac{n+1}{2}+(i+1)}\right) = f^*\left(v_{\frac{n+1}{2}+i}v_{\frac{n+1}{2}+(i+1)}\right) \;;0 \leq i < \frac{n-1}{2}$$
$$g^*\left(u_{(2i+1)}u'_{(2i+1)}\right) = (2n+2i)(2n+2i+1) \;;\; 0 \leq i < \frac{n+1}{2}$$
$$g^*\left(u_{(2i+2)}u'_{(2i+2)}\right) = (2n+2i+1)(2n+2i+2) \;;\; 0 \leq i < \frac{n-1}{2}$$
$$g^*\left(v_{(2i+1)}v'_{(2i+1)}\right) = (3n+2i)(3n+2i+1) \;;\; 0 \leq i < \frac{n+1}{2}$$
$$g^*\left(v_{(2i+2)}v'_{(2i+2)}\right) = (3n+2i+1)(3n+2i+2) \;;\; 0 \leq i < \frac{n-1}{2}$$

Case(ii): Let n be even.

$g\left(u_{\frac{n}{2}+1}\right) = f\left(u_{\frac{n}{2}+1}\right) = 0$

$g\left(u_{\frac{n}{2}+1-i}\right) = f\left(u_{\frac{n}{2}+1-i}\right) \;\; ; 1 \leq i \leq \frac{n}{2}$

$g\left(u_{\frac{n}{2}+1+i}\right) = g\left(u_{\frac{n}{2}+1+i}\right) \;\; ; 1 \leq i < \frac{n}{2}$

$g\left(v_{\frac{n}{2}}\right) = f\left(v_{\frac{n}{2}}\right) = 2$

$g\left(v_{\frac{n}{2}-i}\right) = f\left(v_{\frac{n}{2}-i}\right) \; ; 1 \leq i < \frac{n}{2}$

$g\left(v_{\frac{n}{2}+i}\right) = f\left(v_{\frac{n}{2}+i}\right) \; ; 1 \leq i \leq \frac{n}{2}$

$g\left(u'_{(2i+1)}\right) = (2n+2i)(2n+2i+1) - g\left(u_{(2i+1)}\right) \;\; ; \; 0 \leq i < \frac{n}{2}$

$g\left(u'_{(2i+2)}\right) = (2n+2i+1)(2n+2i+2) - g\left(u_{(2i+2)}\right) \;\; ; \; 0 \leq i < \frac{n}{2}$

$g\left(v'_{(2i+1)}\right) = (3n+2i)(3n+2i+1) - g\left(v_{(2i+1)}\right) \;\; ; \; 0 \leq i < \frac{n}{2}$

$g\left(v'_{(2i+2)}\right) = (3n+2i+1)(3n+2i+2) - g\left(v_{(2i+2)}\right) \;\; ; \; 0 \leq i < \frac{n}{2}$

Let g^* be the induced edge labeling of g.
Then

$g^*\left(u_{\frac{n}{2}+1-i}u_{\frac{n}{2}+1-(i+1)}\right) = f^*\left(u_{\frac{n}{2}+1-i}u_{\frac{n}{2}+1-(i+1)}\right) \;\; ; 0 \leq i < \frac{n}{2}$

$g^*\left(u_{\frac{n}{2}+1+i}u_{\frac{n}{2}+1+(i+1)}\right) = f^*\left(u_{\frac{n}{2}+1+i}u_{\frac{n}{2}+1+(i+1)}\right) \;\; ; 0 \leq i < \frac{n}{2} - 1$

$g^*\left(u_{\frac{n}{2}+1}v_{\frac{n}{2}}\right) = f^*\left(u_{\frac{n}{2}+1}v_{\frac{n}{2}}\right) = 2$

$g^*\left(v_{\frac{n}{2}+i}v_{\frac{n}{2}+(i+1)}\right) = f^*\left(v_{\frac{n}{2}+i}v_{\frac{n}{2}+(i+1)}\right) \;\; ; 0 \leq i < \frac{n}{2}$

$g^*\left(v_{\frac{n}{2}-i}v_{\frac{n}{2}-(i+1)}\right) = f^*\left(v_{\frac{n}{2}-i}v_{\frac{n}{2}-(i+1)}\right) = \; ; 0 \leq i \leq \frac{n}{2} - 1$

$g^*\left(u_{(2i+1)}u'_{(2i+1)}\right) = (2n+2i)(2n+2i+1) \; ; \; 0 \leq i < \frac{n}{2}$

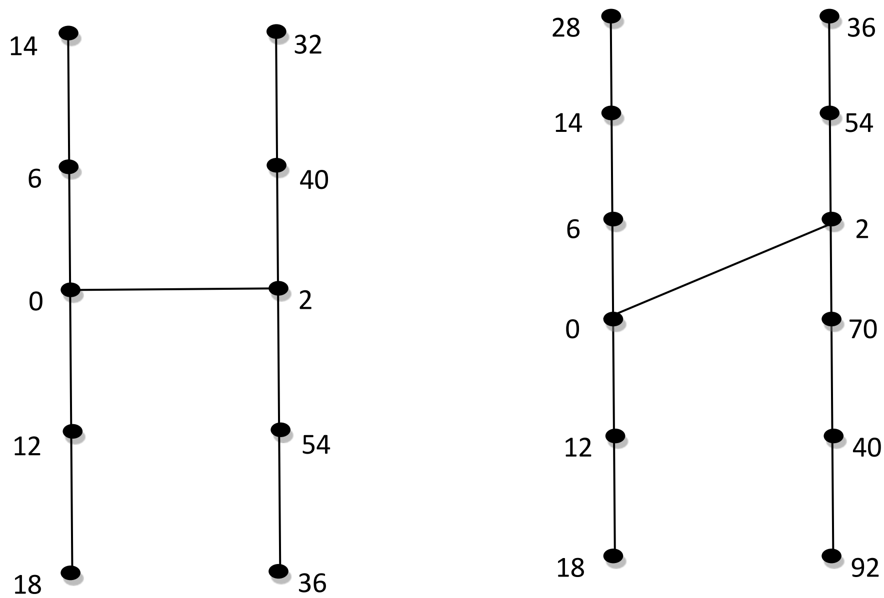
$g^*\left(u_{(2i+2)}u'_{(2i+2)}\right) = (2n+2i+1)(2n+2i+2) \; ; \; 0 \leq i < \frac{n}{2}$

$g^*\left(v_{(2i+1)}v'_{(2i+1)}\right) = (3n+2i)(3n+2i+1) \; ; \; 0 \leq i < \frac{n}{2}$

$g^*\left(v_{(2i+2)}v'_{(2i+2)}\right) = (3n+2i+1)(3n+2i+2) \; ; \; 0 \leq i < \frac{n}{2}$

In both cases, the induced edge labels are $O_1, O_2, O_3, \ldots, O_{(4n-1)}$.
Hence the theorem.

Example 2.6: Oblong sum labeling of the two H-graphs G_1 and G_2 are given in Fig. 2 and the oblong sum labeling of $G_1 \odot S_1$ and $G_2 \odot S_1$ are given in Fig. 3 and Fig. 4 respectively.



G$_1$ **G**$_2$

Fig. 2.

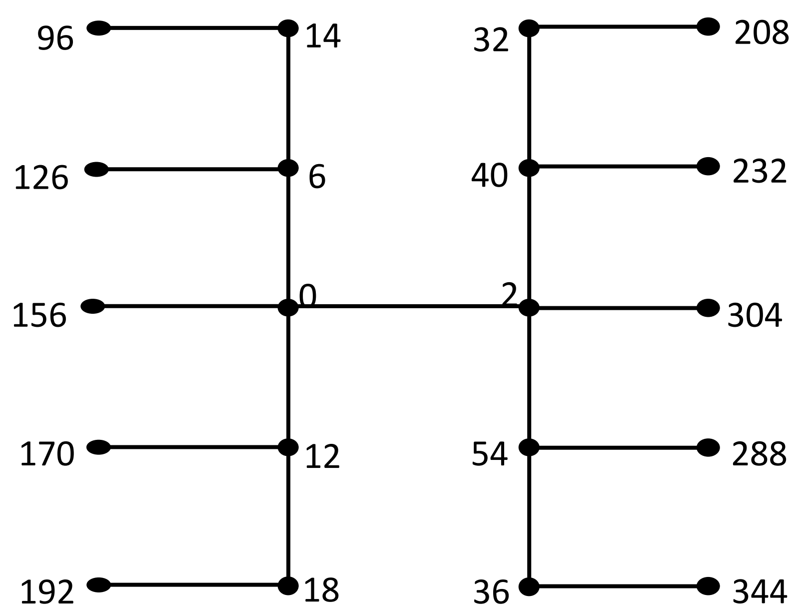


Fig. 3.

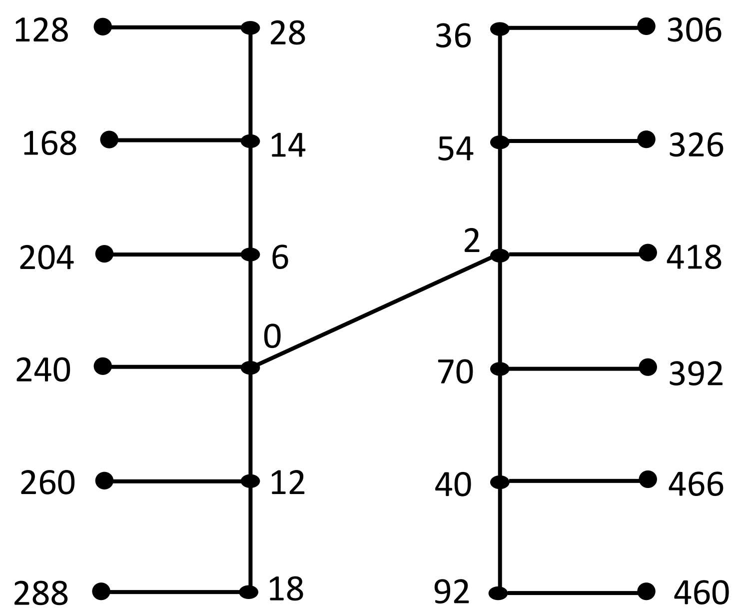


Fig. 4.

Definition 2.7: Shrub $\mathrm{St}(n_1, n_2, \dots, n_m)$ is a graph obtained by connecting a vertex v_0 to the central vertex of each of m number of stars.

Theorem 2.8: Shrub $\mathrm{St}(n_1, n_2, \dots, n_m)$ is oblong sum for all $n_1, n_2, \dots, n_m \geq 1$.

Proof: Let v, v_i, v_{ij} ;$1 \leq i \leq m, 1 \leq j \leq n_i$ be the vertices of $\mathrm{St}(n_1, n_2, \dots, n_m)$.

Then $E[\mathrm{St}(n_1, n_2, \dots, n_m)] = \{vv_i, v_i v_{ij}; 1 \leq i \leq m, 1 \leq j \leq n_i\}$

Let f: $V(\mathrm{St}(n_1, n_2, \dots, n_m)) \to \{0,2,4,6, \dots\}$ be defined as follows.

$f(v) = 0$

$f(v_i) = O_i; 1 \leq i \leq m$

$f(v_{ij}) = O_{m+n_1+n_2+\cdots+n_{i-1}+j} - O_i; 1 \leq i \leq m, 1 \leq j \leq n_i$

Let f^* be the induced edge labeling of f.

Then

$f^*(vv_i) = O_i; 1 \leq i \leq m$

$f^*(v_i v_{ij}) = O_{m+n_1+n_2+\cdots+n_{i-1}+j}$; $1 \leq i \leq m, 1 \leq j \leq n_i$

The induced edge labels are distinct and are $O_1, O_2, \dots, O_{m+n_1+n_2+\cdots+n_m}$.

Hence the theorem.

Example 2.9: Oblong sum labeling of St(2,3,4,5) is given in Fig. 5.

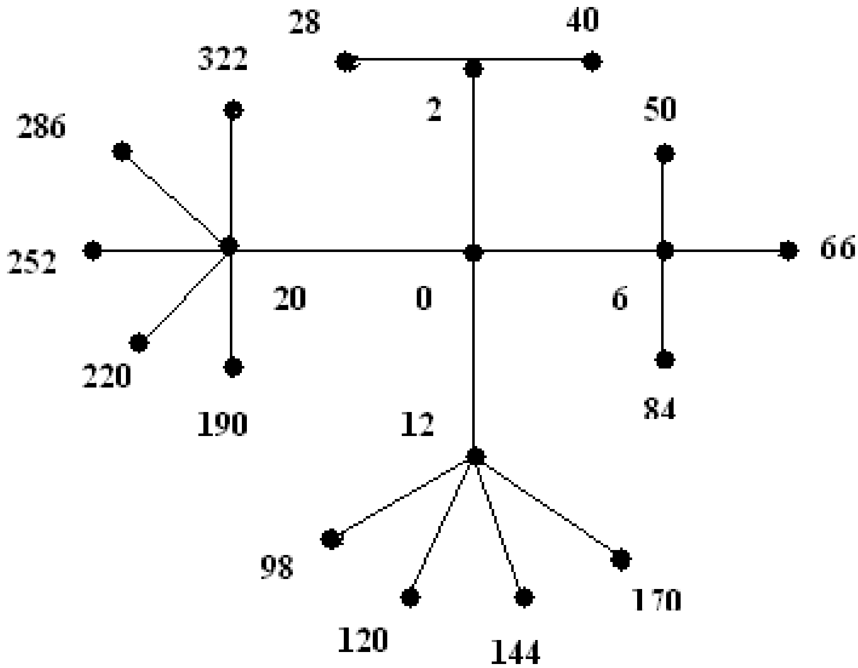


Fig. 5.

Definition 2.10: [5] Banana tree $\mathrm{Bt}(n_1, n_2, \dots, n_m)$ is a graph obtained by connecting a vertex v_0 to one leaf of each of m number of stars.

Theorem 2.11: Banana tree $\mathrm{Bt}(n, n, \dots, n)$(m times) is oblong sum for all $n \geq 1$.

Proof: Let $V\big(\mathrm{Bt}(n, n, \dots, n)\big) = \{v, v_i, u_i, u_{ij}; 1 \leq i \leq m, 1 \leq j \leq n\}$ and $E\big(\mathrm{Bt}(n, n, \dots, n)\big) = \{vv_i, u_i v_i, u_i u_{ij}; 1 \leq i \leq m, 2 \leq j \leq n\}$

Let f: $V\big(\mathrm{Bt}(n, n, \dots, n)\big) \rightarrow \{0,2,4,6, \dots\}$ be defined as follows.

$f(v) = 0$

$f(v_i) = O_i; 1 \leq i \leq m$

$f(u_i) = O_{m+i} - O_i; 1 \leq i \leq m$

$f(u_{ij}) = O_{2m-3+2i+j} - f(u_i); 1 \leq i \leq m, 2 \leq j \leq n$

Let f^* be the induced edge labeling of f.

Then

$f^*(vv_i) = O_i; 1 \leq i \leq m$

$f^*(u_i v_i) = O_{m+i}; 1 \leq i \leq m$

$f^*(u_i u_{ij}) = O_{2m-3+2i+j}; 1 \leq i \leq m, 2 \leq j \leq n$

The induced edge labels are distinct and are $O_1, O_2, \dots, O_{m+mn}$. Hence the theorem.

Example 2.12: Oblong sum labeling of $V(\mathrm{Bt}(3,3,3,3,3,3))$ is given in Fig. 6.

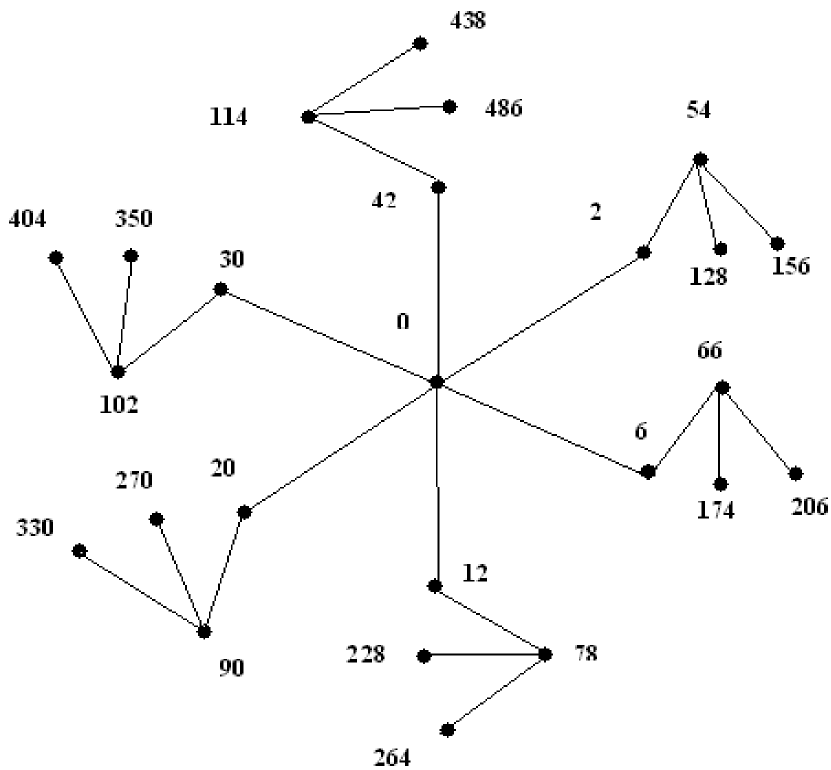


Fig. 6.

Definition 2.13: Let G be a graph with fixed vertex v and let $(P_m: G)$ be the graph obtained from m copies of and the path P_m:$u_1, u_2, \dots, u_m$by joining u_i with the vertex v of the i^{th} copy of G by means of an edge for 1≤ $i \leq n$.

Theorem 2.14:$(P_n: K_{1,m})$ is oblong sum for all $n > 1$ and $m \geq 1$

Proof: Let $V\left(\left(P_n: K_{1,m}\right)\right) = \{v_i, u_i, u_{ij}; 1 \leq i \leq n, 1 \leq j \leq m\}$ and
$E\left(\left(P_n: K_{1,m}\right)\right) = \{v_i v_{i+1}, v_j u_j, u_j u_{jk}; 1 \leq i \leq n-1, 1 \leq j \leq n, 1 \leq k \leq \mathrm{m}\}$

Let f: $V\left(\left(P_n: K_{1,m}\right)\right) \rightarrow \{0,2,4,6, \dots\}$ be defined as follows.

$f(v_1) = 0$
$f(v_i) = O_{i-1} - f(v_{i-1})\,; 2 \leq i \leq n$
$f(u_i) = O_{n-1+i} - f(v_i); 1 \leq i \leq \mathrm{n}$
$f(u_{ij}) = O_{2n-4+3i+j} - f(u_i); 1 \leq \mathrm{i} \leq \mathrm{n}, 1 \leq \mathrm{j} \leq \mathrm{m}$

Let f^* be the induced edge labeling of f. Then

$f^*(v_i v_{i+1}) = O_i; 1 \leq i \leq n-1$

$f^*(v_j u_j) = O_{n-1+j}; 1 \leq j \leq n$
$f^*(u_i u_{ij}) = O_{2n-4+3i+j}; 1 \leq i \leq n, 1 \leq j \leq m$

The induced edge labels are distinct and are $O_1, O_2, \ldots, O_{mn+2n-1}$. Hence the theorem.

Example 2.15: Oblong sum labeling of $(P_5 : K_{1,3})$ is given in Fig. 7.

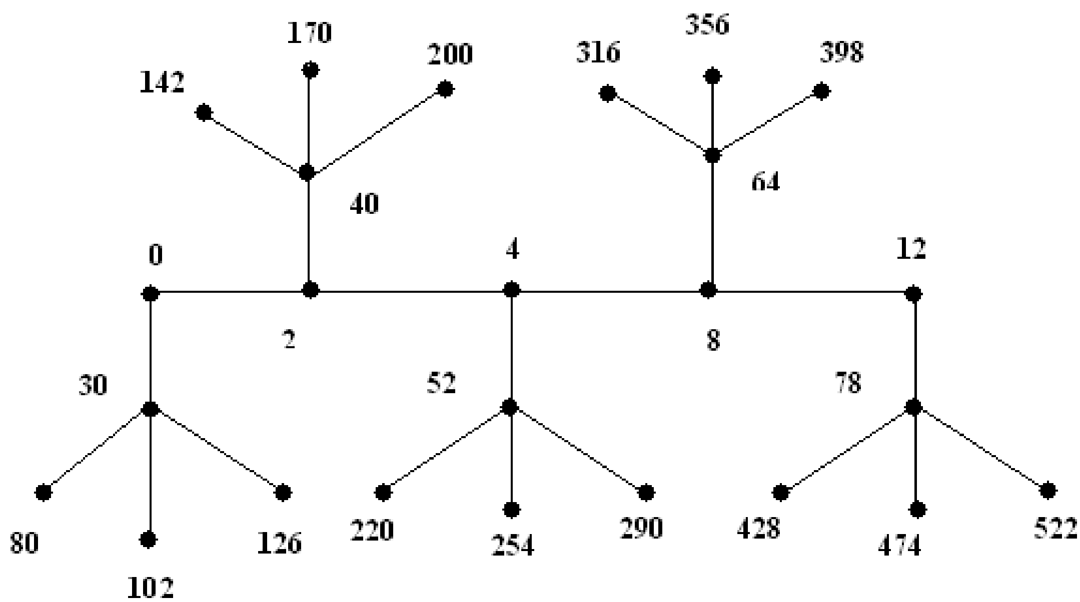


Fig. 7.

3. CONCLUSIONS

In this paper, the authors studied oblong sum labeling of sum special graphs. Similar studies can be made.

Acknowledgement

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References

[1] J. C. Bermond, Graceful graphs, Radio antennae and French wind mills, Graph Theory and Combinatorics, Pitman, London, 1979, pp. 13-37.

[2] G. S. Bloom and S. W. Golomb, Applications of numbered undirected graphs, *Proceedings of the IEEE*, Vol. 65, No. 4, (1977) 562-570

[3] Frank Harary, Graph Theory, Narosa Publishing House, New Delhi (2001)

[4] Joseph A. Gallian, A Dynamic Survey of Graph Labeling, *The Electronic Journal of Combinatorics*, 15 (2008), #DS6

[5] M. Selvi, D. Ramya and P. Jeyanthi, Skolem difference mean graphs, *Proyecciones Journal of Mathematics,* Vol. 34, No.3, (Sep. 2015), 243-254