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Optimization of Channel Profit for Deteriorating Items when Trade Credit Linked to Order Quantity

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ABSTRACT

In this article, an attempt is made to optimize the joint total profit per unit time of vendor – buyer supply chain system where vendor offers different trade credit depends on order quantity to the buyer. Here items in the inventory are deteriorates with constant rate and demand is a linear function of time. Vendor is establishing threshold for allowing trade credit comprehensively to ensure the greatest benefit for both players. By analyzing total channel profit function, we developed the optimal solution to provide buyer's order quantity and replenishment time. Numerical examples and sensitivity analysis are given to illustrate the theoretical results, and some managerial insights are also obtained.

***Keywords*:** Integrated Inventory Model, Time Dependent Demand, Deterioration, Order Quantity Dependent Trade Credit

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1. INTRODUCTION

In traditional EOQ model, as soon as items are received, buyer must pay for procured items. But this is not always true. To attract more customers, vendor offers a promotional tool 'trade credit' to the buyer ie delay in payment without interest. Before the end of the trade credit period, the buyer can sell the goods and accumulate revenue and earn interest. If the payment is not settled by the end of the trade credit period by the buyer then vendor charge higher interest rate. Conversely, to the buyer, the vendor's trade credit reduces his/her purchase cost. Trade credit was first discussed by Haley and Higgins (1973) who examined the effect of a two-part trade credit policy with a cash discount on the optimal inventory policy and payment time. Goyal (1985) established an EOQ model with a constant demand rate under the condition of permissible delay in payments. Aggarwal and Jaggi (1995) extended Goyal's (1985) model to include deteriorating items. Jamal et al. (1997) further generalised the model to allow for shortages and deterioration. Hwang and Shinn (1997) developed the optimal pricing and lot sizing for the retailer under the condition of permissible delay in payments. Chang and Dye (2001) extended the model by Jamal et al.(1997) to allow for not only a varying deterioration rate of time but also the backlogging rate to be inversely proportional to the waiting time. One can refer review article on trade credit by Kawale and Sanas (2017).

In competitive business world, vendors may offer favourable credit terms to encourage buyers to order large quantities. Khouja and Mehrez (1996) were the first to discuss the vendor credit policies on the EOQ model where credit terms are linked to the order quantity. Later, Shinn and Hwang (2003) developed optimal pricing and ordering policies for retailers under order-size-dependent delay in payments. Chung et al. (2005) developed the optimal inventory policies under permissible delay in payments depending on the ordering quantity. Huang (2007) established economic order quantity under conditionally permissible delay in payments. In this paper supplier only offers the retailer fully permissible delay in payments if the retailer ordered a sufficient quantity otherwise he would offer the retailer partially permissible delay in payments. Further Hsu et al. (2008) dealt with optimal inventory planning under permissible delay in payments when retailer order a larger quantity.

Chang et al. (2009) developed the optimal pricing and ordering policy for an integrated inventory model when trade credit linked to order quantity. Ouyang et al. (2009) developed an optimization approach for joint pricing and ordering problem in an integrated inventory system with order-size dependent trade credit. Further Ouyang et al. (2010) derived algebraically the EOQ model with defective items and partially permissible delay in payments linked to order quantity. Teng et al. (2012) developed vendor–buyer inventory models with trade credit financing linked to order quantity under both non-cooperative and integrated environments. Chung (2013) analytically derived the EOQ model with defective items and partially permissible delay in payments linked to order quantity in the supply chain management. Chiu et al. (2013) developed integrated imperfect production inventory model under permissible delay in payments depending on the order quantity. Chung et al. (2013) proposed an integrated inventory model with order-size-dependent trade credit and quality improvement. Chung et al. (2013) extended Huang (2007) model to develop the inventory models under conditional trade credit related to order quantity in a supply chain system. Singh et al. (2014) studied order size dependent trade credit in a three echelon supply chain model. Chung and Liao (2009) developed the optimal ordering policy of the EOQ model under trade credit depending on the ordering quantity from the DCF approach. Kreng and Tan (2010) developed the optimal replenishment decisions under two levels of trade credit policy depending on the order quantity. Guchhait et al. (2013) developed two storage inventory model of a deteriorating item with variable demand under partial credit period linked to order quantity.

Many studies about trade credit focused on determining optimal policy for the buyer or the vendor only. However, these one-sided optimal inventory models neglected the complicated interaction and cooperation opportunity between the buyer and the vendor. In practice, the vendor and buyer may consider how to relieve the conflict relationship and attempt to become partners to create a win–win strategy. Goyal (1976) first developed a single vendor single-buyer integrated inventory model. Subsequently, Banerjee (1986) extended Goyal's (1976) model and assumed that the vendor followed a lot-for-lot shipment policy with respect to a buyer. Goyal (1988) relaxed the lot-for-lot policy and illustrated that the inventory cost can be reduced significantly if the vendor's economic production quantity is an integer multiple of the buyer's purchase quantity. Later Lu (1995), Bhatnagar et al. (1993), Goyal (1995), Viswanathan (1998), Hill (1997, 1999), Kim and Ha (2003), Kelle et al. (2003), Li and Liu (2006) developed an integrated inventory model. These studies on integrated inventory problems did not take the effect of trade credit on the optimal policy between the supplier and buyer into account. Abad and Jaggi (2003) first offered a supplier–buyer integrated model following a lot-for-lot shipment policy under a permissible delay in payment.

In this paper, we develop an integrated vendor–buyer inventory model taking into account the following factors: (i) the demand rate is linearly time dependent; (ii) items in the inventory are deteriorated with constant rate; (iii) the credit terms are linked to the order quantity. To maximize the joint total profit per unit time, two basic issues will be determined in this study. These issues are how large should the replenishment order be, and when should be the order replenished. An interactive procedure is developed to help determine the optimal solution. Finally, numerical examples and sensitivity analysis are presented to illustrate the proposed model.

2. NOTATIONS

The following notations are used in the proposed article:

S_v: Vendor's set up cost per set up.

S_b: Buyer's ordering cost per order.

C_v: Production cost per unit.

C_b: Buyer's purchase cost per unit.

C_c: The unit retail price to customers where $C_c > C_b > C_v$.

I_v: Vendor's inventory holding cost rate per unit per annum, excluding interest charges.

I_b: Buyer's inventory holding cost rate per unit per annum, excluding interest charges.

I_{v0}: Vendor's opportunity cost/$/unit time.

I_{b0}: Buyer's opportunity cost/\$/unit time.

I_{be}: Buyer's interest earned/\$/unit time.

ϱ: Capacity utilisation which is ratio of demand to the production rate, $\varrho < 1$ and known constant.

M: Allowable credit period for the buyer offered by the vendor.

Q: Buyer's order quantity.

Q_d : Pre-specified order quantity to qualify for offer of trade credit.

T: cycle time (decision variable).

T_d : The time length when Q_d units are depleted to zero.

n : Number of shipments from vendor to the buyer.

θ: constant rate of deterioration.

TVP: Vendor's total profit per unit time.

TBP: Buyer's total profit per unit time.

π : TVP + TBP Joint total profit per unit time.

3. ASSUMPTIONS

In addition, the following assumptions are made in derivation of the model:

- The supply chain under consideration comprise of single vendor and single buyer for a single product.
- Shortages are not allowed.
- The demand rate considered is time dependent, increasing demand rate. The constant part of linear demand pattern changes with each cycle.
- Replenishment rate is instantaneous for buyer
- The units in inventory are subject to deteriorate at a constant rate of θ, $0 < \theta < 1$. The deteriorated units can neither be repaired nor replaced during the cycle time.
- Finite production rate.
- Vendor produces the nQ items and then fulfils the buyer's demand, so at the beginning of production item, there is small possibility of deterioration in general. Moreover vendor is a big merchant who can have capacity to prevent deterioration. So in this model, deterioration cost is considered for buyer only at the rate θ is assumed to be constant.
- The vendor sets a threshold Q_d for offering trade credit. If the buyer's order exceeds or equal to Q_d, the buyer will obtain a credit period M. Otherwise, the buyer must pay for the items immediately upon receiving them.

- Vendor offers the buyer a conditional permissible delay period M. During this permissible delay period, the buyer sells the items and uses the sales revenue to earn interest at a rate of I_{be} /unit/annum. At the end of this time period buyer settles the payments due against the purchase made and incurs opportunity cost at a rate of I_{b0} /unit/annum for unsold items in stock.

4. MATHEMATICAL FORMULATION

The inventory level at any instant of time *t* is governed by the differential equation

$$\frac{dI(t)}{dt} + \theta\, I(t) = -(a+bt) \qquad 0 \leq t \leq T$$

With boundary condition I(0) = Q and I(T)= 0 , we get:

$$Q = \left(\frac{a}{\theta} - \frac{b}{\theta^2}\right)(e^{\theta T}-1) + \frac{bT}{\theta}e^{\theta T}, \qquad 0 \leq t \leq T.$$

$$I(t) = \left(\frac{a}{\theta} - \frac{b}{\theta^2}\right)(e^{\theta(T-t)}-1) + \frac{b}{\theta}(Te^{\theta(T-t)} - t), \qquad 0 \leq t \leq T.$$

4. 1. Net profit function for vendor consists of following elements

1. Sales revenue: the total sales revenue per unit time is $(C_b - C_V)\frac{Q}{T}$.

$$= \frac{(C_b - C_v)}{T}\left\{\left(\frac{a}{\theta} - \frac{b}{\theta^2}\right)(e^{\theta T} - 1) + \frac{bTe^{\theta T}}{\theta}\right\}$$

2. Set-up cost : nQ units are manufactured in one production run by the vendor. Therefore the setup cost per unit time is $\frac{S_v}{nT}$

3. Holding cost : Using method given by Joglekar (1988), vendor's average inventory per unit time is

$$\frac{C_v(I_v + I_{v0})}{T}[(n-1)(1-\varrho) + \varrho]\left\{\left(\frac{-a}{\theta^2} + \frac{b}{\theta^3}\right)(1 + \theta T - e^{\theta T}) - \frac{b}{\theta^2}\left(T - Te^{\theta T} + \frac{\theta T^2}{2}\right)\right\}$$

4. Opportunity cost : If Q_d or more units are ordered by the buyer, the credit period of M is offered to settle the account. In this scenario, vendor endures a capital and payment received. Equivalently, when $T \geq T_d$, the delay payment is permitted and corresponding opportunity cost per unit time because of offering permissible delay period is

$$\frac{C_b I_{v0} M Q}{T} = \frac{C_b I_{v0} M}{T}\left\{\left(\frac{a}{\theta} - \frac{b}{\theta^2}\right)(e^{\theta T} - 1) + \frac{bTe^{\theta T}}{\theta}\right\}$$

On the other hand, when $T < T_d$ the vendor receives payments on deliver and so no opportunity cost will occur.

Hence, the total profit per unit time for the vendor is

$$TVP = \begin{cases} TVP_1, & T < T_d \\ TVP_2, & T \geq T_d \end{cases} \quad \textbf{(1)}$$

where:

TVP_1 = Sales revenue – Set up cost – Holding cost.

$$TVP_1 = \frac{(C_b - C_v)}{T}\left\{\left(\frac{a}{\theta} - \frac{b}{\theta^2}\right)(e^{\theta T} - 1) + \frac{bTe^{\theta T}}{\theta}\right\} - \frac{S_v}{nT} - \frac{C_v(I_v + I_{v0})}{T}[(n-1)(1-\varrho) + \varrho]\{\left(\frac{-a}{\theta^2} + \frac{b}{\theta^3}\right)(1 + \theta T - e^{\theta T}) - \frac{b}{\theta^2}(T - Te^{\theta T} + \frac{\theta T^2}{2})\} \quad \textbf{(2)}$$

TVP_2 = Sales revenue – Set up cost – Holding cost – Opportunity cost.

$$TVP_2 = \frac{(C_b - C_v)}{T}\left\{\left(\frac{a}{\theta} - \frac{b}{\theta^2}\right)(e^{\theta T} - 1) + \frac{bTe^{\theta T}}{\theta}\right\} - \frac{S_v}{nT} - \frac{C_v(I_v + I_{v0})}{T}[(n-1)(1-\varrho) + \varrho]\{\left(\frac{-a}{\theta^2} + \frac{b}{\theta^3}\right)(1 + \theta T - e^{\theta T}) - \frac{b}{\theta^2}(T - Te^{\theta T} + \frac{\theta T^2}{2})\} - \frac{C_b I_{v0} M}{T}\left\{\left(\frac{a}{\theta} - \frac{b}{\theta^2}\right)(e^{\theta T} - 1) + \frac{bTe^{\theta T}}{\theta}\right\} \quad \textbf{(3)}$$

4. 2. Net profit function for the buyer consists of following elements

1. Sales revenue: The total sales revenue per unit time is

$$\frac{(C_c - C_b)Q}{T} = \frac{(C_c - C_b)}{T}\left\{\left(\frac{a}{\theta} - \frac{b}{\theta^2}\right)(e^{\theta T} - 1) + \frac{bTe^{\theta T}}{\theta}\right\}$$

2. Ordering cost : Ordering cost per unit time is $\frac{S_b}{T}$

3. Holding cost: The retailer's holding cost (excluding interest charges) per unit time is

$$\frac{C_b I_b}{T}\{\left(\frac{-a}{\theta^2} + \frac{b}{\theta^3}\right)(1 + \theta T - e^{\theta T}) - \frac{b}{\theta^2}(T - Te^{\theta T} + \frac{\theta T^2}{2})\}$$

4. Deteriorating cost : Deteriorating cost per unit time is

$$\frac{C_b}{T}[Q - \int_0^T (a + bt)dt] = \frac{C_b}{T}\left\{\left(\frac{a}{\theta} - \frac{b}{\theta^2}\right)(e^{\theta T} - 1) + \frac{bTe^{\theta T}}{\theta} - aT - \frac{bT^2}{2}\right\}$$

5. Opportunity cost : Based on the values of T, M and T_d, the following four cases to be considered (i) $0 < T < T_d$, (ii) $T_d \leq T \leq M$, (iii) $T_d \leq M \leq T$ and (iv) $M \leq T_d \leq T$

Case 1. 0 < T < Td

When buyer's order quantity Q less than Qd (i.e., T < Td), buyer must pay the purchasing cost as soon as the items are received. The opportunity cost per unit time for these items is

$$\frac{C_b\,\mathrm{I_{bo}}\,\mathrm{Q}}{T} = \frac{C_b\,\mathrm{I_{bo}}}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}\right)(e^{\theta T}-1)+\frac{bTe^{\theta T}}{\theta}\right\}$$

Case 2. $\mathbf{T_d \le T \le M}$

When buyer's order quantity Q greater than or equal to Qd (i.e. $T \geq T_d$,), then a credit period M will be offered. If the permissible payment time expires on or after the inventory is completely depleted (i.e. $M \geq T$), the buyer pays no opportunity cost for the purchased items.

Case 3. $\mathbf{T_d \le M \le T}$

When buyer's order quantity greater than or equal to Qd (i.e., $T \geq T_d$) and permissible payment time expires on or before the inventory is depleted completely (i.e. $M \leq T$), the buyer still has some inventory on hand when paying the total purchasing amount to the vendor. Hence, for the items still in stock, buyer endures a capital opportunity cost at a rate of $\mathrm{I_{bo}}$; the opportunity cost per unit time for these items is

$$\frac{C_b I_{b0}}{T}\int_M^T I(t)dt = \frac{C_b I_{b0}}{T}\{\left(\frac{-a}{\theta^2}+\frac{b}{\theta^3}\right)\left(1+\theta(T-M)-e^{\theta(T-M)}\right)-\frac{b}{\theta^2}(T-Te^{\theta(T-M)}) +\theta(\frac{T^2}{2}-\frac{M^2}{2})\}$$

Case 4. $\mathbf{M \le T_d \le T}$

Case 4 is similar to Case 3 because T is also greater than or equal to M.

1. Interest earned: Same as the opportunity cost, there are four cases to be considered in terms of the interest.

Case 1. 0 < T < Td

In this case, buyer pays the purchasing cost when the items are received, and hence, no interest is earned.

Case 2. $\mathbf{T_d \le T \le M}$

When order quantity greater than or equal to Qd, the vendor offers a credit period M without interest charged to the buyer. During the credit period, buyer sells the products and uses the sales revenue to earn interest at a rate of $\mathrm{I_{be}}$. Thus, the interest earned per unit time is

$$\frac{(I_{be}\,C_c)}{T}\{\int_0^T (a+bt)tdt + Q(M-T)\} = \frac{I_{be}C_c}{T}\left[\frac{aT^2}{2}+\frac{bT^3}{3}\right]+ \frac{I_{be}C}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}\right)(e^{\theta T}-1)+\frac{bTe^{\theta T}}{\theta}\right\}(\mathrm{M-T})$$

Case 3. $T_d \le M \le T$

In this case, the buyer can sell the items and earn interest with rate I_{be} until the end of the credit period M. Thus, the interest earned per unit time is

$$\frac{I_{be}C_c}{T}\int_0^M (a+bt)t\,dt = \frac{I_{be}C_c}{T}\left[\frac{aM^2}{2}+\frac{bM^3}{3}\right]$$

Case 4. $M \le T_d \le T$

In this case, M is less than or equal to T. Thus, Case 4 is similar to Case 3

Hence, the buyer's total profit per unit time is

TBP = TBP_1 , $0 < T < T_d$
TBP_2 , $T_d \le T \le M$
TBP_3 , $T_d \le M \le T$
TBP_4 , $M \le T_d \le T$

TBP_1 = Sales revenue – Ordering cost – Inventory carrying cost – Deteriorating cost – Opportunity cost.

$$TBP_1 = \frac{(C_c-C_b)}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}\right)(e^{\theta T}-1)+\frac{bTe^{\theta T}}{\theta}\right\}-\frac{S_b}{T}-\frac{C_bI_b}{T}\left\{\left(\frac{-a}{\theta^2}+\frac{b}{\theta^3}\right)(1+\theta T-e^{\theta T})-\frac{b}{\theta^2}\left(T-Te^{\theta T}+\frac{\theta T^2}{2}\right)\right\}-\frac{C_b}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}\right)(e^{\theta T}-1)+\frac{bTe^{\theta T}}{\theta}-aT-\frac{bT^2}{2}\right\}-\frac{C_b I_{bo}}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}\right)(e^{\theta T}-1)+\frac{bTe^{\theta T}}{\theta}\right\} \quad \textbf{(4)}$$

TBP_2 = Sales revenue – Ordering cost – Inventory carrying cost – Deteriorating cost + Interest earned.

$$TBP_2 = \frac{(C_c-C_b)}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}\right)(e^{\theta T}-1)+\frac{bTe^{\theta T}}{\theta}\right\}-\frac{S_b}{T}-\frac{C_bI_b}{T}\left\{\left(\frac{-a}{\theta^2}+\frac{b}{\theta^3}\right)(1+\theta T-e^{\theta T})-\frac{b}{\theta^2}\left(T-Te^{\theta T}+\frac{\theta T^2}{2}\right)\right\}-\frac{C_b}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}\right)(e^{\theta T}-1)+\frac{bTe^{\theta T}}{\theta}-aT-\frac{bT^2}{2}\right\}+\frac{I_{be}C_c}{T}\left[\frac{aT^2}{2}+\frac{bT^3}{3}\right]+\frac{I_{be}C_c}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}\right)(e^{\theta T}-1)+\frac{bTe^{\theta T}}{\theta}\right\}(M-T) \quad (5)$$

TBP_3 = TBP_4 = Sales revenue – Ordering cost – Inventory carrying cost – Deteriorating cost + Interest earned – Opportunity cost.

$$TBP_3 = TBP_4 = \frac{(C_c-C_b)}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}\right)(e^{\theta T}-1)+\frac{bTe^{\theta T}}{\theta}\right\}-\frac{S_b}{T}-\frac{C_bI_b}{T}\left\{\left(\frac{-a}{\theta^2}+\frac{b}{\theta^3}\right)(1+\theta T-e^{\theta T})-\frac{b}{\theta^2}\left(T-Te^{\theta T}+\frac{\theta T^2}{2}\right)\right\}-\frac{C_b}{T}\left\{\left(\frac{a}{\theta}-\frac{b}{\theta^2}\right)(e^{\theta T}-1)+\frac{bTe^{\theta T}}{\theta}-aT-\frac{bT^2}{2}\right\}-\frac{C_bI_{b0}}{T}\{\left(\frac{-a}{\theta^2}+\frac{b}{\theta^3}\right)\left(1+\theta(T-M)-e^{\theta(T-M)}\right)-\frac{b}{\theta^2}(T-Te^{\theta(T-M)}+\theta(\frac{T^2}{2}-\frac{M^2}{2})\}+\frac{I_{be}C_c}{T}\left[\frac{aM^2}{2}+\frac{bM^3}{3}\right] \quad \textbf{(6)}$$

4. 3. Joint total profit per unit time

In integrated system, the vendor and the buyer take joint decision which maximizes the profit of the supply chain, the joint total profit per unit time for integrated system is

$$\pi = \begin{cases} \pi 1 = TVP_1 + TBP_1, & 0 < T < T_d \\ \pi 2 = TVP_2 + TBP_2, & T_d \le T \le M \\ \pi 3 = TVP_2 + TBP_3, & T_d \le M \le T \\ \pi 4 = TVP_2 + TBP_4, & M \le T_d \le T \end{cases}$$

Considering $e^{\theta T} = 1+\theta T + \frac{\theta^2 T^2}{2}$

$$TVP_1 = (C_b - C_v)\left(a+\frac{a\theta T}{2} + \frac{bT}{2} + \frac{b\theta T^2}{2}\right) - \frac{S_v}{nT} - C_v(I_v + I_{v0})[(n-1)(1-\varrho) + \varrho]\left(\frac{aT}{2} + \frac{bT^2}{2} + \frac{bT}{\theta}\right)$$

$$TVP_2 = (C_b - C_v - C_b I_{v0} M)\left(a+\frac{a\theta T}{2} + \frac{bT}{2} + \frac{b\theta T^2}{2}\right) - \frac{S_v}{nT} - C_v(I_v + I_{v0})[(n-1)(1-\varrho) + \varrho]\left(\frac{aT}{2} + \frac{bT^2}{2} + \frac{bT}{\theta}\right)$$

$$TBP_1 = (C_c - C_b - C_b I_{b0})\left(a+\frac{a\theta T}{2} + \frac{bT}{2} + \frac{b\theta T^2}{2}\right) - \frac{S_b}{T} - c_b I_b\left(\frac{aT}{2} + \frac{bT^2}{2} + \frac{bT}{\theta}\right) - C_b\left(\frac{a\theta T}{2} + \frac{b\theta T}{2}\right)$$

$$TBP_2 = (C_c - C_b)\left(a+\frac{a\theta T}{2} + \frac{bT}{2} + \frac{b\theta T^2}{2}\right) - \frac{S_b}{T} - c_b I_b\left(\frac{aT}{2} + \frac{bT^2}{2} + \frac{bT}{\theta}\right) - C_b\left(\frac{a\theta T}{2} + \frac{b\theta T}{2}\right) + \frac{I_{be}C_c}{T}\left[\frac{aT^2}{2} + \frac{bT^3}{3}\right] + I_{be}C_c(M-T)\left(a+\frac{a\theta T}{2} + \frac{bT}{2} + \frac{b\theta T^2}{2}\right)$$

$$TBP_3 = TBP_4 = (C_c - C_b)\left(a+\frac{a\theta T}{2} + \frac{bT}{2} + \frac{b\theta T^2}{2}\right) - \frac{S_b}{T} - c_b I_b\left(\frac{aT}{2} + \frac{bT^2}{2} + \frac{bT}{\theta}\right) - C_b\left(\frac{a\theta T}{2} + \frac{b\theta T}{2}\right) + \frac{I_{be}C_c}{T}\left[\frac{aM^2}{2} + \frac{bM^3}{3}\right] - C_b I_{b0}\left\{a\frac{(T-M)^2}{2T} - b\frac{(T-M)^2}{2\theta T} - b\left(-\frac{(T-M)}{\theta} - \frac{(T-M)^2}{2} + \frac{T}{2\theta} - \frac{M^2}{2\theta T}\right)\right\}$$

$$\pi 1 = (C_b - C_v)\left(a+\frac{a\theta T}{2} + \frac{bT}{2} + \frac{b\theta T^2}{2}\right) - \frac{S_v}{nT} - C_v(I_v + I_{v0})[(n-1)(1-\varrho) + \varrho]\left(\frac{aT}{2} + \frac{bT^2}{2} + \frac{bT}{\theta}\right) + (C_c - C_b - C_b I_{b0})\left(a+\frac{a\theta T}{2} + \frac{bT}{2} + \frac{b\theta T^2}{2}\right) - \frac{S_b}{T} - c_b I_b\left(\frac{aT}{2} + \frac{bT^2}{2} + \frac{bT}{\theta}\right) - C_b\left(\frac{a\theta T}{2} + \frac{b\theta T}{2}\right) \tag{7}$$

$$\pi 2 = (C_b - C_v - C_b I_{v0} M)\left(a+\frac{a\theta T}{2} + \frac{bT}{2} + \frac{b\theta T^2}{2}\right) - \frac{S_v}{nT} - C_v(I_v + I_{v0})[(n-1)(1-\varrho) + \varrho]\left(\frac{aT}{2} + \frac{bT^2}{2} + \frac{bT}{\theta}\right) + (C_c - C_b)\left(a+\frac{a\theta T}{2} + \frac{bT}{2} + \frac{b\theta T^2}{2}\right) - \frac{S_b}{T} - c_b I_b\left(\frac{aT}{2} + \frac{bT^2}{2} + \frac{bT}{\theta}\right) - C_b\left(\frac{a\theta T}{2} + \frac{b\theta T}{2}\right) + \frac{I_{be}C_c}{T}\left[\frac{aT^2}{2} + \frac{bT^3}{3}\right] + I_{be}C_c(M-T)\left(a+\frac{a\theta T}{2} + \frac{bT}{2} + \frac{b\theta T^2}{2}\right) \tag{8}$$

$$п3 = п4 = (C_b - C_v - C_b I_{v0} M)\left(a + \frac{a\theta T}{2} + \frac{bT}{2} + \frac{b\theta T^2}{2}\right) - \frac{S_v}{nT} - C_v(I_v + I_{v0})[(n-1)(1-\varrho) + \varrho]\left(\frac{aT}{2} + \frac{bT^2}{2} + \frac{bT}{\theta}\right) + (C_c - C_b)\left(a + \frac{a\theta T}{2} + \frac{bT}{2} + \frac{b\theta T^2}{2}\right) - \frac{S_b}{T} - c_b I_b\left(\frac{aT}{2} + \frac{bT^2}{2} + \frac{bT}{\theta}\right) - C_b\left(\frac{a\theta T}{2} + \frac{b\theta T}{2}\right) + \frac{I_{be}C_c}{T}\left[\frac{aM^2}{2} + \frac{bM^3}{3}\right] - C_b I_{b0}\left\{a\frac{(T-M)^2}{2T} - b\frac{(T-M)^2}{2\theta T} - b\left(-\frac{(T-M)}{\theta} - \frac{(T-M)^2}{2} + \frac{T}{2\theta} - \frac{M^2}{2\theta T}\right)\right\} \quad \textbf{(9)}$$

The optimum value of cycle time can be obtained by setting $\frac{dп}{dT} = 0$ for fixed n. The necessary condition for maximising total profit is $\frac{d^2п}{dT^2} < 0$.

4. 4. Numerical examples

To illustrate the above developed model, an inventory system with the following data is considered a = 1000, b= 50 , θ = 0.1, ϱ=0.7, C_v = \$5/unit, C_b = \$25/ unit, C_c = \$ 55 / unit, S_v = \$1500/setup, S_b = \$100/order, I_v = 1%/unit/annum, I_b = 1%/unit/annum, I_{v0} = 2%/unit/annum, I_{b0} = 5%/unit/annum, I_{be} = 8%/unit/annum and M = 30days. Ordering policy for various Q_d is shown in Table 1.

Table 1. Optimal solutions under different Q_d

Q_d	Q	n	T	Buyer's Profit	Vendor's profit	Joint profit
25,000	59,389	5	16	2,87,510	7,797.2	2,95,310
50,000	59,389	5	16	2,87,510	7,797.2	2,95,310
75,000	1,39,400	5	22	2,98,160	12,637	3,10,800
1,00,000	1,82,390	3	24	2,93,070	14,843	3,07,920

From Table 1, we find that as the value of Q_d increases, joint total profit of integrated system also increases first with increase in buyer's profit and vendor's profit. Then further increase in Q_d decreases buyer's profit and joint total profit. It reveals that the vendor should set the minimum order quantity carefully to make sure that this threshold is effective. If the threshold set by the vendor is too high, the buyer may decide not to order a quantity greater than the threshold to obtain delayed payments because it may decrease his profit.

4. 5. Sensitivity analysis

Sensitivity analysis of the integrated system with respect to parameters: demand scale parameter, demand rate parameter, deterioration rate and capacity utilisation is presented in Table 2, Table 3, Table 4, and Table 5 with Q_d = 25,000. In each analysis the base parameter values are as assumed in Example1 and only the parameter of interest is varied holding all other parameter constant.

Table 2. Sensitive analysis for the demand scale parameter

Parameter a	T(days)	Q	Vendor	Buyer	Joint Profit
1,500	15	68,430	10,846	3,72,840	3,83,680
2,000	14	74,214	13,797	4,59,310	4,73,110
3,000	13	90,583	19,850	6,38,160	6,58,020

Table 3. Sensitive analysis for the demand rate parameter

Parameter b	T(days)	Q	Vendor	Buyer	Joint Profit
60	15	54,262	7,728.7	3,03,110	3,10,830
75	14	50,218	7,696.3	3,24,510	3,32,210
100	12	39,841	7,393.0	3,43,580	3,50,970

Table 4. Sensitive analysis for the deterioration rate

Parameter ɵ	T(days)	Q	Vendor	Buyer	Joint Profit
0.15	18	1,50,970	13,475	3,54,580	3,68,060
0.2	19	3,76,210	19,251	4,18,690	4,37,940
0.3	20	24,62,600	30,835	5,43,745	5,74,580

Table 5. Sensitive analysis for the capacity utilization

Parameter ϱ	T(days)	Q	Vendor	Buyer	Joint Profit
0.6	16	59,389	6,789.3	2,87,510	2,94,300
0.8	16	59,389	8,805.3	2,87,510	2,96,320
0.9	16	59,389	9,813.3	2,87,510	2,97,320

Result show that as the value of a, b and θ increases, vendor's profit , buyer's profit and joint total profit of the entire supply chain increases. As a and θ increases, optimal order quantity Q also increases and optimal cycle time T decreases but as b increases, optimal order quantity Q decreases and optimal cycle time T increases. From Table 5, we can conclude that if there is increase in capacity utilisation parameter ϱ, then vendor's profit and joint total profit increases but buyer's profit, optimal order quantity Q and optimal cycle time T remain same. Above tables shows that, trade credit linked to order quantity is beneficial for players of the supply chain.

5. CONCLUSION

We formulate an integrated vendor–buyer inventory model in this paper with the assumptions that the market demand is time dependent and the credit terms are linked to the order quantity. Items in the inventory deteriorates with constant rate. The objective is to determine the buyer's optimal ordering quantity and the optimal length of order cycle for maximizing the joint profit of supply chain system. This paper also presented practical application example and a sensitivity analysis where the proposed inventory model is utilized to support business decision making. This paper finds that vendor should set threshold quantity more carefully to ensure greatest benefits for both players and to attract the sales more effective.

In future, model can be further generalised by the by taking weibull distributed deterioration rate and trapezoidal fuzzy deterioration. Consideration of some more realistic form of demand for imperfect products produced, one would also be a worthful contribution.

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