Dependence of energy of heat radiation of molecules from its kinetic energy

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ABSTRACT

A formula is derived for the dependence of the thermal radiation energy of a gas molecule on its kinetic energy. From this formula it follows that the radiation energy of a molecule is not equal to its kinetic energy. Since the mean free path of molecules is larger in more rarefied gases, the energy of thermal radiation in such gases is, accordingly, greater. For the same reason, the smaller the size of the gas molecules, the volume of empty space in such a gas will be greater, and, consequently, the gas will be more discharged, so that the energy of emission of gas molecules in comparison with the kinetic energy of the molecule will be even greater.

Keywords: radiation energy of a molecule, kinetic energy of a molecule, temperature of a molecule, gas cell

1. INTRODUCTION

Imagine a closed shell, isolated from the surrounding space and located at a constant temperature, and inside the shell - an ideal vacuum. Despite this, it will not be completely
"empty". The cavity bounded by the shell will be filled with electromagnetic thermal radiation emanating from the molecules surrounding the shell of the gas, whose bulk energy density is determined by the Stefan-Boltzmann law and the radiation of the molecule itself. If, however, this is attributed to only one molecule, then the gas cell [1-4] in which the selected molecule is located will play the role of such a closed shell, and other molecules of the surrounding gas medium protrude as the walls of the cell, colliding with the given gas molecule. Since the molecule in the gas cell moves, it has kinetic energy $E_i$. In [1, 2], on the basis of the assumption that these energies are equal, 

$$\varepsilon_i = E_i, \quad (1)$$

a formula has been derived for the distribution of the temperatures of gas molecules as a function of the total gas temperature.

The purpose of this paper is to verify the correctness of this assumption (1) and, accordingly, the truth of the formula obtained [1, 2] for the distribution of temperatures of molecules.

2. FORMULA FOR DEPENDENCE OF ENERGY OF RADIATION OF MOLECULES FROM ITS KINETIC ENERGY

For definiteness, we shall assume that the gas molecules are monatomic and can be represented in the form of a sphere of radius $r$. According to [4], a gas cell in which there is only one gas molecule is a cylinder with a base area $\pi r^2$ and a height equal to the mean free path of the $i$-th molecule $l_i$, which corresponds to the volume of the gas cell

$$W_i = \pi r^2 l_i, \quad (2)$$

and the temperature of the molecule in this cell is

$$\tau_i = \frac{l_i}{k_r} E_i, \quad (3)$$

where: $E_i$ — the kinetic energy of this molecule.

Due to the equilibrium between the gas temperature and the thermal radiation temperature $\theta$ of the gas, we have

$$\theta = T \quad (4)$$

Then the radiation energy $\varepsilon$ in a volume $V$, containing gas molecules is determined by the Stefan-Boltzmann law:

$$\varepsilon = \frac{\pi^2 k^\beta}{15(hc)^3} VT^\beta, \quad (5)$$
where: $k$ – the Boltzmann constant, $\hbar$ – Planck’s constant, $c$ – velocity of light, $\beta \approx 4$ (for most substances, except for metals, in which $\beta \geq 4$). Inside the gas cell of any molecule the radiation temperature will be equal $\theta$, despite the fact that the temperature of the molecule in this cell may be different. lies in the fact that the propagation velocity of electromagnetic radiation waves is much higher than the velocity of the molecules.

If the volume of the gas cell $V$ is chosen as the volume $W_i$, then formula (5) for the $i$-cell, taking into account (2), we obtain the dependence of the total energy of the thermal radiation (i.e. the total energy of radiation from the $i$-molecule and molecules of the environment) from the size of the gas molecules, its temperature and the mean free path of the molecule in a given cell:

$$
\varepsilon_i^0 = \frac{\pi^2 k \beta r^2}{15(hc)^3} l_i T^{\beta}. \tag{6}
$$

In order to estimate the contribution of the $i$-molecule itself to this thermal radiation in this cell, we will conventionally assume that there is no gas outside the cell, and the cell walls are made of a material that does not emit thermal radiation. Obviously, under such conditions, equality

$$
\theta_i = r_i. \tag{7}
$$

Then for this case formula (5) will look like

$$
\varepsilon_i = \frac{\pi^2 k \beta}{15(hc)^3} W_i \theta_i^{\beta}, \tag{8}
$$

where: $\varepsilon_i$ and $\theta_i$ the energy and temperature of the thermal radiation of the $i$-th gas molecule, respectively. From (8) we obtain the formula for the dependence of the temperature of the thermal radiation of a particular molecule on its energy.

$$
\theta_i = \frac{1}{k} \beta \frac{15(hc)^3}{\pi^2} \frac{\varepsilon_i}{W_i}. \tag{9}
$$

Substituting expressions (3) and (7) into (9), we obtain:

$$
\frac{\beta}{\sqrt{\frac{15(hc)^3}{\pi^2}}} \frac{\varepsilon_i}{W_i} = \frac{l_i}{r} E_i, \tag{10}
$$

whence, taking into account (2), we find the required dependences between the energy of the thermal radiation of a particular molecule and its kinetic energy

$$
E_i = \frac{\beta}{\sqrt{\frac{15(hc)^3 r^{\beta-2}}{\pi^3 l_i^{\beta+1}}}}. \tag{11}
$$
\[ \mathcal{E}_i = \frac{\pi^3 l_i^{\beta+1}}{15(hc)^3 r^{\beta-2}} E_i^\beta. \]  

(12)

3. CONCLUSIONS

a) The formulas obtained for the interdependence of the kinetic energy of a molecule and the energy of its thermal radiation indicate the inequality of these energies.

b) From the form of these formulas it follows that, since the mean free path of molecules is larger in more rarefied gases, the energy of thermal radiation in such gases is also greater. Accordingly, the smaller the size of the gas molecules, the volume of empty space in such a gas is larger, so the gas will be more sparse, so that the energy of emission of gas molecules will also be greater.

c) The formula presented earlier in [1, 2] for the temperature distribution of gas molecules as a function of the total gas temperature is erroneous.

References