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Apparent and Absolute Magnitudes of Stars: A Simple Formula

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ABSTRACT

An empirical formula for estimating the apparent and absolute magnitudes of stars in terms of the parameters radius, distance and temperature is proposed for the first time for the benefit of the students. This reproduces successfully not only the magnitudes of solo stars having spherical shape and uniform photosphere temperature but the corresponding Hertzsprung-Russell plot demonstrates the main sequence, giants, super-giants and white dwarf classification also.

***Keywords*:** Stars, apparent magnitude, absolute magnitude, empirical formula, Hertzsprung-Russell diagram

1. INTRODUCTION

The visible brightness of a star is expressed in terms of its apparent magnitude [1] as well as absolute magnitude [2]; the absolute magnitude is in fact the apparent magnitude while it is observed from a distance of $32.6\ ly$. The apparent magnitude of a celestial object having flux F in the visible band is expressed as [1, 3, 4]

$$m = -2.5\ log_{10}\left(\frac{F}{2.56 \cdot 10^{-6}}\right) \qquad (1)$$

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Here $2.56 \cdot 10^{-6}$ is the reference luminous flux per unit area in the same band such as that of star Vega having apparent magnitude almost zero. Here the flux F is the magnitude of starlight the Earth intercepts in a direction normal to the incidence over an area of one square meter. The condition that the Earth intercepts in the direction normal to the incidence is normally fulfilled for stars which are far away from the Earth.

This note is committed to develop a formula for estimation of apparent as well as absolute magnitudes of stars in terms of its three parameters radius, distance and temperature so that students are able to reproduce the numbers associated with many known stars making them comfortable with the magnitude scale. This will also expose the students to the topics such as Hertzsprung-Russell diagram, evolution of stars, their end of life and so on.

2. THEORY

Let us begin with the Planck's law [5]

$$I(\lambda,T)d\lambda = \frac{\varepsilon(\lambda,T)\cdot A\cdot 2\pi hc^2\cdot d\lambda}{\lambda^5[\exp(hc/\lambda kT)-1]}\,\mathrm{W} \tag{2}$$

This describes the power emitted between the wavelengths (in meters) λ and $\lambda + d\lambda$ from an hot surface having uniform temperature T Kelvin, area A square meters and emissivity ε . Here h and k are Planck's constant and Boltzmann's constant, respectively. The above power within the wavelengths $\lambda_i = 380$ nm and $\lambda_f = 760$ nm is visible to our eyes; on the other hand, the eyes are not equally sensitive [6] to all wavelengths in this region. Rather, its spectral efficiency is optimum at $\lambda_m = 555$ nm and becomes vanishingly small outside this interval. This behavior is quantified [3, 7] by spectral luminous efficiency $V(\lambda)$ for photopic vision as a function of λ

$$V(\lambda) \approx \exp(-\mathrm{a}z^2 + \mathrm{b}z^3) \tag{3}$$
$$\mathrm{z} \equiv \lambda/\lambda_\mathrm{m} - 1; \lambda_\mathrm{m} = 555\ \mathrm{nm},\ a = 88.90, b = 112.95$$

Considering a star to be an ideal blackbody ($\varepsilon = 1$) the luminous flux emitted from its unit surface area ($A = 1\ \mathrm{m}^2$) is obtained by multiplying the power $I(\lambda,T)d\lambda$ by $683V(\lambda)$ and integrating it over from λ_i to λ_f

$$Q(\lambda_i \to \lambda_f) = \int_{\lambda_f}^{\lambda_i} \frac{683V(\lambda)\cdot 2\pi hc^2\cdot d\lambda}{\lambda^5\{exp(\mathrm{hc}/\lambda\mathrm{kT})-1\}} \tag{4}$$

The factor 683 occurs for the reason that at $\lambda_m = 555$ nm the electromagnetic radiation of one watt provides a luminous flux of 683 lumens. It is worth reminding students here that throughout this paper we will be dealing with luminous flux per unit area which is also called illuminance [8] having SI unit lumen (ℓm). When the luminous flux $Q(\lambda_i \to \lambda_f)$ approaches the surface of the Earth it gets diluted by the factor α [9]

$$\alpha = \frac{R_{Star}^2}{d^2}; d \equiv \text{ mean distance between the Earth and Star.} \tag{5}$$

The value of d will be $32.6\ ly$ for estimating absolute magnitude. This gives the amount of Star Luminous Constant On the Earth ($SLCOE$) as

$$SLCOE = Q(\lambda_i \to \lambda_f) \cdot \alpha \tag{6}$$

The above acronym variable $SLCOE$ represents the amount of starlight arriving at right angle on the Earth surface covering an area of one square meter. This is the desired flux to be substituted in the expression (1) for the apparent magnitude of a star

$$m_{Star} = -2.5\ log_{10}\left(\frac{SLCOE}{2.56\cdot 10^{-6}}\right) = -2.5\ log_{10}\left(\frac{Q(\lambda_i \to \lambda_f)\cdot\alpha}{2.56\cdot 10^{-6}}\right) \tag{7}$$

This can be converted into natural logarithmic scale

$$m_{Star} = -2.5 \cdot log_{10}(e)\left\{\ln\{Q(\lambda_i \to \lambda_f)\} + \ln\left(\frac{R_{Star}^2}{d^2}\right) - \ln(2.56 \cdot 10^{-6})\right\} \tag{8}$$

The expression $\ln\{Q(\lambda_i \to \lambda_f)\}$ has been parameterized as

$$\ln\{Q(\lambda_i \to \lambda_f)\} = 22.053T^{0.0212} - \frac{23342}{T} \tag{9}$$

This has been achieved by evaluating the integral (4) using Simpson rule in the temperature range $2000 \le T \le 50000$ and fit them through the expression (9). Thus, the formula for the apparent magnitude of a star is

$$m_{Star} = -2.5 \cdot log_{10}(e) \cdot \left\{22.053\mathrm{T}^{0.0212} - \frac{23342}{T} + \ln\left(\frac{R_{Star}^2}{d^2}\right) - \ln(2.56 \cdot 10^{-6})\right\} \tag{10}$$

In this system, the brighter an object appears it has lower magnitude. The faintest objects which could be detected with the naked eyes must have apparent magnitude $\lesssim 6$ while the Hubble Space Telescope can capture the image up to apparent magnitudes of 28. Under the presumption that the star is located at a distance $d = 32.6\ ly = 3.08 \cdot 10^{17}\ m$ the absolute magnitude of a star would be

$$M_{Star} = -2.5 \cdot log_{10}(e) \cdot \left\{22.053\mathrm{T}^{0.0212} - \frac{23342}{T} + \ln\left(\frac{R_{Star}^2}{(3.08\cdot 10^{17})^2}\right) - \ln(2.56 \cdot 10^{-6})\right\} \tag{11}$$

These expressions are appropriate to estimate apparent and absolute magnitudes of stars which are solo in nature having predominantly spherical surface and uniform photosphere temperature; the multi star systems such as Polaris, Antares, Achernar, Epsilon Canis Majoris, and so on will not be considered in the present note. Apparent magnitude of a star is a number that tells us how bright the specific star appears from the Earth whereas the absolute magnitudes provide a method to compare the brightness of the stars being clustered at far away distance $10\ pc = 32.6\ ly$.

3. NUMERICAL ILLUSTRATION

It will be worth demonstrating the use of the formulae (10) and (11) by considering one typical case. Let us consider the star Sun [10] which is nearest to us having parameters

$Radius\ of\ the\ Sun = R_{\odot} = 6.957 \cdot 10^{8}\ m$

$Its\ distance\ from\ the\ Earth = 1.496 \cdot 10^{11}\ m$

$Its\ photosphere\ temperature = 5776\ K$

The estimated apparent and absolute magnitudes would be $m_{Sun} = -26.70$ and $M_{Sun} = 4.87$, respectively; they match very well with the observed magnitudes [10] -26.74 and 4.83. The formula (11) has also been applied to other relevant stars compiled from the available literature and listed in Table 1; it lists their radii (in the unit of solar radius $R_{\odot}$), distances from the Earth (in the unit of $light\ years$), temperatures of the photosphere (in $Kelvin$) and the observed absolute magnitudes $M_{Observed}$. The estimated absolute magnitudes $M_{Estimated}$ (11) of each star are also listed in 7th column of the table. For each case the absolute value of percentage slip-up is also calculated through the expression $\{(M_{Observed} - M_{Estimated})/M_{Observed}\} \cdot 100$ and listed in the 8th column. An examination of these numbers reveals that in majority of cases the percentage error is marginal showing that the model presented here is successful. There are couples of cases such as star Vega, Arcturus, Betelgeuse and Regulus A where the percentage error is substantial; this is because in these cases the errors associated with the data are not marginal and once they are accounted for the two magnitudes $M_{Observed}$ and $M_{Estimated}$ will do overlap.

Table 1. List of solo stars, their radii, distances, photosphere temperatures, observed and estimated absolute magnitudes and percentage deviations.

S. No.	Name of Star	Radius	Distance	Temp	Observed $M_{Observed}$	Estimated $M_{Estimated}$	Percentage error
1	Sun	$1.0R_{\odot}$ [5]	$1.496 \cdot 10^{11} m$ [5]	5776 [5]	4.83 [10]	4.87	0.8
2	Canopus	$71R_{\odot}$ [11]	310 ly [11]	7350 [11]	-5.53 [12]	-5.47	1.1
3	Alpha Centauri A	$1.2234R_{\odot}$ [13]	4.37 ly [14]	5790 [15]	4.38 [16]	4.42	0.9
4	Alpha Centauri B	$0.8632R_{\odot}$ [13]	4.37 ly [14]	5260 [15]	5.71 [16]	5.68	0.5
5	Arcturus	$25.4R_{\odot}$ [17]	36.7 ly [17]	4286 [17]	-0.30 [18]	-0.44	47
6	Capella Aa [19]	$11.98R_{\odot}$	42.919 ly	4970	0.296	0.28	5.4
7	Capella Ab [19]	$8.83R_{\odot}$	42.919 ly	5730	0.167	0.18	7.8

8	Capella H [19]	$0.54R_{\odot}$	45 ly	3700	9.53	8.94	6.2
9	Rigel	$78.9R_{\odot}$ [20]	860 ly [20]	12100 [21]	−7.84 [22]	−7.36	6.1
10	Procyon A	$2.048R_{\odot}$ [23]	11.46 ly [23]	6530 [23]	2.66 [24]	2.73	2.6
11	Procyon B	$0.01234R_{\odot}$ [25]	11.46 ly [25]	7740 [25]	13.0 [24]	13.12	0.9
12	Betelgeuse	$1075R_{\odot}$ [26]	642 ly [27]	3590 [28]	−6.02 [29]	−7.32	22
13	Altair	$1.83R_{\odot}$ [30]	16.73 ly [30]	7700 [30]	2.22 [31]	2.29	3.2
14	Spica	$7.40R_{\odot}$ [32]	250 ly [32]	22400 [33]	−3.55 [33]	−3.57	0.6
15	Pollux	$8.8R_{\odot}$ [34]	33.78 ly [34]	4666 [34]	1.08 [35]	1.32	22
16	Fomalhaut [36]	$1.842R_{\odot}$	25.13 ly	8590	1.72	1.87	8.7
17	TW Piscis Austrini	$0.629R_{\odot}$ [37]	24.9 ly [37]	4711 [37]	7.07 [36]	6.99	1.1
18	Deneb [38]	$203R_{\odot}$	2614 ly	8525	−8.38	−8.32	7.2
19	Castor A	$2.4R_{\odot}$ [39]	51 ly [39]	10286 [40]	0.986 [41]	0.69	30
20	Castor C [42]	$0.6191R_{\odot}$	51 ly	3820	8.95	8.41	6.0
21	BI 253	$10.7R_{\odot}$ [43]	164000 ly [44]	50100 [45]	−5.7 [45]	−5.5	8.8
22	HD 93250 [46]	$15.9R_{\odot}$	7661 ly	46000	−6.14	−6.27	2.1
23	10 Lacertae [47]	$8.27R_{\odot}$	2331 ly	36000	−4.40	−4.54	3.2
24	Alpha Ursae Minoris Aa	$37.5R_{\odot}$ [48]	433 ly [49]	6015 [50]	−3.6 [51]	−3.20	11
25	Alpha Ursae Minoris B	$1.38R_{\odot}$ [52]	433 ly [49]	6900 [52]	3.1 [51]	3.35	8.1
26	Mu Columbae Ostar [53]	$6.58R_{\odot}$	1300 ly	33000	−3.64	−3.92	7.7
27	AE Aurigae [53]	$7.47R_{\odot}$	1741 ly	33000	−3.92	−4.20	7.1
28	Regulus A	$3.092R_{\odot}$ [54]	79.2 ly	12460 [55]	−0.57 [56]	−0.41	28
29	Regulus B	$0.5R_{\odot}$	79.2 ly	4885	6.3 [57]	7.28	16

30	Zeta^1 Scorpii [58]	$103R_\odot$	$2608\ ly$	17200	-8.5	-8.78	3.3
31	Zeta^2 Scorpii [59]	$21R_\odot$	$132\ ly$	4169	0.30	0.15	50
32	Sirius A	$1.711R_\odot$ [60]	$8.6\ ly$	9940 [61]	1.42 [62]	1.53	7.7
33	Sirius B	$0.0084R_\odot$ [63]	$8.6\ ly$	25200 [60]	11.18 [64]	10.96	2.0
34	Bernad's Star	$0.196R_\odot$ [37]	$5.97\ ly$	3134 [65]	13.21 [66]	12.48	5.5
35	Proxima Centauri	$0.1542R_\odot$ [67]	$4.24\ ly$	3042 [68]	15.60 [69]	13.26	15
36	Vega [70]	$2.527R_\odot$	$25.04\ ly$	9106	0.58	0.98	69

4. HERTZSPRUNG-RUSSELL DIAGRAM

The next step is plotting estimated absolute magnitudes against the temperatures of the stars (Figure 1) showing estimated absolute magnitudes on the ordinate and temperatures on the abscissa; opposite to the majority of graphs typically displaying the temperature scale from the lowest to highest values, the scaled temperatures here decreases from left to right and the y-axis denotes the absolute magnitudes ranging from the dimmest to the brightest.

Such a plot is called Hertzsprung-Russell diagram (Figures 2 and 3); it is a scatter plot of stars' absolute magnitudes against the effective temperatures irrespective of their locations and it is an important tool for studying stellar evolution.

This shows that there are four main types of stars: main sequence, white dwarfs, giants and super-giants. The central diagonal spanning across from the upper left corner to the bottom right comprises the main sequence, accommodating approximately 90% of all stars; the fuel of such stars are generated through hydrogen burning that is the fusion of hydrogen into helium.

In our Figure 2 the stars on the main sequence are BI 253, HD 93250, 10 Lacertae, Mu Columbae Ostar, Spica, Regulus A, Castor A, Vega, Sirius A, Procyon A, Fomalhaut, Altair, Alpha Ursae Minoris B, Alpha Centauri A, Sun, Alpha Centauri B, TW Piscis Austrini, Regulus B, Castor C, Capella H, Bernard's star and Proxima Centauri.

The characteristics of all these stars are that they are generating their energy by nuclear fusion deep in their cores. In low mass stars, fusion proceeds by hydrogen being burned into helium while in high mass stars, fusion proceeds through the carbon-nitrogen-oxygen cycle. In each case the net effect is the conversion of mass into energy powering the star's luminosity.

For main sequence stars, their luminosity, temperature and radius are governed by their mass. The most massive stars are the hottest and most luminous, and the least massive stars are the coolest and least luminous.

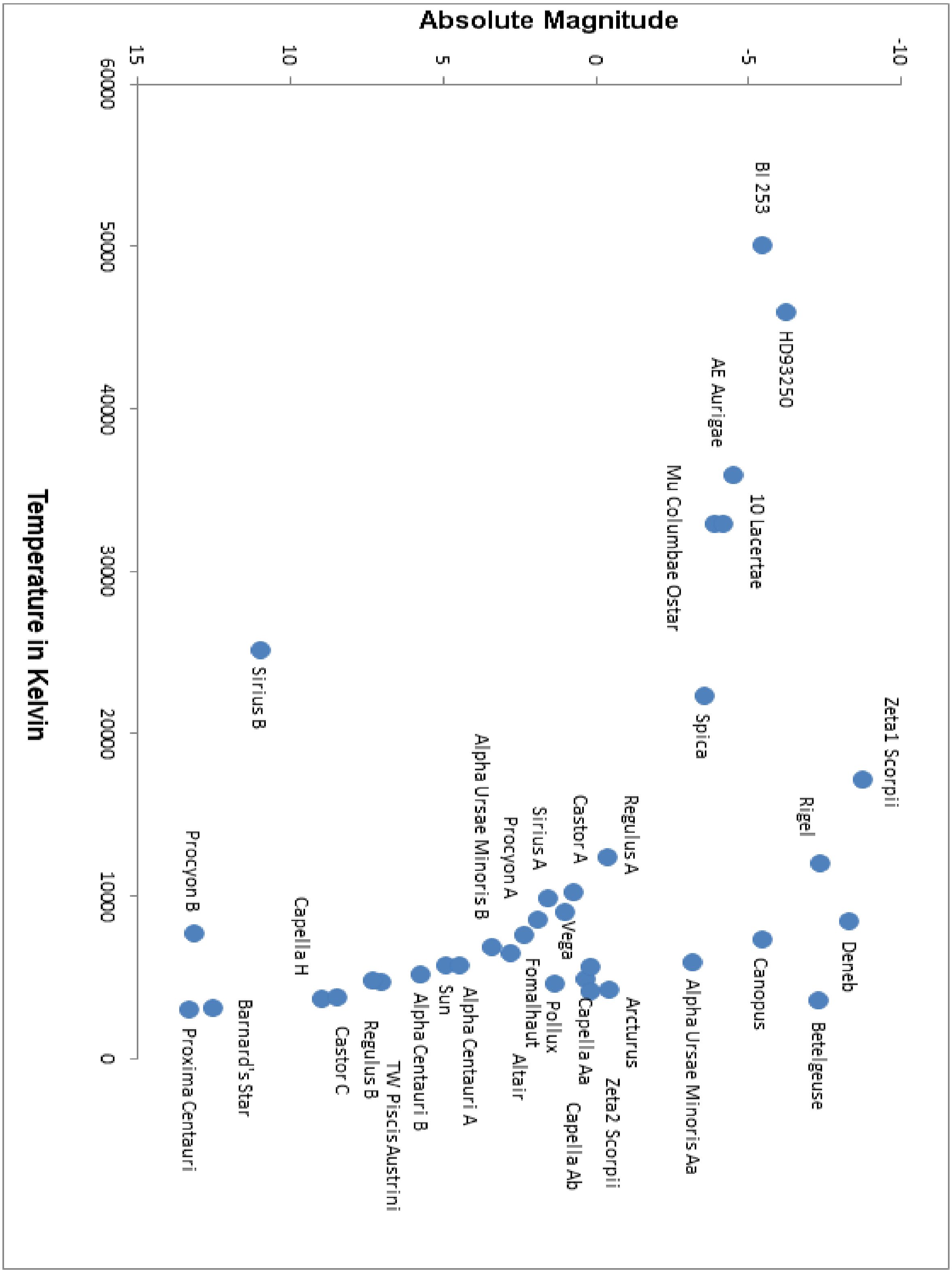

Figure 1. Plot of estimated absolute magnitudes (11) against the temperatures of the stars listed in Table I. Contrary to general practice, on both the axes the assigned values go down as one moves away from the origin.

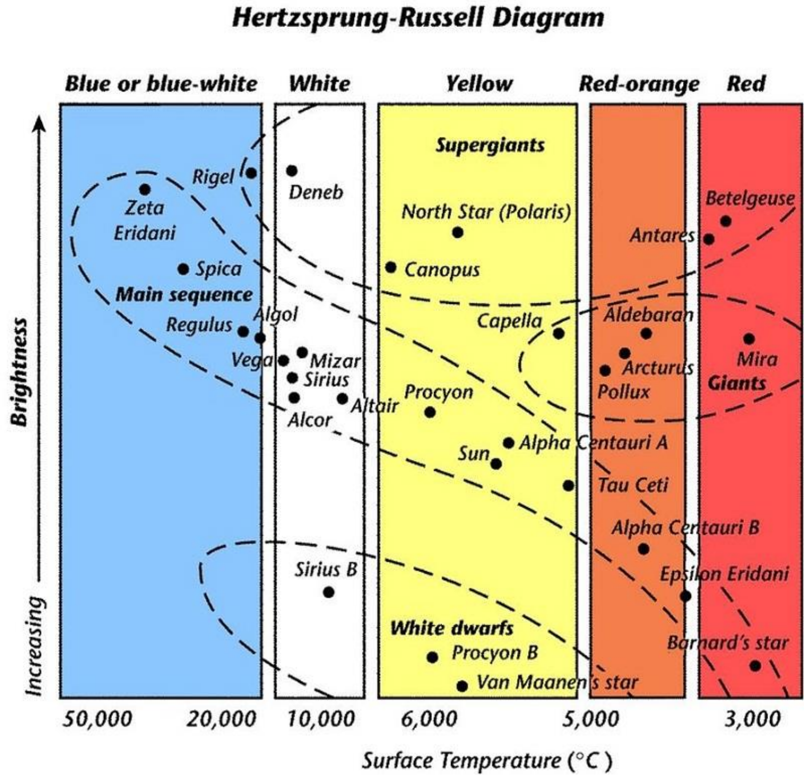

Figure 2. Hertzsprung-Russell diagram

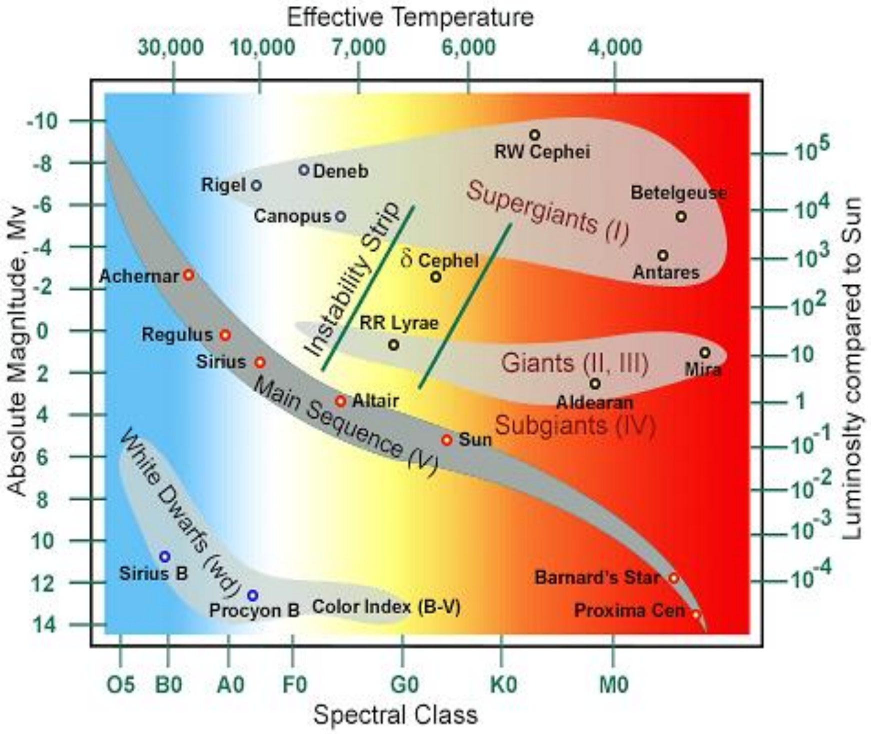

Figure 3. Hertzsprung-Russell diagram

5. CONCLUSIONS

Once a star has consumed all the hydrogen in its core, it moves away from the main sequence to the giants branch where names of stars present are Alpha Ursae Minoris Aa, Arcturus, Zeta^2 Scorpii, Capella Aa, Capella Ab and Pollux; these are the low mass stars near the end of their lives. Their cores are now filled with helium. There is not enough hydrogen left in the core so burning occurs in a shell surrounding the core, where it is mostly helium. As the helium fusion occurs at substantially higher temperatures this causes the star to grow in size thereby becoming cooler but more luminous due to large radius.

The more massive stars after finishing their time on the main sequence evolve as super-giants. These stars grow in radius, and can change temperature dramatically, but they do not change much in luminosity. Examples are Zeta^1 Scorpii, Rigel, Deneb, Betelgeuse and Canopus; these are high mass stars approaching the end of their lives.

Finally, the fusion stops in the core of a massive star and it becomes a White dwarf; the name coined by Willem Luyten in 1922. A low or medium mass star (with mass less than about 8 times the mass of our Sun) will become a white dwarf. A typical white dwarf is about as massive as the Sun, yet only slightly bigger than the Earth; stars that have a lot of mass may end their lives as black holes or neutron stars.

A white dwarf is highly dense and its mass is comparable to that of the Sun while its volume is comparable to that of Earth. In the absence of the fusion White dwarfs are low in luminosity. But they are hot, since they were once the core of a star where the temperature was many millions of degrees K, which is much hotter than the outer surface of any star. The white dwarf will slowly cool over time becoming fainter and redder as it radiates away its heat. Examples are Sirius B and Procyon B (Figure 1).

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