Available online at www.worldscientificnews.com

World Scientific News

An International Scientific Journal

WSN 95 (2018) 159-166 EISSN 2392-2192

# On some triple sequence spaces of $\chi^3$

**Ayhan Esi**[1] and **Subramanian Nagarajan**[2]

[1]Department of Mathematics, Faculty of Arts and Sciences, Adiyaman University, Turkey

[2]Department of Mathematics, Sastra University, Thanjavur, Tamil Nadu, India

[1,2]E-mail address: aesi23@hotmail.com , nsmaths@gmail.com

**ABSTRACT**

We introduce the concepts in probability of rough lacunary statistical convergence and $N_\theta$ - rough convergence of a triple sequence spaces of real numbers and discuss general properties of above rough convergence.

***Keywords*****:** Rough lacunary statistical convergence, triple sequences, $N_\theta$ - convergence, chi sequence

## 1. INTRODUCTION

The idea of rough convergence was introduced by Phu [11], who also introduced the concepts of rough limit points and roughness degree. The idea of rough convergence occurs very naturally in numerical analysis and has interesting applications. Aytar [1] extended the idea of rough convergence into rough statistical convergence using the notion of natural density just as usual convergence was extended to statistical convergence. Pal et al. [10] extended the notion of rough convergence using the concept of ideals which automatically extends the earlier notions of rough convergence and rough statistical convergence.

A triple sequence (real or complex) can be defined as a function $x: \mathbb{N} \times \mathbb{N} \times \mathbb{N} \to \mathbb{R}(\mathbb{C})$, where $\mathbb{N}, \mathbb{R}$ and $\mathbb{C}$ denote the set of natural numbers, real numbers and complex numbers respectively. The different types of notions of triple sequence was introduced and investigated

( Received 22 February 2018; Accepted 05 March 2018; Date of Publication 06 March 2018 )

at the initial by Sahiner et al. [12,13], Esi et al. [2-4], Datta et al. [5], Subramanian et al. [14], Debnath et al. [6], Esi et al. [16] and many others.

A triple sequence $x = (x_{mnk})$ is said to be triple analytic if

$$sup_{m,n,k}|x_{mnk}|^{\frac{1}{m+n+k}} < \infty.$$

The space of all triple analytic sequences are usually denoted by $\Lambda^3$. A triple sequence $x = (x_{mnk})$ is called triple gai sequence if

$$\left((m+n+k)!\,|x_{mnk}|\right)^{\frac{1}{m+n+k}} \to 0 \text{ as } m,n,k \to \infty.$$

The space of all triple gai sequences are usually denoted by $\chi^3$.

## 2. DEFINITIONS AND PRELIMINARIES

### 2. 1. Definition

An Orlicz function (see [7]) is a function $M: [0,\infty) \to [0,\infty)$ which is continuous, non-decreasing and convex with $M(0) = 0$, $M(x) > 0$, for $x > 0$ and $M(x) \to \infty$ as $x \to \infty$. If convexity of Orlicz function $M$ is replaced by $M(x+y) \le M(x) + M(y)$, then this function is called modulus function.

Lindenstrauss and Tzafriri ([8]) used the idea of Orlicz function to construct Orlicz sequence space.

A sequence $g = (g_{mn})$ defined by

$$g_{mn}(v) = sup\{|v|u - (f_{mnk})(u): u \ge 0\}, m,n,k = 1,2,\cdots$$

is called the complementary function of a Musielak-Orlicz function $f$. For a given Musielak-Orlicz function $f$, [see [9] ] the Musielak-Orlicz sequence space $t_f$ is defined as follows

$$t_f = \left\{x \in w^3 : I_f(|x_{mnk}|)^{1/m+n+k} \to 0 \quad as \quad m,n,k \to \infty\right\},$$

where: $I_f$ is a convex modular defined by

$$I_f(x) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{k=1}^{\infty} f_{mnk}(|x_{mnk}|)^{1/m+n+k}, x = (x_{mnk}) \in t_f.$$

We consider $t_f$ equipped with the Luxemburg metric

$$d(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{k=1}^{\infty} f_{mnk}\left(\frac{|x_{mnk}|^{1/m+n+k}}{mnk}\right)$$

is an extended real number.

**2. 2. Definition**

Let $X, Y$ be a real vector space of dimension $w$, where $n \leq m$. A real valued function $d_p(x_1, \dots, x_n) = \| (d_1(x_1, 0), \dots, d_n(x_n, 0)) \|_p$ on $X$ satisfying the following four conditions:

**(i)** $\| (d_1(x_1, 0), \dots, d_n(x_n, 0)) \|_p = 0$ if and only if $d_1(x_1, 0), \dots, d_n(x_n, 0)$ are linearly dependent,

**(ii)** $\| (d_1(x_1, 0), \dots, d_n(x_n, 0)) \|_p$ is invariant under permutation,

**(iii)** $\| (\alpha d_1(x_1, 0), \dots, d_n(x_n, 0)) \|_p = |\alpha| \, \| (d_1(x_1, 0), \dots, d_n(x_n, 0)) \|_p, \alpha \in \mathbb{R}$

**(iv)** $d_p((x_1, y_1), (x_2, y_2) \cdots (x_n, y_n)) = (d_X(x_1, x_2, \cdots x_n)^p + d_Y(y_1, y_2, \cdots y_n)^p)^{1/p} for 1 \leq p < \infty$; (or)

(v) $d((x_1, y_1), (x_2, y_2), \cdots (x_n, y_n)) := \sup\{d_X(x_1, x_2, \cdots x_n), d_Y(y_1, y_2, \cdots y_n)\}$,

for $x_1, x_2, \cdots x_n \in X, y_1, y_2, \cdots y_n \in Y$ is called the $p$ product metric of the Cartesian product of $n$ metric spaces (see [15]) .

**2. 3. Definition**

The triple sequence $\theta_{i,\ell,j} = \{(m_i, n_\ell, k_j)\}$ is called triple lacunary if there exist three increasing sequences of integers such that

$$m_0 = 0, h_i = m_i - m_{r-1} \to \infty \text{ as } i \to \infty \text{ and}$$
$$n_0 = 0, \overline{h_\ell} = n_\ell - n_{\ell-1} \to \infty \text{ as } \ell \to \infty.$$
$$k_0 = 0, \overline{h_j} = k_j - k_{j-1} \to \infty \text{ as } j \to \infty.$$

Let $m_{i,\ell,j} = m_i n_\ell k_j, h_{i,\ell,j} = h_i \overline{h_\ell h_j}$, and $\theta_{i,\ell,j}$ is determine by

$$I_{i,\ell,j} = \{(m, n, k) : m_{i-1} < m < m_i \quad and \quad n_{\ell-1} < n \leq n_\ell \quad and \quad k_{j-1} < k \leq k_j\}, q_i = \frac{m_i}{m_{i-1}}, \overline{q_\ell} = \frac{n_\ell}{n_{\ell-1}}, \overline{q_j} = \frac{k_j}{k_{j-1}}.$$

**2. 4. Definition**

Let $\theta = \{m_r n_s k_t\}_{(rst) \in \mathbb{N} \cup o}$ be the triple lacunary sequence spaces. A triple sequence spaces of $(X_{mnk})$ is said to be triple lacunary statistically convergent to a number $X$ if for any $\varepsilon > 0$,

$$lim_{rst \to \infty} \frac{1}{h_{rst}} |\{(m, n, k) \in I_{rst} : |X_{mnk}, \bar{0}| \geq \varepsilon\}| = 0 \,,$$

where:

$I_{r,s,t} = \{(m,n,k): m_{r-1} < m < m_r \quad and \quad n_{s-1} < n \leq n_s \quad and \quad k_{t-1} < k \leq k_t\}, q_r = \frac{m_r}{m_{r-1}}, \overline{q_s} = \frac{n_s}{n_{s-1}}, \overline{q_t} = \frac{k_t}{k_{t-1}}.$

In this case write

$$\left\|\chi_f^3(X_{mnk}),\left(d(x_1),d(x_2),\cdots,d(x_{n-1})\right)\right\|_p \to^{S_\theta} \left\|\chi_f^3(X),\left(d(x_1),d(x_2),\cdots,d(x_{n-1})\right)\right\|_p.$$

**2. 5. Definition**

Let $\theta = \{m_r n_s k_t\}_{(rst)\in\mathbb{N}\cup o}$ be the triple sequence spaces of lacunary. A number $(X_{mnk})$ is said to be $N_\theta -$ convergent to a real number $X$ if for every $\varepsilon > 0$,
$lim_{rst\to\infty} \frac{1}{h_{rst}} \sum_{m\in I_r} \sum_{n\in I_s} \sum_{k\in I_t} |X_{mnk}, \bar{0}| = 0$. In this case we write $X_{mnk} \to^{N_\theta} \bar{0}$.

**2. 6. Definition**

Let $\alpha$ be a nonnegative real number. A triple sequence spaces of random variables $(X_{mnk})$ is said to be rough lacunary statistically convergent in probability to $X: W \times W \times W \to \mathbb{R}\times\mathbb{R}\times\mathbb{R}$ with respect to the roughness of degree $\alpha$ if for any $\varepsilon, \delta > 0$, $lim_{rst\to\infty} \frac{1}{h_{rst}} \left|\left\{(m,n,k) \in I_{rst}: P\left(\left[f_{mnk}\left(\left\|\mu_{mnk}(X),\left(d(x_1),d(x_2),\cdots,d(x_{n-1})\right)\right\|_p\right)\right] \geq \alpha + \varepsilon\right) \geq \delta\right\}\right| = 0$ and we write $\left\|\chi_f^3(X_{mnk}),\left(d(x_1),d(x_2),\cdots,d(x_{n-1})\right)\right\|_p \to_\alpha^{S^P} \bar{0}$. It will be denoted by $\alpha S_\theta^P$.

**2. 7. Definition**

Let $\alpha$ be a nonnegative real number. A triple sequence spaces of random variables $(X_{mnk})$ is said to be rough $N_\theta -$ convergent in probability to $X: W \times W \times W \to \mathbb{R}\times\mathbb{R}\times\mathbb{R}$ with respect to the roughness of degree

$\alpha$ if for any $\varepsilon > 0$, $lim_{rst\to\infty} \frac{1}{h_{rst}} \sum_{m\in I_r} \sum_{n\in I_s} \sum_{k\in I_t} \left|\left\{P\left(\| \left[f_{mnk}\left(\left(\mu_{mnk}(X) - \mu(X)\right),\left(d(x_1),d(x_2),\cdots,d(x_{n-1})\right) \|_p\right)\right] \geq \alpha + \varepsilon\right)\right\}\right| = 0$, and we write

$$\left\|\chi_f^3(X_{mnk}),\left(d(x_1),d(x_2),\cdots,d(x_{n-1})\right)\right\|_p \to_\alpha^{N_\theta^P} \left\|\chi_f^3(X),\left(d(x_1),d(x_2),\cdots,d(x_{n-1})\right)\right\|_p$$

The class of all $\alpha - N_\theta -$ convergent triple sequence spaces of random variables in probability will be denoted by $\alpha N_\theta^P$.

**2. 8. Definition**

Let $\theta = \{m_r n_s k_t\}_{(rst)\in\mathbb{N}\cup o}$ be lacunary triple sequence spaces of lacunary refinement of $\theta$ is a triple lacunary sequence spaces of $\theta' = \left\{m_r' n_s' k_t'\right\}_{(rst)\in\mathbb{N}\cup o}$ satisfying

$$\theta = \{m_r n_s k_t\}_{(rst)\in\mathbb{N}\cup o} \subset \left\{m_r' n_s' k_t'\right\}_{(rst)\in\mathbb{N}\cup o}$$

**2. 9. Note**

Let $f$ be an Musielak-Orlicz function and triple sequence spaces of $\left\|\chi_f^3, \left(d(x_1), d(x_2), \cdots, d(x_{n-1})\right)\right\|_p = \left[f_{mnk}\left(\left\|\mu_{mnk}(X), \left(d(x_1), d(x_2), \cdots, d(x_{n-1})\right)\right\|_p\right)\right]$, where $\mu_{mnk}(X) = \left(\left((m+n+k)!\, X_{mnk}\right)^{1/m+n+k}, \bar{0}\right)$.

## 3. MAIN RESULTS

**3. 1. Theorem**

Let $\theta = \{m_r n_s k_t\}_{(rst)\in \mathbb{N}\cup o}$ be a triple lacunary sequence spaces. Then the followings are equivalent:

**(i)** $\left\|\chi_f^3(X_{mnk}), \left(d(x_1), d(x_2), \cdots, d(x_{n-1})\right)\right\|_p$ is $\alpha -$ triple lacunary statistically convergent in probability to $\bar{0}$.

**(ii)** $\left\|\chi_f^3(X_{mnk}), \left(d(x_1), d(x_2), \cdots, d(x_{n-1})\right)\right\|_p$ is $\alpha - N_\theta$ convergent in probability to $\bar{0}$.

**Proof: (i)** $\Rightarrow$ (ii) First suppose that $\left\|\chi_f^3(X_{mnk}), \left(d(x_1), d(x_2), \cdots, d(x_{n-1})\right)\right\|_p \rightarrow_{\alpha}^{S_\theta^P} \bar{0}$. Then we can write

$$\frac{1}{h_{rst}} \sum_{m\in I_r} \sum_{n\in I_s} \sum_{k\in I_t} \left|\left\{P\left(\| \left[f_{mnk}\left(\left(\mu_{mnk}(X)-\mu(X)\right), \left(d(x_1), d(x_2), \cdots, d(x_{n-1})\right) \|_p\right)\right] \geq \alpha+\varepsilon\right)\right\}\right| =$$
$$\frac{1}{h_{rst}} \sum_{m\in I_r} \sum_{n\in I_s} \sum_{k\in I_t, P\left(\|\left[f_{mnk}\left(\left(\mu_{mnk}(X)-\mu(X)\right),\left(d(x_1),d(x_2),\cdots,d(x_{n-1})\right)\|_p\right)\right]\geq \alpha+\varepsilon\right)\geq \frac{\delta}{2}} \left|\left\{P\left(\| \left[f_{mnk}\left(\left(\mu_{mnk}(X)-\mu(X)\right), \left(d(x_1), d(x_2), \cdots, d(x_{n-1})\right) \|_p\right)\right] \geq \alpha+\varepsilon\right)\right\}\right| +$$
$$\frac{1}{h_{rst}} \sum_{m\in I_r} \sum_{n\in I_s} \sum_{k\in I_t, P\left(\|\left[f_{mnk}\left(\left(\mu_{mnk}(X)-\mu(X)\right),\left(d(x_1),d(x_2),\cdots,d(x_{n-1})\right)\|_p\right)\right]\geq \alpha+\varepsilon\right)< \frac{\delta}{2}} \left|\left\{P\left(\| \left[f_{mnk}\left(\left(\mu_{mnk}(X)-\mu(X)\right), \left(d(x_1), d(x_2), \cdots, d(x_{n-1})\right) \|_p\right)\right] \geq \alpha+\varepsilon\right)\right\}\right| \leq \frac{1}{h_{rst}} \left|\left\{P\left(\| \left[f_{mnk}\left(\left(\mu_{mnk}(X)-\mu(X)\right), \left(d(x_1), d(x_2), \cdots, d(x_{n-1})\right) \|_p\right)\right] \geq \alpha+\varepsilon\right) \geq \frac{\delta}{2}\right\}\right| + \frac{\delta}{2}.$$

**(ii)** $\Rightarrow$ (i) Next suppose that condition (ii) holds. Then

$$\sum_{m\in I_r} \sum_{n\in I_s} \sum_{k\in I_t} \left|\left\{P\left(\| \left[f_{mnk}\left(\left(\mu_{mnk}(X)-\mu(X)\right), \left(d(x_1), d(x_2), \cdots, d(x_{n-1})\right) \|_p\right)\right] \geq \alpha+\varepsilon\right)\right\}\right| \geq$$
$$\sum_{m\in I_r} \sum_{n\in I_s} \sum_{k\in I_t} \left|\left\{P\left(\| \left[f_{mnk}\left(\left(\mu_{mnk}(X)-\mu(X)\right), \left(d(x_1), d(x_2), \cdots, d(x_{n-1})\right) \|_p\right)\right] \geq \alpha+\varepsilon\right) \geq \delta\right\}\right| \geq \delta\left|\left\{P\left(\| \left[f_{mnk}\left(\left(\mu_{mnk}(X)-\mu(X)\right), \left(d(x_1), d(x_2), \cdots, d(x_{n-1})\right) \|_p\right)\right] \geq \alpha+\varepsilon\right) \geq \delta\right\}\right|.$$

Therefore

$\frac{1}{\delta}\frac{1}{h_{rst}}\sum_{m\in I_r}\sum_{n\in I_s}\sum_{k\in I_t}\left|\left\{P\left(\| \left[f_{mnk}\left(\left(\mu_{mnk}(X)-\mu(X)\right),\left(d(x_1),d(x_2),\cdots,d(x_{n-1})\right)\|_p\right)\right]\geq\alpha+\varepsilon\right)\right\}\right|\geq\frac{1}{h_{rst}}\left|\left\{P\left(\| \left[f_{mnk}\left(\left(\mu_{mnk}(X)-\mu(X)\right),\left(d(x_1),d(x_2),\cdots,d(x_{n-1})\right)\|_p\right)\right]\geq\alpha+\varepsilon\right)\geq\delta\right\}\right|.$

Hence $\left\|\chi_f^3(X_{mnk}),\left(d(x_1),d(x_2),\cdots,d(x_{n-1})\right)\right\|_p\rightarrow_{\alpha}^{S_\theta^P}\bar{0}.$

**3. 2. Theorem**

If $\left\|\chi_f^3(X_{mnk}),\left(d(x_1),d(x_2),\cdots,d(x_{n-1})\right)\right\|_p\rightarrow_{\alpha}^{S_\theta^P}\bar{0}$ and

$\left\|\chi_f^3(Y_{mnk}),\left(d(x_1),d(x_2),\cdots,d(x_{n-1})\right)\right\|_p\rightarrow_{\alpha}^{S_\theta^P}\bar{0}$ then

$P(|\{\left\|\chi_f^3(X_{mnk}-Y_{mnk}),\left(d(x_1),d(x_2),\cdots,d(x_{n-1})\right)\right\|_p|\geq\alpha+\varepsilon)\}=\bar{0}.$

**Proof:** Omitted.

**3. 3. Theorem**

Let $\theta'=\{m_r'n_s'k_t'\}_{(rst)\in\mathbb{N}\cup o}$ be a triple lacunary refinement of the triple lacunary sequence spaces of $\theta=\{m_rn_sk_t\}_{(rst)\in\mathbb{N}\cup 0}$. Let $h_r=(m_{r-1},m_r],h_s=(n_{s-1},n_s],h_t=(k_{t-1},h_r],r,s,t=1,2,3\cdots$. If there exists a $\eta>0$ such that $\frac{|h_{rst}|}{|I_{rst}|}>\eta$ for every $h_{rst}\subseteq I_{rst}$.

Then

$\left\|\chi_f^3(X_{mnk}),\left(d(x_1),d(x_2),\cdots,d(x_{n-1})\right)\right\|_p\rightarrow_{\alpha}^{S_\theta^P}\bar{0}\Rightarrow$

$\left\|\chi_f^3(X_{mnk}),\left(d(x_1),d(x_2),\cdots,d(x_{n-1})\right)\right\|_p\rightarrow_{\alpha}^{S_{\theta'}^P}\bar{0}.$

**Proof:** Let $\left\|\chi_f^3(X_{mnk}),\left(d(x_1),d(x_2),\cdots,d(x_{n-1})\right)\right\|_p\rightarrow_{\alpha}^{S_\theta^P}\bar{0}$ and $\varepsilon,\delta>0$. Therefore $lim_{rst\to\infty}\frac{1}{|I_{rst}|}\left|\left\{P\left(\| \left[f_{mnk}\left(\left(\mu_{mnk}(X)-\mu(X)\right),\left(d(x_1),d(x_2),\cdots,d(x_{n-1})\right)\|_p\right)\right]\geq\alpha+\varepsilon\right)\geq\delta\right\}\right|=0$. For every $h_{rst}$ we can find $I_{rst}$ such that $h_{rst}\subseteq I_{rst}$. We obtain $\frac{1}{|h_{rst}|}\left|\left\{P\left(\| \left[f_{mnk}\left(\left(\mu_{mnk}(X)-\mu(X)\right),\left(d(x_1),d(x_2),\cdots,d(x_{n-1})\right)\|_p\right)\right]\geq\alpha+\varepsilon\right)\geq\delta\right\}\right|=\frac{|I_{rst}|}{|h_{rst}|}\frac{1}{|I_{rst}|}\left|\left\{P\left(\| \left[f_{mnk}\left(\left(\mu_{mnk}(X)-\mu(X)\right),\left(d(x_1),d(x_2),\cdots,d(x_{n-1})\right)\|_p\right)\right]\geq\alpha+\varepsilon\right)\geq\delta\right\}\right|\leq\frac{1}{\eta}\frac{1}{|I_{rst}|}\left|\left\{P\left(\| \left[f_{mnk}\left(\left(\mu_{mnk}(X)-\mu(X)\right),\left(d(x_1),d(x_2),\cdots,d(x_{n-1})\right)\|_p\right)\right]\geq\alpha+\varepsilon\right)\geq\delta\right\}\right|.$

### 3. 4. Remark

In this refinements $\theta'$ of $\theta$ exists. The following example, let $(u, v, w) \in \mathbb{N} \backslash \{1,1,1\}$ and introducing $(u-1, v-1, w-1)$ points in the interval $h_r = (m_{r-1}, m_r], h_s = (n_{s-1}, n_s], h_t = (k_{t-1}, h_r], r, s, t = 1,2,3 \cdots$.

Then

$$h_1 = \left((m_0, n_0, k_0), (m_0, n_0, k_0) + \frac{j_1}{(uvw)}\right] h_2$$

$$= \left((m_0, n_0, k_0) + \frac{j_1}{(uvw)}, (m_0, n_0, k_0) + \frac{2j_1}{(uvw)}\right] 0.5cm \vdots h_{(uvw)}$$

$$= \left((m_0, n_0, k_0) + \frac{(u-1)(v-1)(w-1)j_1}{(uvw)}, (m, n, k)\right] h_{(u+1,v+1,w+1)}$$

$$= \left((m_1, n_1, k_1), (m_1, n_1, k_1) + \frac{j_2}{(uvw)}\right] h_{(u+2,v+2,w+2)}$$

$$= \left((m_1, n_1, k_1) + \frac{j_2}{(uvw)}, (m_1, n_1, k_1) + \frac{2j_2}{(uvw)}\right] 0.5cm \vdots h_{(2u,2v,2w)}$$

$$= \left((m_1, n_1, k_1) + \frac{(u-1)(v-1)(w-1)j_2}{(uvw)}, (m_2, n_2, k_2)\right] 0.5cm$$

$$\vdots h_{r-1(u+1,v+1,w+1)}$$

$$= \left((m_{r-1}, n_{r-1}, k_{r-1}), (m_{r-1}, n_{r-1}, k_{r-1}) + \frac{h_r}{(uvw)}\right] h_{r-1(u+2,v+2,w+2)}$$

$$= \left((m_{r-1}, n_{r-1}, k_{r-1}) + \frac{h_r}{(uvw)}, (m_{r-1}, n_{r-1}, k_{r-1}) + \frac{2h_r}{(uvw)}\right] 0.5cm$$

$$\vdots h_{r-1(2u,2v,2w)}$$

$$= \left((m_{r-1}, n_{r-1}, k_{r-1}) + \frac{(u-1)(v-1)(w-1)h_{r-1}}{(uvw)}, (m_r, n_r, k_r)\right]. 0.5cm \vdots$$

Then $|h_{rst}| \to \infty$ as $r, s, t \to \infty$ and $\frac{|h_{rst}|}{|I_{rst}|} \geq \frac{1}{(uvw)}$ for every $h_{rst} \subseteq I_{rst}$.

## 4. CONCLUSIONS

We introduced triple sequence spaces of rough lacunary statistical convergence in probability and triple sequence spaces of $N_\theta -$ rough convergence with respect sequence of Musielak-Orlicz function. For the reference sections, consider the following introduction described the main results are motivating the research.

**References**

[1] S. Aytar. Rough statistical Convergence, *Numerical Functional Analysis Optimization*, 29 (3) (2008) 291-303.

[2] A. Esi, On some triple almost lacunary sequence spaces defined by Orlicz functions, *Research and Reviews: Discrete Mathematical Structures*, 1 (2) (2014) 16-25.

[3] A. Esi and M. Necdet Catalbas,Almost convergence of triple sequences, *Global Journal of Mathematical Analysis*, 2 (1) (2014) 6-10.

[4] A. Esi and E. Savas, On lacunary statistically convergent triple sequences in probabilistic normed space, *Appl. Math. and Inf. Sci.* 9 (5) (2015) 2529-2534.

[5] A. J. Datta A. Esi and B.C. Tripathy, Statistically convergent triple sequence spaces defined by Orlicz function, *Journal of Mathematical Analysis*, 4 (2) (2013) 16-22.

[6] S. Debnath, B. Sarma and B.C. Das ,Some generalized triple sequence spaces of real numbers, *Journal of nonlinear analysis and optimization*, 6 (1) (2015) 71-79.

[7] P.K. Kamthan and M. Gupta, Sequence spaces and series, Lecture notes, Pure and Applied Mathematics, 65 Marcel Dekker, In c. New York , 1981.

[8] J. Lindenstrauss and L. Tzafriri, On Orlicz sequence spaces, *Israel J. Math.* 10 (1971) 379-390.

[9] J. Musielak, Orlicz Spaces, Lectures Notes in Math., 1034, Springer-Verlag, 1983.

[10] S.K. Pal, D. Chandra and S. Dutta. Rough ideal Convergence, *Hacee. Jounral Mathematics and Statistics*, 42 (6) (2013) 633-640.

[11] H.X. Phu. Rough convergence in normed linear spaces, *Numerical Functional Analysis Optimization*, 22 (2001) 201-224.

[12] A. Sahiner, M. Gurdal and F.K. Duden, Triple sequences and their statistical convergence, *Selcuk J. Appl. Math.* 8 (2) (2007) 49-55.

[13] A. Sahiner, B.C. Tripathy, Some $I$ related properties of triple sequences, *Selcuk J. Appl. Math.* 9 (2) (2008) 9-18.

[14] N. Subramanian and A. Esi, The generalized tripled difference of $\chi^3$ sequence spaces, *Global Journal of Mathematical Analysis*, 3 (2) (2015) 54-60.

[15] N. Subramanian and C. Murugesan, The entire sequence over Musielak $p-$ metric space, *Journal of the Egyptian Mathematical Society*, 24 (2) (2016) 233-238.

[16] Ayten Esi, N. Subramanian and Ayhan Esi, Triple rough statistical convergence of sequence of Bernstein operators. *International Journal of Advanced and Applied Sciences*, 4 (2) (2017) 28-34.