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Determination of the Position and Orientation of Aerial Photogrammetry Sensors: A Practical Implementation

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ABSTRACT

Surveying engineering aims at determining the three-dimensional positions of points on the earth surface. One of the techniques of achieving this is by making measurements on two-dimensional aerial photographs. This is known as Aerial Photogrammetry. Aerial Photogrammetry employs the use of aircraft such as aeroplane, helicopter, or in recent times, drone, to take photographs of places to be mapped from the air. In determining the required ground coordinates of an object by Aerial Photogrammetry, the position and attitude (orientation) of the sensor (or camera) during exposure must be known. The recovery of these parameters (exterior orientation parameters) is known as space resection. Although several texts and papers have described various methods of retrieving the parameters, and how to go about the solutions, the descriptions are vivid, and usually unclear to a large percentage of students. The solution involves tedious computational tasks and is generally time-consuming and brain-tasking. Hence, there is need to proffer a step-by-step solution to a real-world problem that students can study. This paper aims to implement this by employing images captured by an Unmanned Aerial Vehicle (UAV) and measured ground coordinates at the University of Lagos, Nigeria. With a view to achieving quick, less stressful and gross error-free solutions, a computer program was developed in C# programming language for the problem using the principle of collinearity condition

***Keywords*:** Photogrammetry, Aerial, Space Resection, UAV, Exterior Orientation, Position

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1. INTRODUCTION

The fundamental objective of surveying engineering is to determine the spatial positions of points on the earth surface. The word "spatial" indicates that the location of the information is known in three-dimensional space. In other words, the information is geographically referenced in a three-dimensional coordinate system (Derenyi, 1996). There are different techniques of determining these spatial positions. These include traversing, triangulation, trilateration, etc. Photographs can also be used for spatial positioning, as done in Photogrammetry.

Photogrammetry, as defined by the American Society of Photogrammetry, is the art, science, and technology of obtaining reliable information about physical objects and the environment through processes of recording, measuring, and interpreting photographic images and patterns of recorded radiant electromagnetic energy and other phenomena. (Wolf, 1983).

The principal application of photogrammetry is to determine the spatial positions of the natural and man-made features situated on the earth's surface (topographic application). Photogrammetry is also used as a measuring tool in architecture, industrial design, deformation studies of structures, accident investigation, medicine etc. These operations are referred to as non-topographic applications (Derenyi, 1996).

In photogrammetry, the three-dimensional coordinates (spatial positions) of objects are determined by the utilization of two-dimensional photographic images. However, in determining the required ground coordinates of an object, the position and attitude of the camera (camera exterior orientation elements) during the exposure must be known. The recovery of these parameters is known as space resection.

This paper proffers a step-by-step solution to the space resection problem of an aerial photogrammetry project carried out at the University of Lagos, Nigeria, in a bid to make the steps involved in solving such problem clearer to photogrammetry students.

2. MATERIALS AND METHODS

The adopted method for solving the space resection problem is the collinearity equations method, based on the collinearity condition, to determine the exterior orientation parameters of a camera at the instance of exposure.

2. 1. The Collinearity Condition

The collinearity condition states that the exposure station of any photograph, an object point, and its photo image all lie on a straight line. The equations which express the collinearity condition are the collinearity equations, expressed thus:

$$x_a = x_o - f\left[\frac{m_{11}(X_A - X_L) + m_{12}(Y_A - Y_L) + m_{13}(Z_A - Z_L)}{m_{31}(X_A - X_L) + m_{32}(Y_A - Y_L) + m_{33}(Z_A - Z_L)}\right] \tag{2.1}$$

$$y_a = y_o - f\left[\frac{m_{21}(X_A - X_L) + m_{22}(Y_A - Y_L) + m_{23}(Z_A - Z_L)}{m_{31}(X_A - X_L) + m_{32}(Y_A - Y_L) + m_{33}(Z_A - Z_L)}\right] \tag{2.2}$$

where:
X_L, Y_L, Z_L are the coordinates of exposure station;
X_A, Y_A, Z_A are the coordinates of object point A;
x_a, y_a, z_a are the coordinates of object point A on the image, with respect to xy photo coordinate system (of which the principal point is the origin);
x_o, y_o are the coordinates of the principal point;
f is the focal length of the camera;
m_{11}, m_{12}, m_{13}, m_{21}, ... are elements of the rotation matrix M;

$$M = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \tag{2.3}$$

The elements of M are functions of the orientation angles ω, φ and κ of the camera as expressed below.

$$m_{11} = \cos\varphi \cos\kappa \tag{2.4}$$

$$m_{12} = \cos\omega \cos\kappa + \sin\omega \sin\varphi \cos\kappa \tag{2.5}$$

$$m_{13} = \sin\omega \sin\kappa - \cos\omega \sin\varphi \cos\kappa \tag{2.6}$$

$$m_{21} = -\cos\varphi \sin\kappa \tag{2.7}$$

$$m_{22} = \cos\omega \cos\kappa - \sin\omega \sin\varphi \sin\kappa \tag{2.8}$$

$$m_{23} = \sin\omega \cos\kappa + \cos\omega \sin\varphi \sin\kappa \tag{2.9}$$

$$m_{31} = \sin\varphi \tag{2.10}$$

$$m_{32} = -\sin\omega \cos\varphi \tag{2.11}$$

$$m_{33} = \cos\omega \cos\varphi \tag{2.12}$$

2. 2. Space Resection by Collinearity

Space resection by collinearity involves formulating the collinearity equations for a number of control points whose X_A, Y_A, Z_A ground coordinates are known and whose images appear in the tilted photograph.

The solutions are obtained by method of least squares. Each control point gives 2 equations. Since there are 6 unknowns, then, at least 3 control points are required. These control points should not be on or near straight line.

Collinearity equations, when applied to the space resection problem, involve 6 unknowns. These are the orientation angles of the camera, ω, φ and κ (inherent in the rotation matrix M) and the exposure station coordinates of the camera (X_L, Y_L, Z_L).

These equations are non-linear. Hence, they need to be linearized before they can be used for solving for the six parameters listed above

The equations can be re-written as:

$$F = x_o - f\left[\frac{r}{q}\right] = x_a \tag{2.13}$$

$$G = y_o - f\left[\frac{s}{q}\right] = y_a \tag{2.14}$$

where:

$$q = m_{31}(X_A - X_L) + m_{32}(Y_A - Y_L) + m_{33}(Z_A - Z_L) \tag{2.15}$$

$$r = m_{11}(X_A - X_L) + m_{12}(Y_A - Y_L) + m_{13}(Z_A - Z_L) \tag{2.16}$$

$$s = m_{21}(X_A - X_L) + m_{22}(Y_A - Y_L) + m_{23}(Z_A - Z_L) \tag{2.17}$$

Applying Taylor's theorem to these equations (using only up to first order partial derivatives), we get:

$$F_o + \left(\frac{\partial F}{\partial \omega}\right)_o d\omega + \left(\frac{\partial F}{\partial \varphi}\right)_o d\varphi + \left(\frac{\partial F}{\partial \kappa}\right)_o d\kappa + \left(\frac{\partial F}{\partial X_L}\right)_o dX_L + \left(\frac{\partial F}{\partial Y_L}\right)_o dY_L + \left(\frac{\partial F}{\partial Z_L}\right)_o dZ_L = x_a \tag{2.18}$$

$$G_o + \left(\frac{\partial G}{\partial \omega}\right)_o d\omega + \left(\frac{\partial G}{\partial \varphi}\right)_o d\varphi + \left(\frac{\partial G}{\partial \kappa}\right)_o d\kappa + \left(\frac{\partial G}{\partial X_L}\right)_o dG_L + \left(\frac{\partial G}{\partial Y_L}\right)_o dY_L + \left(\frac{\partial G}{\partial Z_L}\right)_o dZ_L = y_a \tag{2.19}$$

where:
F_o, G_o are functions of F and G evaluated at the initial approximations for the 6 unknowns;
$\left(\frac{\partial F}{\partial \omega}\right)_o, \left(\frac{\partial F}{\partial \varphi}\right)_o, \left(\frac{\partial G}{\partial \omega}\right)_o, \left(\frac{\partial G}{\partial \varphi}\right)_o, etc.$, are partial derivatives of F and G with respect to the indicated unknowns evaluated at the initial approximations;
$d\omega, d\varphi, d\kappa, etc.$, are unknown corrections to be applied to the initial approximations.

Applying least squares principle:

$$X = (A^T PA)^{-1}(A^T PL) \tag{2.20}$$

where:
A is design matrix that contains the partial derivatives of collinearity equations with respect to the six unknown exterior orientation parameters for each control point. Hence, the dimension

of the matrix is 2n × 6, where n is the number of control points, and 6 denotes the six unknowns.

$$A = \begin{bmatrix} \left(\frac{\partial F}{\partial X_L}\right)_1 & \left(\frac{\partial F}{\partial Y_L}\right)_1 & \left(\frac{\partial F}{\partial Z_L}\right)_1 & \left(\frac{\partial F}{\partial \omega}\right)_1 & \left(\frac{\partial F}{\partial \varphi}\right)_1 & \left(\frac{\partial F}{\partial \kappa}\right)_1 \\ \left(\frac{\partial G}{\partial X_L}\right)_1 & \left(\frac{\partial G}{\partial Y_L}\right)_1 & \left(\frac{\partial G}{\partial Z_L}\right)_1 & \left(\frac{\partial G}{\partial \omega}\right)_1 & \left(\frac{\partial G}{\partial \varphi}\right)_1 & \left(\frac{\partial G}{\partial \kappa}\right)_1 \\ \left(\frac{\partial F}{\partial X_L}\right)_2 & \left(\frac{\partial F}{\partial Y_L}\right)_2 & \left(\frac{\partial F}{\partial Z_L}\right)_2 & \left(\frac{\partial F}{\partial \omega}\right)_2 & \left(\frac{\partial F}{\partial \varphi}\right)_2 & \left(\frac{\partial F}{\partial \kappa}\right)_2 \\ \left(\frac{\partial G}{\partial X_L}\right)_2 & \left(\frac{\partial G}{\partial Y_L}\right)_2 & \left(\frac{\partial G}{\partial Z_L}\right)_2 & \left(\frac{\partial G}{\partial \omega}\right)_2 & \left(\frac{\partial G}{\partial \varphi}\right)_2 & \left(\frac{\partial G}{\partial \kappa}\right)_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \left(\frac{\partial F}{\partial X_L}\right)_n & \left(\frac{\partial F}{\partial Y_L}\right)_n & \left(\frac{\partial F}{\partial Z_L}\right)_n & \left(\frac{\partial F}{\partial \omega}\right)_n & \left(\frac{\partial F}{\partial \varphi}\right)_n & \left(\frac{\partial F}{\partial \kappa}\right)_n \\ \left(\frac{\partial G}{\partial X_L}\right)_n & \left(\frac{\partial G}{\partial Y_L}\right)_n & \left(\frac{\partial G}{\partial Z_L}\right)_n & \left(\frac{\partial G}{\partial \omega}\right)_n & \left(\frac{\partial G}{\partial \varphi}\right)_n & \left(\frac{\partial G}{\partial \kappa}\right)_n \end{bmatrix} \tag{2.21}$$

P is the weight matrix, usually taken as an identity matrix (i.e. the diagonal elements of the matrix are 1s). Its dimension is 2n × 2n, where n is the number of control points.

L is the misclosure matrix formed as a result of the collinearity equations not equalling zero due to random errors involved in observations. Its dimension is always 2n × 1 (i.e. column vector), where n is the number of control points.

$$L = \begin{bmatrix} F_1 \\ G_1 \\ F_2 \\ G_2 \\ \vdots \\ F_n \\ G_n \end{bmatrix} \tag{2.22}$$

The result of the matrix operations gives a 6 × 1 correction vector X which has to be added to the initial approximate values to give new values. The result is iterated until the solution converges.

$$X = \begin{bmatrix} dX_L \\ dX_L \\ dX_L \\ d\omega \\ d\varphi \\ d\kappa \end{bmatrix} \tag{2.23}$$

The precision of the adjusted quantities is ascertained by calculating the a-posteriori variance thus:

$$V = AX - L \tag{2.24}$$

$$\sigma_0^2 = \frac{V^T PV}{r} \tag{2.25}$$

where:
V is the matrix of residuals;
σ_0^2 is the a-posteriori variance and r is the number of degrees of freedom in the adjustment, which usually equals the number of observations minus the number of unknowns.

The square root of σ_0^2 is referred to as the reference standard deviation (σ_0).

Standard deviations of the individual adjusted quantities are:

$$\sigma_{x_i} = \sigma_0 \sqrt{q_{x_i x_i}} \tag{2.26}$$

σ_{x_i} is the standard deviation of the ith adjusted unknown x_i, i.e. the value in the ith row of the X matrix;
σ_0 is the standard deviation of unit weight and;
$q_{x_i x_i}$ is the diagonal element in the ith row and ith column of matrix $(A^T PA)^{-1}$.

The following are the steps for solving the space resection problem.

1. Determination of a set of initial values for the unknown quantities $(X_L^0, Y_L^0, Z_L^0, \omega^0, \varphi^0, \kappa^0)$
2. Determination of the values of matrices A and L.
3. Solving the linearized equations for the differential corrections
4. Adding these corrections to the initial approximation values
5. Iterating the solution of these equations (step 2-4) until it converges.

2. 2. 1. Derivation of Expressions for Elements of A and L Matrices

From collinearity equations, assuming that the photo coordinates are refined, x_o and y_o can be neglected in the equations. This leaves us with:

$$x_a = -f \left[\frac{m_{11}(X_A - X_L) + m_{12}(Y_A - Y_L) + m_{13}(Z_A - Z_L)}{m_{31}(X_A - X_L) + m_{32}(Y_A - Y_L) + m_{33}(Z_A - Z_L)} \right] \tag{2.27}$$

$$y_a = -f \left[\frac{m_{21}(X_A - X_L) + m_{22}(Y_A - Y_L) + m_{23}(Z_A - Z_L)}{m_{31}(X_A - X_L) + m_{32}(Y_A - Y_L) + m_{33}(Z_A - Z_L)} \right] \tag{2.28}$$

Substituting (2.15), (2.16), and (2.17), into (2.27) and (2.28), we have:

$$x_a = -f \left[\frac{r}{q} \right] \tag{2.29}$$

$$y_a = -f \left[\frac{s}{q} \right] \tag{2.30}$$

Simplifying further, we have:

$$\frac{-(fr + qx_a)}{q} = 0 \tag{2.31}$$

$$\frac{-(fs + qy_a)}{q} = 0 \tag{2.32}$$

(2.31) and (2.32) are expressions for elements of L matrix.

Recalling (2.3) to (2.12), we can easily obtain our partial derivatives:

$$\frac{\partial F_x}{\partial X_L} = -\frac{x_a}{q} m_{31} - \frac{f}{q} m_{11} = -\frac{x_a}{q} \sin\varphi - \frac{f}{q} (\cos\varphi \cos\kappa) \tag{2.33}$$

$$\frac{\partial F_x}{\partial Y_L} = -\frac{x_a}{q} m_{32} - \frac{f}{q} m_{12} = \frac{x_a}{q} \sin\omega \cos\varphi - \frac{f}{q} (\cos\omega \cos\kappa + \sin\omega \sin\varphi \cos\kappa) \tag{2.34}$$

$$\begin{aligned} \frac{\partial F_x}{\partial Z_L} &= -\frac{x_a}{q} m_{33} - \frac{f}{q} m_{13} \\ &= -\frac{x_a}{q} \cos\omega \cos\varphi - \frac{f}{q} (\sin\omega \sin\kappa - \cos\omega \sin\varphi \cos\kappa) \end{aligned} \tag{2.35}$$

$$\begin{aligned} \frac{\partial F_x}{\partial \omega} = \frac{x_a}{q} &[(-\cos\omega \cos\varphi)(Y_A - Y_L) + (-\sin\omega \cos\varphi)(Z_A - Z_L)] \\ &+ \frac{f}{q} [(-\sin\omega \cos\kappa + \cos\omega \sin\varphi \cos\kappa)(Y_A - Y_L) \\ &+ (\cos\omega \sin\kappa + \sin\omega \sin\varphi \cos\kappa)(Z_A - Z_L)] \end{aligned} \tag{2.36}$$

$$\begin{aligned} \frac{\partial F_x}{\partial \varphi} = \frac{x_a}{q} &[(\cos\varphi)(X_A - X_L) + (\sin\omega \sin\varphi)(Y_A - Y_L) \\ &+ (-\cos\omega \sin\varphi)(Z_A - Z_L)] \\ &+ \frac{f}{q} [(-\sin\varphi \cos\kappa)(X_A - X_L) + (\sin\omega \cos\varphi \cos\kappa)(Y_A - Y_L) \\ &+ (-\cos\omega \cos\varphi \cos\kappa)(Z_A - Z_L)] \end{aligned} \tag{2.37}$$

$$\begin{aligned} \frac{\partial F_x}{\partial \kappa} = \frac{f}{q} &[(-\cos\varphi \sin\kappa)(X_A - X_L) + (-\cos\omega \sin\kappa - \sin\omega \sin\varphi \sin\kappa)(Y_A - Y_L) \\ &+ (\sin\omega \cos\kappa + \cos\omega \sin\varphi \sin\kappa)(Z_A - Z_L)] \end{aligned} \tag{2.38}$$

$$\frac{\partial F_y}{\partial X_L} = -\frac{y_a}{q} m_{31} - \frac{f}{q} m_{21} = -\frac{y_a}{q} \sin\varphi + \frac{f}{q} (\cos\varphi \sin\kappa) \tag{2.39}$$

$$\frac{\partial F_y}{\partial Y_L} = -\frac{y_a}{q} m_{32} - \frac{f}{q} m_{22} = \frac{y_a}{q} \sin\omega \cos\varphi - \frac{f}{q} (\cos\omega \cos\kappa + \sin\omega \sin\varphi \sin\kappa) \tag{2.40}$$

$$\begin{aligned}\frac{\partial F_y}{\partial Z_L} &= -\frac{y_a}{q} m_{33} - \frac{f}{q} m_{23} \\ &= -\frac{y_a}{q} \cos\omega\cos\varphi - \frac{f}{q}(\sin\omega\cos\kappa - \cos\omega\sin\varphi\sin\kappa)\end{aligned} \tag{2.41}$$

$$\begin{aligned}\frac{\partial F_y}{\partial \omega} &= \frac{y_a}{q}[(-\cos\omega\sin\varphi)(Y_A - Y_L) + (-\sin\omega\cos\varphi)(Z_A - Z_L)] \\ &+ \frac{f}{q}[(-\sin\omega\cos\kappa - \cos\omega\sin\varphi\sin\kappa)(Y_A - Y_L) \\ &+ (\cos\omega\cos\kappa + \sin\omega\sin\varphi\sin\kappa)(Z_A - Z_L)]\end{aligned} \tag{2.42}$$

$$\begin{aligned}\frac{\partial F_y}{\partial \varphi} &= \frac{y_a}{q}[(\cos\varphi)(X_A - X_L) + (\sin\omega\sin\varphi)(Y_A - Y_L) \\ &+ (-\cos\omega\sin\varphi)(Z_A - Z_L)] \\ &+ \frac{f}{q}[(\sin\varphi\sin\kappa)(X_A - X_L) + (-\sin\omega\cos\varphi\sin\kappa)(Y_A - Y_L) \\ &+ (\cos\omega\cos\varphi\sin\kappa)(Z_A - Z_L)]\end{aligned} \tag{2.43}$$

$$\begin{aligned}\frac{\partial F_y}{\partial \kappa} &= \frac{f}{q}[(-\cos\varphi\cos\kappa)(X_A - X_L) \\ &+ (-\cos\omega\sin\kappa - \sin\omega\sin\varphi\cos\kappa)(Y_A - Y_L) \\ &+ (-\sin\omega\sin\kappa + \cos\omega\sin\varphi\cos\kappa)(Z_A - Z_L)]\end{aligned} \tag{2.44}$$

(2.33) to (2.44) form the elements of the A matrix.

2. 2. 2. Initial Approximations

Since the non-linear collinearity equations are linearized by taking the partial derivatives of the equations with respect to the parameters (Taylor's theorem), approximations to the required parameters are needed.

Z_L^0 can be taken as the flying height of the aircraft. It can be calculated using the scale equation:

$$H = \frac{AB}{ab} f + h_{AB} \tag{2.45}$$

ω^0, φ^0 can be assumed as zero (near vertical photograph).
X_L^0, Y_L^0 and κ^0can be approximated by first calculating the ground coordinates of the image points (X_m', Y_m') using the following equations:

$$X'_m = \frac{x_m}{f}(Z_L - Z_m) \tag{2.46}$$

$$Y'_m = \frac{y_m}{f}(Z_L - Z_m) \tag{2.47}$$

x_m, y_m are the image coordinates of point m, Z_L is the approximate camera Z position, and Z_m is the Z coordinate of point m.

Using the known and computed ground coordinates, a two-dimensional conformal coordinate transformation is performed and the parameters are computed using a least-squares solution. The translation factors from this solution are used as initial approximations for, Z_L and Y_L, while the computed rotation angle is used as an approximation for κ.

The equations for 2D conformal transformation are given as:

$$\mathrm{X} = (\mathrm{S}\cos\theta)\mathrm{x} - (\mathrm{S}\sin\theta)\mathrm{y} + T_X \qquad (2.48)$$

$$\mathrm{Y} = (\mathrm{S}\sin\theta)\mathrm{x} + (\mathrm{S}\cos\theta)\mathrm{y} + T_Y \qquad (2.49)$$

Let $\mathrm{S}\cos\theta = a$, $\mathrm{S}\sin\theta = b$, $T_x = c$, and $T_y = d$

where: S is the scale, θ is the rotation angle, T_x and T_y are the translations in the X and Y directions respectively.

Adding residuals to the equations to develop observation equations, we have:

$$\mathrm{ax} - \mathrm{by} + \mathrm{c} = \mathrm{X} + v_X \qquad (2.50)$$

$$\mathrm{ay} + \mathrm{bx} + \mathrm{d} = \mathrm{Y} + v_Y \qquad (2.51)$$

The unknowns (S, θ, T_x and T_y) are found using unweighted least squares method.

Thus:

$$X = (A^T A)^{-1}(A^T L) \qquad (2.52)$$

where:

$$A = \begin{bmatrix} x_1 & -y_1 & 1 & 0 \\ y_1 & x_1 & 0 & 1 \\ x_2 & -y_2 & 1 & 0 \\ y_2 & x_2 & 0 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ x_m & -y_m & 1 & 0 \\ y_m & x_m & 0 & 1 \end{bmatrix} \qquad X = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \qquad L = \begin{bmatrix} X_1 \\ Y_1 \\ X_2 \\ Y_2 \\ \vdots \\ X_m \\ Y_m \end{bmatrix}$$

m is the number of points to be transformed.

In using the above expressions to find the initial approximations of X_L, Y_L and κ, (x,y) are the computed ground control coordinates given by the equations above, and (X,Y) are the known ground control coordinates.

The initial approximations are given as:

$X_L = T_x = c$

$Y_L = T_y = d$

$\kappa = \theta = \tan^{-1}\left(\frac{b}{a}\right)$

2. 3. Data Acquisition

An aerial photograph of part of University of Lagos (UNILAG), Akoka campus taken with the Unmanned Aerial Vehicle (UAV), with control points established on the ground within the area, and their corresponding image positions clearly marked out on the photo was obtained from a secondary source.

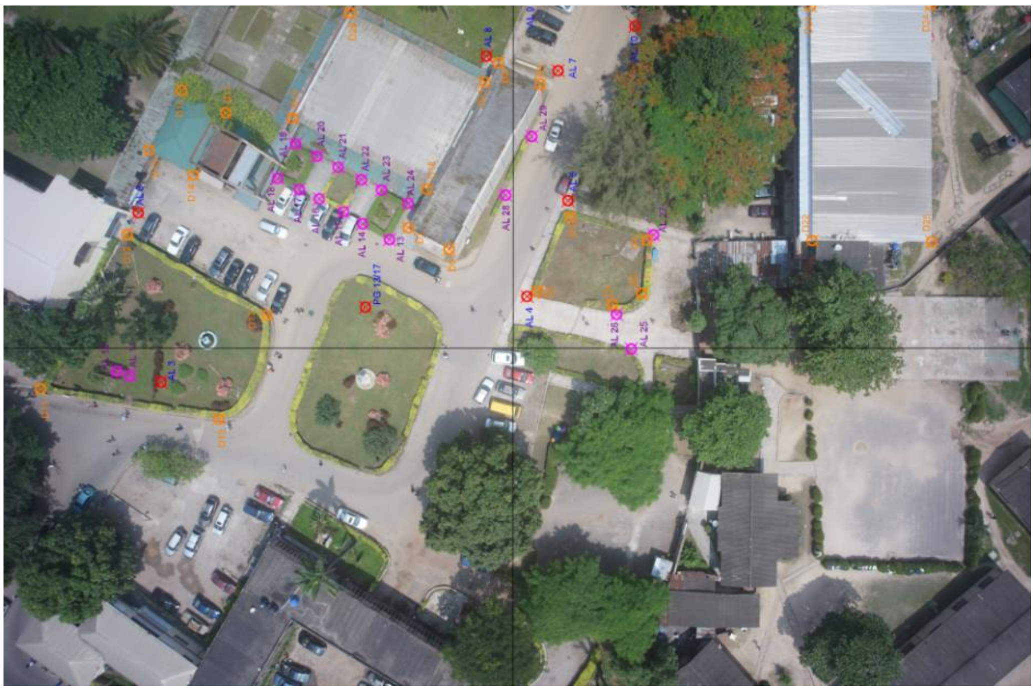

Figure 2.1. Aerial photo of part of UNILAG campus

The coordinates of the ground control points and corresponding image positions were measured and presented in the table below.

Table 2.1. Coordinates of ground controls and their corresponding image coordinates

Point ID	Ground Control Point Coordinates			Image Coordinates			
	X(m)	*Y(m)*	*Z(m)*	*X_L(mm)*	*Y_L(mm)*	*X_R(mm)*	*Y_R(mm)*
400/PG13_17	542595.566	720363.461	6.242	11.600	41.540	-61.420	30.460
402/AL3	542622.636	720383.420	6.497	-9.530	99.080	-83.510	88.610
404/AL4	542572.935	720355.142	5.929	14.62	-3.91	-57.490	-15.320
406/AL5	542571.208	720339.823	5.900	41.630	-15.460	-30.430	-26.160
408/AL6	542632.938	720359.523	5.240	38.210	105.510	-35.150	98.380

410/AL7	542577.324	720321.538	6.219	78.220	-12.690	6.760	-22.630
412/AL8	542589.016	720322.327	6.004	82.330	7.510	10.780	-1.870
414/AL9	542584.176	720311.746	5.916	98.510	-7.080	27.910	-16.320

Flying height of the UAV: 100m

2. 4. Computer Programming

The computer application for solving space resection is developed in C# programming language, which is a general-purpose, type-safe, object-oriented programming language. The figure below shows the flowchart for the adopted algorithm used for developing the program.

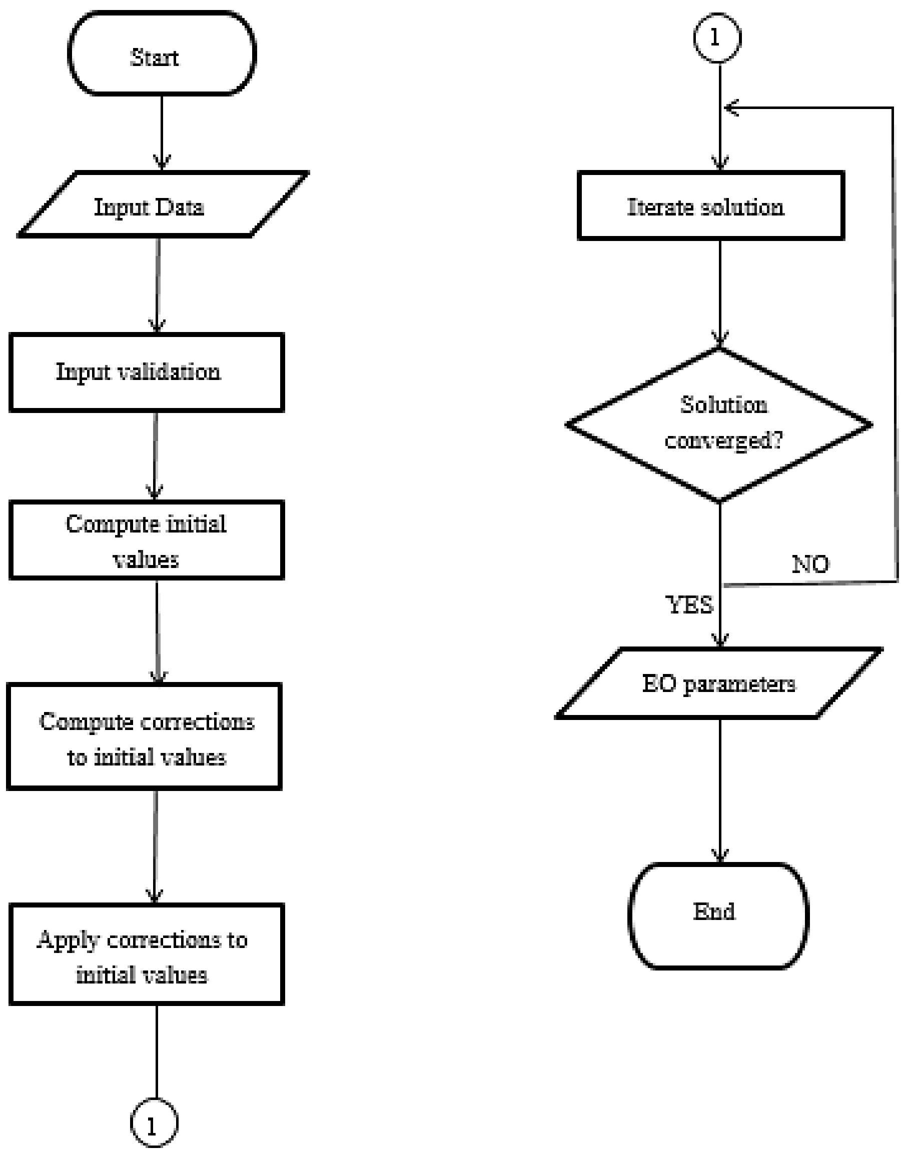

Figure 2.2. Flowchart for the computer program

3. RESULTS

Solving the space resection problem by following the steps described above, the following results were obtained.

Table 3.1. Results obtained for left and right photographs

Result		Left Photo	Right Photo
Number of Iterations		5	11
A-posteriori Variance		0.3979	0.7689
X	Initial Value (m)	542572.7699	542588.4357
	Final Value (m)	542572.6000	542590.1553
	Standard Deviation	2.1832	1.8258
Y	Initial Value (m)	720363.5228	720328.1507
	Final Value (m)	720362.5515	720323.9503
	Standard Deviation	1.8051	2.6830
Z	Initial Value (m)	93.6098	92.3372
	Final Value (m)	91.3426	88.5159
	Standard Deviation	1.4415	0.6726
Omega	Initial Value (°)	0.0000	0.0000
	Final Value (°)	0.5591	2.7867
	Standard Deviation	0.0208	0.0312
Phi	Initial Value (°)	0.0000	0.0000
	Final Value (°)	-0.0542	1.1605
	Standard Deviation	0.0257	0.0207
Kappa	Initial Value (°)	286.1562	285.1392
	Final Value (°)	286.3387	285.3822
	Standard Deviation	0.0057	0.0082

Table 3.2. Breakdown of adjustment process for the left photo.

Iteration No.		**1**	**2**	**3**	**4**	**5**
X Initial Value (m): 542572.7699	Correction (m)	-0.2261	0.0511	0.0050	5.3741E-05	4.5075E-05
	Updated Value (m)	542572.5438	542572.5949	542572.6000	542572.6000	542572.6000
Y Initial Value (m): 720363.5228	Correction (m)	-0.9191	-0.0471	-0.0049	-6.4666E-05	-3.8424E-05
	Updated Value (m)	720362.6036	720362.5565	720362.5516	720362.5515	720362.5515
Z Initial Value (m): 93.6098	Correction (m)	-2.4922	0.2087	0.0156	0.0006	8.4595E-05
	Updated Value (m)	91.1176	91.3263	91.3419	91.3425	91.3426
ω Initial Value (°): 0.0000	Correction (°)	0.5228	0.0338	0.0025	-4.9714E-06	2.5690E-05
	Updated Value (°)	0.5228	0.5565	0.5590	0.5590	0.5591
φ Initial Value (°): 0.0000	Correction (°)	-0.1560	0.0940	0.0076	0.0003	4.4858E-05
	Updated Value (°)	-0.1560	-0.0620	-0.0545	-0.0542	-0.0542
κ Initial Value (°): 286.1562	Correction (°)	0.1699	0.0119	0.0006	4.4216E-05	-4.4510E-07
	Updated Value (°)	286.3262	286.3380	286.3386	286.3387	286.3387

Table 3.3. Breakdown of adjustment process for the right photo

Iteration No.		**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
X Initial Value (m): 542588.4357	Correction (m)	0.5652	0.6398	0.3177	0.1257	0.0459	0.0163	0.0058	0.0020	0.0007	0.0002	8.9174E-05
	Update (m)	542589.0008	542589.6407	542589.9585	542590.0842	542590.1301	542590.1465	542590.1522	542590.1542	542590.1550	542590.1552	542590.1553

κ Initial Value(°): 285.1392		**φ** Initial Value (°): 0.0000		**ω** Initial Value (°): 0.0000		**Z** Initial Value (m): 92.3372		**Y** Initial Value (m): 720328.1507	
Update (°)	Correction (°)	Update (°)	Correction (°)	Update (°)	Correction (°)	Update (m)	Correction (m)	Update (m)	Correction (m)
285.1747	0.0356	0.3023	0.3023	1.6196	1.6196	89.5987	-2.7385	720325.5467	-2.6040
285.2791	0.1044	0.7926	0.4903	2.3431	0.7235	88.8851	-0.7136	720324.5655	-0.9812
285.3399	0.0608	1.0239	0.2313	2.6265	0.2834	88.6365	-0.2485	720324.1716	-0.3939
285.3663	0.0264	1.1120	0.0880	2.7299	0.1035	88.5551	-0.0814	720324.0285	-0.1432
285.3765	0.0101	1.1435	0.0315	2.7667	0.0368	88.5291	-0.0261	720323.9778	-0.0507
285.3802	0.0037	1.1546	0.0111	2.7797	0.0130	88.5204	-0.0086	720323.9599	-0.0178
285.3815	0.0013	1.1584	0.0039	2.7843	0.0046	88.5175	-0.0029	720323.9536	-0.0063
285.3820	0.0005	1.1598	0.0014	2.7859	0.0016	88.5165	-0.0010	720323.9514	-0.0022
285.3821	0.0002	1.1603	0.0005	2.7864	0.0006	88.5161	-0.0004	720323.9506	-0.0008
285.3822	5.7532E-05	1.1605	0.0002	2.7866	0.0002	88.5160	-0.0001	720323.9504	-0.0003
285.3822	2.0302E-05	1.1605	6.0055E-05	2.7867	7.1017E-05	88.5159	-4.5385E-05	720323.9503	-9.7580E-05

Given below are the convergence graphs of the solutions. Plotting the X, Y and Z corrections against the iteration number resulted in the linear convergence graph. In the same manner, plotting the omega, phi and kappa corrections against the iteration number resulted in angular convergence graph. Thus:

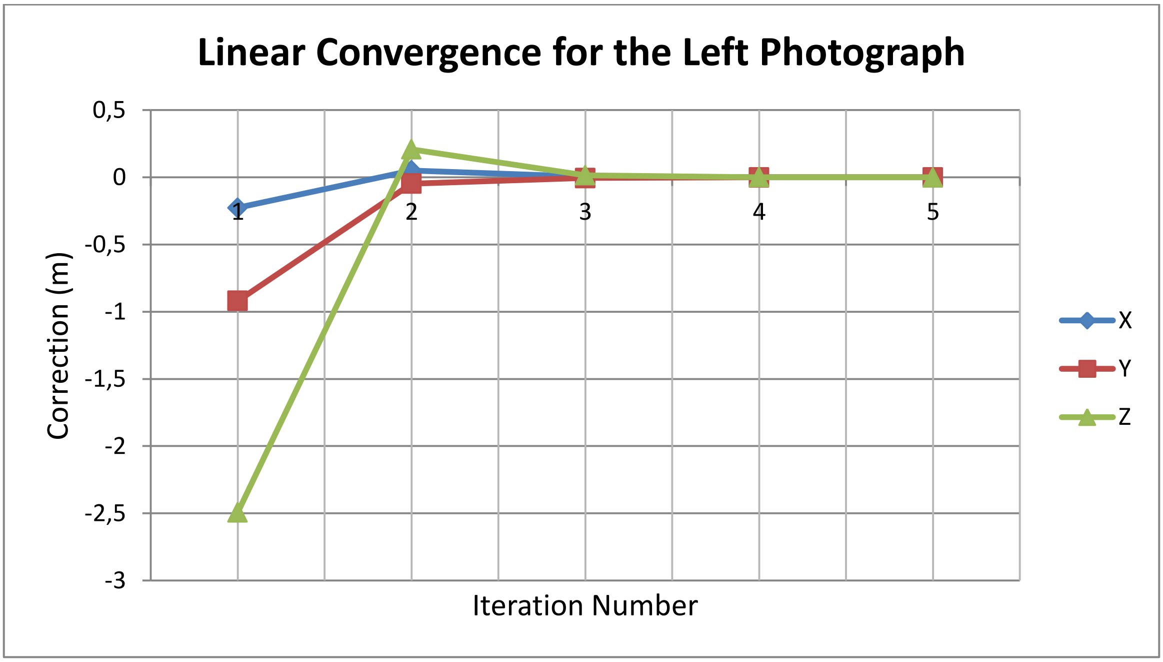

Figure 3.1. Linear Convergence Graph for the left photograph

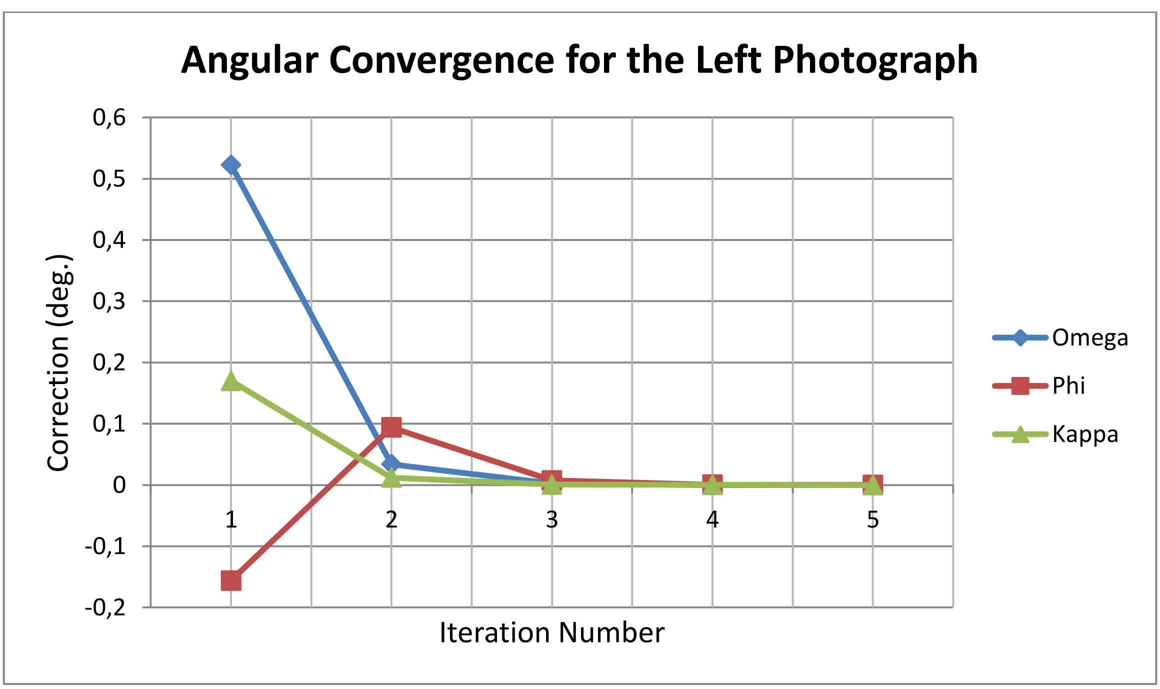

Figure 3.2. Angular Convergence Graph for the left photograph

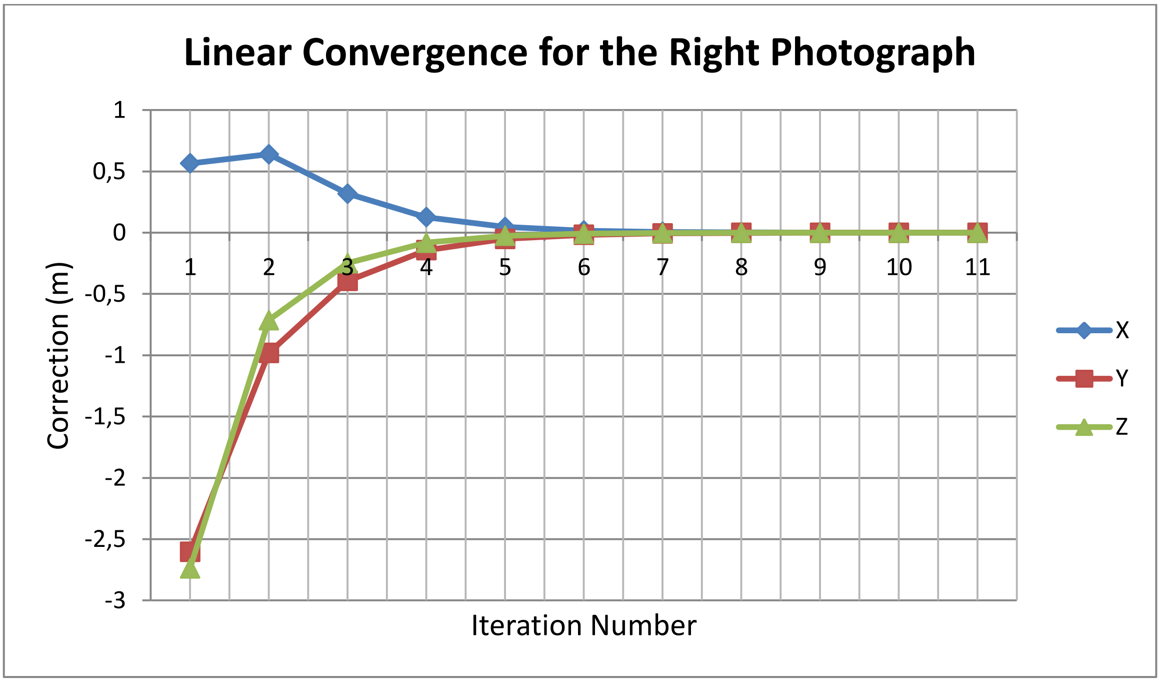

Figure 3.3. Linear Convergence Graph for the right photograph

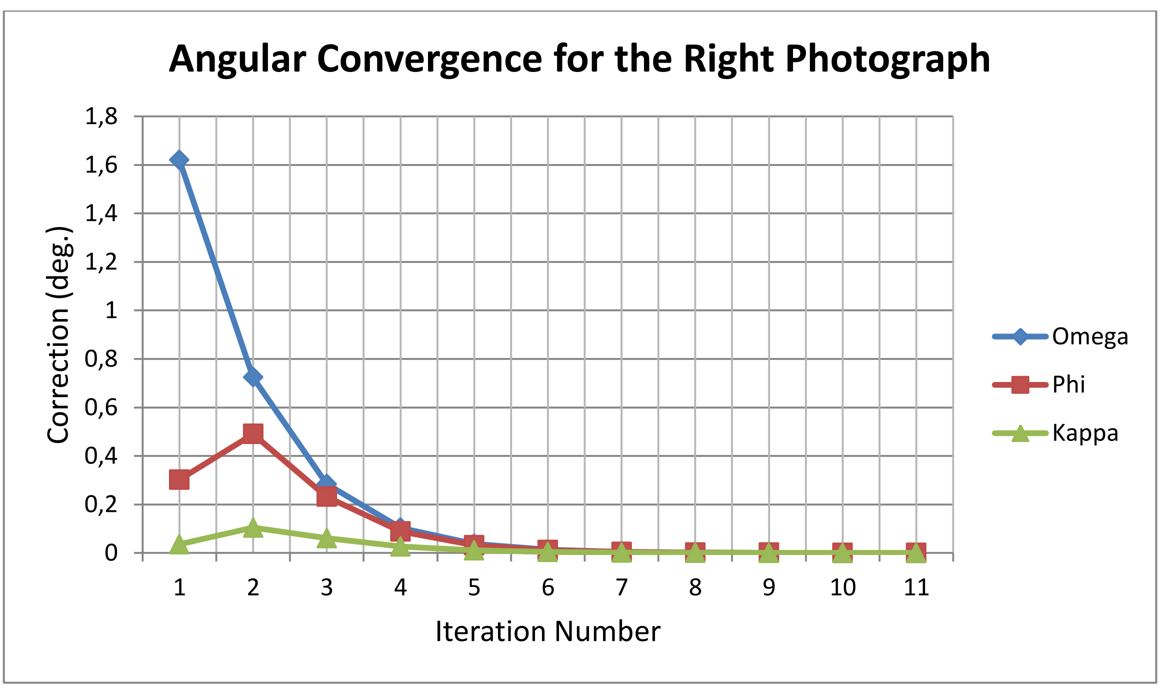

Figure 3.4. Angular Convergence Graph for the right photograph

4. CONCLUSION

The recovery of exterior orientation parameters of a camera during exposure is a necessity in analytical photogrammetry. This task, undertaken with the aim of carrying out space resection computation for the aerial camera was a complete success. The aim was achieved and the result obtained was satisfactory.

The minimum standard error of adjusted parameter obtained for the left image was 0.0057 (standard error for kappa) and the maximum was 2.1832 (standard error for X). For the right image, the minimum was 0.0082 (standard error for kappa) and the maximum was 2.6830 (standard error for Y).

In the absence of known exterior orientation parameters to compare the obtained results with, the accuracy of the values obtained can only be reliably verified if the values are used to compute point coordinates through spatial intersection. This would of course presuppose that the intersection calculations are without errors. Such a test is outside the scope of this paper.

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