Energodynamic Substantiaton of the Principle Least Action

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ABSTRACT

It is shown that the absence of a justification for the principle of least action is due to attempts to do this on the basis of the mechanics of conservative systems. It is proposed to substantiate it from the most general positions of energodynamics as the nonequilibrium thermodynamics of multivariant energy-transforming systems, which for the first time introduced specific parameters of the spatial inhomogeneity of such systems as a measure of their deviation from equilibrium. With this approach, the principle of least action becomes a consequence of the aspiration of such systems to equilibrium and the ensuing condition of a minimum moment of momentum distribution in an inhomogeneous velocity field, i.e. work required to maintain the moving system in a stationary nonequilibrium state throughout the process. This determines the validity of this principle for non-conservative systems and for all forms of energy, which greatly expands the scope of its applicability and makes it a universal tool for analyzing real processes.

Keywords: the principle of least action, nonequilibrium systems, dissipation and work "against balance", criteria of equilibrium and stationarity, parameters of heterogeneity, evolution and involution

1. INTRODUCTION

The history of establishing the principle of least action (PLA) dates back to that period in the development of the natural sciences, when such concepts as force, impulse of force,

work, power, energy still did not exist, and the very notion of "action" was uncertain. and the law of its conservation. This principle was first formulated by P. Maupertuis in 1744, proceeding from the theological notions of that time that all processes occurring in nature take place for a specific purpose and proceed in the most rational (economical) way [1]. Only after a rather considerable time by the efforts of mathematicians (L. Euler [2], J. Lagrange [3], etc.), this principle acquired a concrete analytic expression, which in modern language corresponds to the assertion that for the real path of a material point in a conservative force field, the integral of momentum of a particle, taken along a segment of a trajectory between any two of its points, is minimal in comparison with the same integrals taken over segments of other curves.

This and other formulations of the above-mentioned principle did not come from the physical meaning of "action" or from any fundamental laws of natural science, but were based more on conviction in the rationality of the laws of nature, by which it sets goals and finds the simplest means for achieving them. Thus, Laplace believed that "the true purpose of nature is the saving of manpower" (ie, in modern terminology, work). The same point of view was also taken by Lagrange, who believed that this principle "with more justification should be called the principle of extreme manpower" [3].

The first person who gave the principle of least action the status of a general law of mechanics was Helmholtz [4]. Retaining the essence of the principle, unlike other researchers, he took as the initial, primary value the Lagrangian function of the state of the object of investigation L as the difference between his kinetic energy E^k and the potential energy E^p. This function was expressed in terms of the generalized coordinates $\mathbf{r}_i(t)$ and momenta $\mathbf{p}_i(t)$ of all N particles of the system ($i = 1,2, ..., n$) considered as a function of time, which made the Lagrangian $L[\mathbf{r}_i, \mathbf{p}_i, t]$ time t. In accordance with this, the principle of least action began to be written down in the form of the requirement of minimality of some functional

$$\hat{S}(t) = \int L[\mathbf{r}_i(t), \mathbf{p}_i(t)]dt = \min. \tag{1}$$

From the properties of the extremum of this function, Helmholtz succeeded in deriving the laws of motion of a number of systems. Gradually, his idea of "finding wordings for the laws of new classes of phenomena" [4] and the ability to derive all the basic laws of classical physics from a single mathematical construction called action, made this method one of the most widely used and most important physical principles. Thus, M. Planck considered this principle to be a "higher physical law" [5], more universal than the energy-momentum conservation principle [6, p. 85]. A. Einstein believed that the general theory of relativity can be developed on the basis of only this variational principle [7, p. 524]. According to R. Feynman, even the fundamental relationship between the laws of symmetry and conservation, established by E. Noether, is based on the principle of minimum action [8, p. 93]. In the skilful hands of physicists and mathematicians, this principle has been successfully applied not only in mechanics, but also in most sections of theoretical physics [9]. Today, extreme principles close to the PLA play a significant role in the fields of knowledge that are not related either to mechanical motion, to geometry, or to the concept of "action", including in biology, information theory, and optimal control problems.

Nevertheless, up to now, not only the attempts to derive the PLA from the general principles of the mechanics of conservative systems, but even to understand the physical meaning of the Lagrange function, have not been successful. Until now, this principle

operates with concepts more familiar to philosophers than to natural scientists. It is not accidental that Feynman's opinion that, in its essence, the principle of least action is a philosophical principle [10, p. 18].

Therefore, it is of interest to show that the PLA could not be obtained on the basis of the mechanics of conservative systems, since the latter deals with internally equilibrium systems and excludes dissipative processes that lead steady flows to the state closest to equilibrium.

This becomes quite obvious only with the introduction into the nonequilibrium thermodynamics of the missing extensive spatial inhomogeneity parameters associated with temperature gradients, pressures, chemical, electrical, etc. potentials as well as these potentials themselves - with entropy, volume, number of moles of k-th substances, charge, etc. [11]. The purpose of this article is to show this most clearly.

2. METHODOLOGY OF THE ENERGODYNAMIC APPROACH TO NONEQUILIBRIUM SYSTEMS

The simplest way of describing the proposed approach is to proceed from a unified theory of transport processes and the transformation of any forms of energy, the foundations of which are laid down in [11]. In applied terms, this theory represents a generalization of equilibrium [12,13] and nonequilibrium [14-16] thermodynamics, first to non-static transport processes of various forms of energy [17], and then to real processes of their transformation [18]. This made it possible to synthesize thermodynamics, hydrodynamics and electrodynamics with mechanics, obtaining as its consequences all of their basic laws and equations [19], and to substantiate the unity of the laws of transformation of thermal and nonthermal forms of energy, developing on this basis the theory of similarity of energy-transforming systems [20].
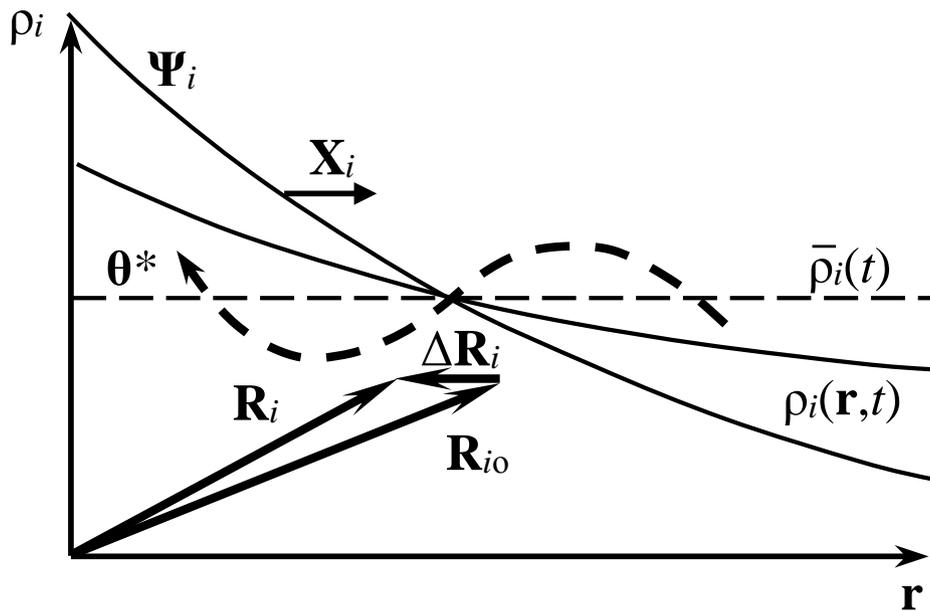

Fig. 1. To the formation of the moment distribution.

In methodological terms, energy dynamics is similar to classical thermodynamics, which is based on the principles of the excluded perpetual motion machine, i.e. adheres to the method of principles and the system approach, which is a kind of deductive research method ("from the general to the particular"). Like classical thermodynamics, energy dynamics operates on the parameters of the system as a whole, considering its internal (own) energy U as the characteristic function of the carriers of its various i-forms Θ_i (mass M, moles kth of substances N_k, entropy S, charge Q, momentum components $\mathbf{P}$, its moment $\mathbf{L}$, etc.). However, in contrast to the existing theories [14-16], it does not rely on the local equilibrium hypothesis and does not divide the inhomogeneous systems into an infinite set of conditionally homogeneous volume elements.

Instead, it introduces specific parameters for the spatial heterogeneity of such systems as a whole. They are moments of the distribution $\mathbf{Z}_i$ of the parameters Θ_i as additional characteristic functions of the state of such systems [21]. To understand their meaning, let us consider a medium with the density $\rho_i(\mathbf{r},t) = \partial\Theta_i/\partial V$ continuously distributed over its volume V of any extensive quantity Θi at an arbitrary time instant t (Fig. 1). This distribution corresponds to the curves $\rho_i(\mathbf{r},t)$, which are presented for clarity as a function of the radius vector of the point of the field $\mathbf{r}$.

As follows from the figure, if the distribution of Θi deviates from a homogeneous one with density, some quantity of this quantity Θi * is transferred from one part of the system to another in the direction indicated by the arrow. This "redistribution" causes the center of this magnitude to shift from the initial position of $\mathbf{R}_{io}$ to the current $\mathbf{R}_i$, determined in a known way:

$$\mathbf{R}_{io} = \Theta_i^{-1}\int\bar{\rho}_i(t); \quad \mathbf{R}_i = \Theta_i^{-1}\int\rho_i(\mathbf{r},t)\,\mathbf{r}dV. \tag{1}$$

Comparing these two propositions, we find that the state of the inhomogeneous system is characterized by the appearance of specific "distribution moments" $\mathbf{Z}_i$ of energy carriers Θ_i [12]:

$$\mathbf{Z}_i = \Theta_i\Delta\mathbf{R}_i = \int_V\left[\rho_i(\mathbf{r},t) - \bar{\rho}_i(t)\right]\mathbf{r}dV. \tag{2}$$

where: $\Delta\mathbf{R}_i = \mathbf{R}_i - \mathbf{R}_{io}$ is the deviation of the center of the value of Θ_i from its position at equilibrium, which we called the "displacement vector"; $\mathbf{r}$ - "running" (Euler) coordinate of the point of the field; $i = 1,2, ..., n$ is the number of energy forms in the system; t is time.

Setting $\mathbf{R}_{io} = 0$, we represent the total time derivative of the displacement vector $\mathbf{R}_i$ by the known expression:

$$d\mathbf{R}_i/dt \equiv \mathbf{v}_i = \mathbf{w}_i + \mathbf{\omega}_i\times\mathbf{R}_i, \tag{3}$$

where: $\mathbf{w} = d\mathbf{r}_i/dt = \mathbf{e}\,d|\mathbf{R}_i|/dt$ is the translational component of the transfer velocity $\mathbf{v}_i$ of the center of the quantity Θ_i; $\mathbf{e}$ is the unit vector in the direction of this transfer; $\mathbf{\omega}_i = d\mathbf{\varphi}_i/dt$ is the angular velocity of rotation of the vector $\mathbf{R}_i$; $\mathbf{\varphi}_i$ is the spatial angle of its orientation.

It immediately follows that the total derivative of the moment of the distribution $\mathbf{Z}_i$ of any energy carrier Θ_i has the meaning of its momentum $\mathbf{P}_i = \Theta_i\mathbf{v}_i$, and its partial derivative

under the conditions of the constancy of the direction of its transfer (angle φ_i) is the meaning of its flux

$$\mathbf{J}_i = (\partial \mathbf{Z}_i/\partial t)_\varphi = \Theta_i \mathbf{w}_i. \tag{4}$$

From the analogous term in Onsager's theory J_i as derivatives with respect to time from certain parameters of the deviation of the system from the state of equilibrium, this quantity differs in its unambiguous meaning and vector nature, which makes it impossible to identify it with the concept of "energy consumption" or "generalized speed" of the i-th process $d\Theta_i/dt$.

Being a function of three independent parameters $\Theta_i d\mathbf{r}_i \Theta_i$, $\mathbf{r}_i$ и φ_i, the distribution time $\mathbf{Z}_i$ can vary in the course of three independent processes:

$$d\mathbf{Z}_i = \mathbf{r}_i\,d\Theta_i + d\varphi_i \times \mathbf{Z}_i. \tag{5}$$

The first of these processes is due to the introduction into the system of the energy carrier Θ_i, (mass, k-th substance, charge, volume, entropy, momentum, etc.) from the environment while maintaining their distribution throughout the system; the second is the process of their "redistribution" according to the system, i.e. its "polarization", understood in the most general sense as the creation in it of a spatial heterogeneity; the third is the process of "reorientation" of the energy carrier Θ_i, (including its rotation with the system) [21].

According to the theorem proved in energy dynamics, the number of independent arguments of the energy U of any system is equal to the number of independent processes taking place in it. This means that if we like the i-th form of the U_i energy of the system $U = \Sigma_i U_i$ as a function of the state of the system has the form $U_i = U_i (\Theta_i, \mathbf{r}_i$ и $\varphi_i)$, and the total energy differential of the system dU can be represented in the form of the identity [18,21]:

$$dU \equiv \Sigma_i \Psi_i d\Theta_i - \Sigma_i \mathbf{F}_i \cdot d\mathbf{r}_i - \Sigma_i \mathbf{M}_i \cdot d\varphi_i, \tag{6}$$

where: $\Psi_i \equiv (\partial U/\partial \Theta_i)$ are the averaged over the system values of its generalized potentials (absolute temperature T and pressure p, chemical μ_k, electric φ, gravitational ψ_g, etc. potential); $\mathbf{F}_i \equiv - (\partial U/\partial \mathbf{r}_i)$ - forces in their general physical sense; $\mathbf{M}_i \equiv - (\partial U/\partial \varphi_i)$ are the moments of these forces.

Identity (6) generalizes the concept of work, allowing to distinguish between three categories of work performed by the same i-th form of energy: the "input work" of the energy carrier $dW^{\text{в}} = \Psi_i d\Theta_i$ [12,18]; "useful" (technical) work $dW^{\text{в}} = \Psi_i d\Theta_i$, and the work of "reorientation" (rotation) of the system $dW^{\text{т}} = \Sigma_i \mathbf{M}_i \cdot d\varphi_i$.

Another fundamentally important difference in energy dynamics is the introduction into thermodynamics of an initially alien concept of the resultant force $\mathbf{F} = \Sigma_i \mathbf{F}_i$ in its classical (Newtonian) understanding. In this case, energy dynamics generalizes this concept, giving a unified definition of forces of any nature Fi as a gradient of the corresponding form of energy taken with the opposite sign. This not only reveals the diversity of forces in nature, but also their unity. Thanks to this unity, it is possible to synthesize a number of technical disciplines that study the dynamics of various processes [19]. In particular, the reason for the absence of the concept of force in equilibrium thermodynamics becomes obvious, where all $\mathbf{Z}_i$ and $\mathbf{R}_i = 0$. Here is also the "key" to understanding the meaning of the term "thermodynamic" force $\mathbf{X}_i$. Indeed, since for $d\mathbf{R}_i = d\mathbf{r}_i$ the identity (6) takes the form:

$$dU \equiv \Sigma_i \Psi_i d\Theta_i - \Sigma_i \mathbf{X}_i \cdot d\mathbf{Z}_i, \tag{7}$$

then

$$\mathbf{X}_i \equiv - (\partial U / \partial \mathbf{Z}_i) = - \Theta_i^{-1} (\partial U / \partial \mathbf{r}_i) = \mathbf{F}_i / \Theta_i. \tag{8}$$

It follows that the so-called "thermodynamic force" X_i, which does not have an unambiguous meaning in nonequilibrium thermodynamics (TIP) and in energodynamics is the specific force $\mathbf{F}_i$ in its usual sense, and for "internal" forces generalizes the concept of "strength" of long-range force fields (electric, magnetic, gravitational) on the temperature, pressure, chemical, etc. fields. potentials, also expressed in terms of their negative gradients [18]. The uniqueness of the meaning of the thermodynamic force $\mathbf{X}_i$ and the flow $\mathbf{J}_i$ in the identity (6) obtained in this way eliminates the known arbitrariness in their choice on the basis of the expression "entropy production" dS/dt, due to the ambiguous splitting of it into the factors X_i and $J_i^{1)}$. This eliminates the need to compose cumbersome entropy balance equations used in consumer goods to find them [15,16]. However, this uniqueness is even more important when finding the criteria for the evolution, equilibrium, and stability of thermodynamic systems.

3. THE ENERGODYNAMIC THEORY OF THE EVOLUTION AND STABILITY OF SYSTEMS

The fundamental difference of energy dynamics from equilibrium and nonequilibrium thermodynamics is the refusal to use entropy as a criterion of equilibrium and irreversibility. C positions of the theory of evolution, understood as the development of systems, their complexity and the acquisition of new properties by them, it is difficult to find a more unsuccessful parameter for analyzing its problems than the entropy of R. Clausius. Indeed, this parameter remains, as we know, unchanged, when the system as a whole moves away from equilibrium by performing a useful (reversible) work $dW^{\textpi}$ over it. In the absence of heat transfer, this entropy can only increase; can reflect only the degradation (involution) of the system, which is confirmed by the second law of thermodynamics. This is what gave Prigogine the reason to declare "a glaring contradiction of the second principle with the theory of the evolution of biological systems" [22].

Realizing this, many researchers try to substitute the Clausius thermodynamic entropy [12,13] with Boltzmann statistico-mechanical entropy, which has the sense of chaos measure or impart an even deeper meaning to the Shannon measure of information uncertainty [13, 23], which in principle can vary as in that, and in the other direction. However, neither one nor the other does not remove the contradictions noted by I. Prigogine, since they have a different meaning and are not related to the concept of heat exchange and work as Clausius

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[1] In particular, only for diffusion processes of k-th substance in multicomponent systems separated by a membrane or a valve, TIP allows selection as the driving forces of the difference in the concentration of this substance Δc_k, its enthalpy - Δh_k, chemical potential $-\Delta \mu_k$; their negative gradients $- \nabla c_k$; $- \nabla h_k$; $-\nabla \mu_k$; and their functions: $- \Delta(c_k/T)$ and $- \Delta c_k/T$; $- \Delta(h_k/T)$ and $- \Delta h_k/T$; $- \Delta(\mu_k/T)$ and $- \Delta \mu_k/T$ and their negative gradients $- \nabla(c_k/T)$ and $- \nabla c_k/T$; $- \nabla(h_{k:}/T)$ and $\nabla h_k /T$); $- \nabla(\mu_k/T)$ and; $-\nabla \mu_k/T$.

entropy. In addition, the increase in entropy reflects only the effect, and not the cause of irreversibility. It does not indicate to what extent this or that degree of freedom of the system degrades, since it characterizes only the general result of "any and all" irreversibility. Finally, the entropy criteria of evolution can not reflect the existence of a "partial" (incomplete) equilibrium, characterized by the cessation of only some of the processes taking place in the system. It is known that equilibrium never occurs simultaneously over all degrees of freedom of a multivariant system. Due to the hierarchy of relaxation times [24], first comes the equilibrium of any one (i-th) kind, characterized by the cessation of the i-th process, then - the equilibrium of the j-th kind, etc. However, the principle of increasing entropy can not reflect this feature of multivariant systems. Even in TIP, the disappearance of any of the forces X_i does not mean the disappearance of the associated flow Ji (i.e., the termination of the i-th process), since in accordance with the Onsager postulate, each of them disappears only when all the thermodynamic forces acting in the system X_i [14,15].

The use of entropy as a criterion of irreversibility was due historically to the absence at that time of other coordinates that did not obey the conservation law in isolated systems, and also the limitations of classical thermodynamics by equilibrium processes for which the concept of the $\mathbf{F}_i \equiv - (\partial U/\partial \mathbf{r}_i)$ is meaningless in view of $d\mathbf{r}_i = d\mathbf{R}_i = 0$. Meanwhile, the equilibrium theory, first developed by Lagrange [3], was based on the concept of force. According to it, the mechanical system is in equilibrium, if the sum of the work of all forces for any virtual displacement of the system is zero. It would seem that in the thermodynamics of irreversible processes (TIP), also using the concept of force, the equilibrium condition should be expressed through forces. However, not only the "quasi-thermodynamics" of Onsager [25] and the nonequilibrium thermodynamics of I. Gyamati [15] based on variational principles, but also "extended" irreversible thermodynamics [16] did not go along this natural path.

By introducing uniformly the forces and their moments as derivatives of energy according to the corresponding generalized coordinates, the energy dynamics corrects this situation. In this case, entropy occupies the proper place for the extensive measure of thermal motion, which has the meaning of a "thermoimpulse" (particle momentum that has lost its vector nature due to the chaotic nature of the thermal motion) and conjugate with temperature, just like a mechanical impulse - with the speed of ordered motion [19]. The moment of its distribution is one of the set of parameters $\mathbf{Z}_i$ characterizing the removal of the system as a whole from equilibrium of a given kind. Derivatives of such parameters with respect to time $\mathbf{J}_i = d\mathbf{Z}_i/dt$ in isolated systems determine the rate of removal of the system from internal thermal equilibrium or approximation to it. In this case, the entropy pulse $\mathbf{J}_s = d\mathbf{Z}_s/dt = S\mathbf{v}_s$, like any other flow $\mathbf{J}_i$, indicates the nature of the irreversible process and the speed of energy dissipation caused by it $\Sigma_j\mathbf{X}_i\cdot\mathbf{J}_i$, and its direct cause $\mathbf{X}_j$. From this standpoint, the attempt of nonequilibrium thermodynamics to apply entropy as a criterion of "any and all" irreversibility looks at least strange.

The concept of thermodynamic force as an intensive measure of nonequilibrium, energodynamics brings the thermodynamic theory of equilibrium closer to the mechanical one. In this case, the equilibrium criterion of the i-th kind becomes, as usual, the vanishing of the same force $\mathbf{X}_i$, which in accordance with the identity (6) corresponds to the termination of the i-th process, even in the presence of other j-forces $\mathbf{X}_j$. The work $đW^{\pi} = \mathbf{X}_i\cdot d\mathbf{Z}_i$ performed here is expressed by the second sum of the identity (6) and is used not only to overcome the "scattering" forces, but also to the useful work "against equilibrium" in other degrees of

freedom of the system, which removes them from equilibrium. Thus, energy dynamics, unlike the "pseudo-thermostatics" of Thompson [26] and the "quasithermodynamics of L. Onzager [25], does not exclude from consideration any (irreversible or reversible) part of the real processes, which makes it suitable for any non-conservative systems.

At the same time, the dependence of the work of $đW^\text{т}$ on the nature of the process (scattering forces) makes it dependent on the path (character) of the process and deprives the $đW^\text{т}$ properties of the total differential[1], it is more convenient to express the equilibrium condition directly by vanishing the corresponding moment $\mathbf{Z}_i$, characterizing, as shown above, from the whole on the inner balance of the i-th kind. This makes it easy to distinguish the evolution of the system from its involution, understood as degradation (loss of efficiency), not only of the system as a whole, but of each of its degrees of freedom separately:

$$d\mathbf{Z}_i > 0 \text{ (evolution)}; \quad d\mathbf{Z}_i < 0 \text{ (involution)}. \tag{9}$$

It is equally easy to distinguish a state of equilibrium, characterized by the termination of the process, from the stationary state inherent in the established processes:

$$d\mathbf{Z}_i = 0 \text{ for } \mathbf{J}_i = 0 \text{ (equilibrium)}; \quad d\mathbf{Z}_i = 0 \text{ for } \mathbf{J}_i \neq 0 \text{ (stationary state)}. \tag{10}$$

This state is stable if the deviation of the parameter $\mathbf{Z}_i$ has reached its minimum, since the deviation from it requires the cost of some work $đW^\text{т}$:

$$d^2\mathbf{Z}_i > 0; \quad d\mathbf{J}_i/dt > 0. \tag{11}$$

Thus, the condition for stationarity of any i-th process (its steady-state nature) is the minimum of the corresponding moment of distribution $\mathbf{Z}_i$ of the energy carrier Θ_i. It follows that if the system remains stable, evolving in a certain time interval Δt from the equilibrium state to the current nonequilibrium state, then the change in the moment $\mathbf{Z}_i$ will be also minimal in this case. Taking the initial time instant $t = 0$, this condition can be expressed by the integral:

$$\mathbf{Z}_i = \int_o^t \mathbf{J}_i \, dt = \min. \tag{12}$$

This condition has a universal character and is valid for any form of energy. Below we show that the principle of least action is a particular case of this rather obvious condition.

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[1] This is emphasized by the sign of the incomplete differential $đ$.

4. THE PRINCIPLE OF LEAST ACTION AS CONSEQUENCE OF ENERGODYNAMIC EVOLUTIONARY CRITERIA

We apply the energy-dynamic method of analyzing partial equilibria to the justification of the principle of least action. To this end, we take into account that for problems related to the study of the motion of material bodies, the moment of distribution $\mathbf{Z_p}$ of the impulse $\mathbf{P}$ plays the role of the inhomogeneity parameter. The formation of this moment does not depend on what it is caused by: external long-range forces $\mathbf{F}_i$ or internal stresses (thermodynamic forces $\mathbf{X}_i$). Therefore, the energy-dynamic approach is applicable to both closed and non-closed systems.

We now turn our attention to the fact that in the case of the moment of momentum distribution $\mathbf{Z_p}$, its flux $\mathbf{J_p} \equiv (\partial Z_{\mathbf{p}}/\partial t) = \mathbf{P}\upsilon = 2E^{\kappa}$, i.e. is the double value of the kinetic energy of the system $E^{\kappa} = M\upsilon^2$, where υ is the average flow velocity. In accordance with this, the momentum distribution in the flow $Z_{\mathbf{p}} = |\mathbf{Z_p}|$ is a scalar, and in accordance with (12), the criterion for the stability of the system's motion becomes the expression:

$$\mathbf{Z_p} = \smallint \mathbf{J_p}dt = \min, \qquad (13)$$

It is easy to see that the integrand in (13) has the meaning of Maupertuis's "action" [1]. Thus the principle of the least action of Maupertuis appears to be a special case of the energy dynamic criteria of evolution in their application to mechanical motion:

$$\hat{S}(t) = Z_{\mathbf{p}}/2 = \smallint E^{\kappa}dt = \min. \qquad (14)$$

Since under stationary conditions the potential energy of the system E^{π} remains unchanged, expression (18) can be extended to the Lagrange function $L = E^{\kappa} - E^{\pi}$, and to the Hamiltonian function as their sum. In this case, it will still express the condition for the minimum of the moment modulus of momentum distribution $\mathbf{Z_p}$ throughout the entire process. This means that, thanks to dissipation, from all the trajectories of the system's motion, "chooses" the one at which it remains closer to equilibrium, which, under conditions of unchanged external "compulsion", corresponds to the established character of the movement. This removes any cover of "mystery" with the PLA.

Moreover, it becomes clear that the PLA has nothing to do with the notion of "action" in mechanics as the product of the force $\mathbf{F}_i$ for the time Δt of its action, since the parameter $\mathbf{Z_p}$ is a function of the state, and not of the process, and remains unchanged under stationary conditions. This does not at all mean that its magnitude can not be different when the external "coercion" changes. A clear example is the change in the velocity profile in a viscous fluid flow from laminar to turbulent with an increase in the gradient or pressure gradient (the thermodynamic force $\mathbf{X_p} = \mathbf{F}_i/M$ in the same channel section. In this case, the ratio of the maximum velocity in the core of the flow to its average value υ, which determines along with it the value of $\mathbf{Z_p}$ as a measure of the inhomogeneity of the velocity field, depends on the "coercive" force $\mathbf{X_p}$. It is quite natural that $\mathbf{Z_p}$ remains unchanged with its constancy and increases with its increase according to the law inherent in laminar or turbulent motion. This law can be obtained both from experiment and by the successful selection (construction) of the integrand in (15), so that its variation leads to the result corresponding to the experiment.

Fundamentally new in the proposed justification for the principle of least action is the fact that it reveals the groundlessness of the assertion that the PLA is not applicable to non-

conservative systems. In contrast, the energy dynamics asserts that the minimum of any of the moments of the distribution of $\mathbf{Z}_i$, including $\mathbf{Z_p}$, is due precisely to dissipation. Thus, it emphasizes the true meaning of this principle as a condition for a minimum of coercion that maintains a dynamic system in a state of steady motion, which corresponds to the minimum value of its deviation from the state of internal equilibrium. This is what determines the striking universality of the principle of least action, which can not be explained from the standpoint of the mechanics of conservative systems. This eliminates any obstacles to applying the principle of least action to real processes.

5. CONCLUSIONS

1. The proposed energy-dynamic justification for the principle of least action (PLA) removes from it a veil of mystery and makes it a natural consequence of the system's striving for equilibrium. At the same time, the meaning of the concept "action" and its relation to the measurable parameters of the system are revealed;

2. The lack of evidence of the PLA is due to attempts to do this on the basis of the classical mechanics of conservative systems. A more general approach from the standpoint of energy dynamics makes it possible to obtain it as a special case of energy criteria for the evolution of inhomogeneous systems;

3. From the energyodynamic point of view, the PLA expresses the condition of minimality of the "anti-equilibrium" operation required to maintain the flow in a stationary nonequilibrium state (with an inhomogeneous velocity field);

4. The energodynamic method of justifying the HDPE from the condition of a minimum of dissipation determines the validity of this principle for non-conservative systems, which considerably expands its applicability;

5. Universality of the PLA is due to the unity of the energy criteria of evolution, equilibrium and stability for all forms of energy of material systems, regardless of the degree of their removal from equilibrium and complexity. This makes the PLA a reliable tool for analyzing nonequilibrium systems.

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