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Generalized nonsingular model for compact stars electrically charged

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ABSTRACT

We found new class of solutions to the Einstein-Maxwell system of equations for charged quark matter within the framework of MIT Bag Model considering a particular form for the measure of anisotropy and a metric function which depends on an adjustable parameter α. Variables as the energy density, charge density, radial pressure and the metric functions are written in terms of elementary and polynomial functions. We show that the form chosen for the metric function allows obtain physically acceptable solutions with any value of the adjustable parameter.

***Keywords*:** Adjustable parameter, Einstein-Maxwell system, anisotropy, metric functions, MIT Bag Model

1. INTRODUCTION

One of the fundamental problems in the general theory of relativity is finding exact solutions of the Einstein field equations [1,2]. Some solutions found fundamental applications in astrophysics, cosmology and more recently in the developments inspired by string theory [2]. Different mathematical formulations that allow to solve Einstein´s field equations have been used to describe the behavior of objects submitted to strong gravitational fields known as neutron stars, quasars and white dwarfs [3-5].

From the development of Einstein´s theory of general relativity, the description of compact objects has been a central issue in relativistic astrophysics in the last few decades

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[2,6]. Recent experimental observations in binary pulsars [6] suggest that could be quark stars. The existence of quark stars in hydrostatic equilibrium was first by Itoh [7] in a seminal treatment. Recently, the study of strange stars consisting of quark matter has stimulated much interest since could represent the most energetically favorable state of baryon matter.

In theoretical works of realistic stellar models, is important include the pressure anisotropy [8-10]. The existence of anisotropy within a star can be explained by the presence of a solid core, phase transitions, a type III super fluid, a pion condensation [11] or another physical phenomenon as the presence of the electrical field [12]. The physics of ultrahigh densities is not well understood and many of the strange stars studies have been performed within the framework of the MIT-Bag model [13]. In this model, the strange matter equation of state has a simple linear form given by $p = \frac{1}{3}(\rho - 4B)$ where ρ is the energy density, p is the isotropic pressure and B is the bag constant. Many researchers have used a great variety of mathematical techniques to try to obtain exact solutions for quark stars within the framework of MIT-Bag model; Komathiraj and Maharaj [13] found two new classes of exact solutions to the Einstein-Maxwell system of equations with a particular form of the gravitational potential and isotropic pressure.

Malaver [14,15] also has obtained some models for quark stars considering a potential gravitational that depends on an adjustable parameter. Thirukkanesh and Maharaj [16] studied the behavior of compact relativistic objects with anisotropic pressure in the presence of the electromagnetic field. Maharaj et al. [17] generated new models for quark stars with charged anisotropic matter considering a linear equation of state. Thirukkanesh and Ragel [18] obtained new models for compact stars with quark matter. Sunzu et al. found new classes of solutions with specific forms for the measure of anisotropy [19].

With then use of Einstein´s field equations, important advances has been made to model the interior of a star. In particular, Feroze and Siddiqui [20,21] and Malaver [22-25] consider a quadratic equation of state for the matter distribution and specify particular forms for the gravitational potential and electric field intensity. Mafa Takisa and Maharaj [26] obtained new exact solutions to the Einstein-Maxwell system of equations with a polytropic equation of state. Thirukkanesh and Ragel [27] have obtained particular models of anisotropic fluids with polytropic equation of state which are consistent with the reported experimental observations. Malaver [28,29] generated new exact solutions to the Einstein-Maxwell system considering Van der Waals modified equation of state with and without polytropical exponent and Thirukkanesh and Ragel [30] presented a anisotropic strange quark matter model by imposing a linear barotropic equation of state with Tolman IV form for the gravitational potential. Mak and Harko [31] found a relativistic model of strange quark star with the suppositions of spherical symmetry and conformal Killing vector. Bhar et al. [32,33] have studied extensively the behavior of static spherically symmetric relativistic objects with anisotropic matter distribution considering the Tolman VII space-time. Pant et al. [34] have found new exact solutions of the field equations for anisotropic neutral fluid in isotropic coordinates. Sunzu and Danford [35] generated exact models for the Einstein-Maxwell field equations with a new choice of measure of anisotropy that I physically reasonable.

Our objective in this paper is to generate a new class for charged isotropic matter with the bag equation of state that presents a linear relation between the energy density and the radial pressure in static spherically symmetric spacetime using a particular form a metric function *y(x)* which depends on an adjustable parameter *α*. We have obtained some new classes of static spherically symmetrical models of charged matter where the variation of the parameter α modifies the radial pressure, charge density, energy density and the metric functions of the compact objects. This article is organized as follows, in Section 2, we present Einstein´s field equations as an equivalent set of differential equations using a transformations due to Durgapal and Bannerji [36]. In Section 3, we make a particular choice of the function *y(x)* that allows solving the field equations and we have obtained new models for charged anisotropic matter. In Section 4, a physical analysis of the new solutions is performed. Finally in Section 5, we conclude.

2. EINSTEIN FIELD EQUATIONS

We consider a spherically symmetric, static and homogeneous and anisotropic spacetime in Schwarzschild coordinates given by

$$ds^2 = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \qquad (1)$$

using the transformations, $x = Cr^2$, $Z(x) = e^{-2\lambda(r)}$ and $A^2 y^2(x) = e^{2\nu(r)}$ with arbitrary constants A and c, suggested by Durgapal and Bannerji [36], the Einstein field equations as given in (1) are

$$\frac{1-Z}{x} - 2\dot{Z} = \frac{\rho}{C} + \frac{E^2}{2c} \qquad (2)$$

$$4Z\frac{\dot{y}}{y} - \frac{1-Z}{x} = \frac{p_r}{C} - \frac{E^2}{2C} \qquad (3)$$

$$\Delta = \frac{4xCZ\ddot{y}}{y} + C(2x\dot{Z} + 6Z)\frac{\dot{y}}{y} + C\left[2\left(\dot{Z} + \frac{B}{C}\right) + \frac{Z-1}{x}\right] \qquad (4)$$

$$\sigma^2 = \frac{4CZ}{x}\left(x\dot{E} + E\right)^2 \qquad (5)$$

where: ρ is the energy density, p_r is the radial pressure, E is electric field intensity, σ is the charge density, Δ is the anisotropy and dot denote differentiations with respect to x.

The equation (4) is the measure of anisotropy. We can replace the system of field equations, including the bag equation of state by the system

$$\rho = 3p_r + 4B \qquad (6)$$

$$\frac{p_r}{C} = Z\frac{\dot{y}}{y} - \frac{1}{2}\dot{Z} - \frac{B}{C} \tag{7}$$

$$p_t = p_r + \Delta \tag{8}$$

$$\frac{E^2}{2C} = \frac{1-Z}{x} - 3Z\frac{\dot{y}}{y} - \frac{1}{2}\dot{Z} - \frac{B}{C} \tag{9}$$

$$\sigma = 2\sqrt{\frac{CZ}{x}}\left(x\dot{E} + E\right) \tag{10}$$

The eq. (6) is the bag equation of state and (8) is the tangential pressure. The equations (6), (7), (8) , (9) and (10) governs the gravitational behavior of a charged quark star.

3. NEW CLASS OF SOLUTIONS

Using the method suggested by Komathiraj and Maharaj [13], it is possible to obtain a exact solution of the Einstein-Maxwell system. In this paper, motivated by Malaver [37], we take the particular form of the metric function

$$y(x) = (a + \alpha x)^2 \tag{11}$$

where: a is a real constant and α is an adjustable parameter. This potential is regular at the origin and well behaved in the interior of the sphere. The substitution of (11) in (4) allows to obtain the equation of the first order

$$\dot{Z} + \frac{\left[8x^2\alpha^2 + 12\alpha x(a+\alpha x) + (a+\alpha x)^2\right]}{2x\left[2x\alpha(a+\alpha x) + (a+\alpha x)^2\right]}Z = \frac{\left(\frac{x\Delta}{C} + 1 - \frac{2xB}{C}\right)(a+\alpha x)^2}{2x\left[2\alpha x(a+\alpha x) + (a+\alpha x)^2\right]} \tag{12}$$

We specify the measure of anisotropy of the following form:

$$\Delta = A_0 + A_1 x \tag{13}$$

where: A_0 and A_1 are arbitrary constants. With $A_0 = A_1 = 0$, we obtain the condition of isotropic pressure.

We have considered the particular cases for $\alpha = 2,4$. For the case $\alpha = 2$, the equation (12) can be written as

$$\dot{Z}+\frac{\left[32x^2+24x(a+2x)+(a+2x)^2\right]}{2x\left[4x(a+2x)+(a+2x)^2\right]}Z=\frac{\left(\frac{x(A_0+A_1x)}{C}+1-\frac{2xB}{C}\right)(a+2x)^2}{2x\left[4x(a+2x)+(a+2x)^2\right]} \quad (14)$$

Integrating (14), we obtain

$$Z(x)=\frac{\left(\begin{array}{l}693A_1a^3x^2+2970A_1a^2x^3+4620A_1ax^4+2520A_1x^5-2310Ba^3x-8316Ba^2x^2-11880Bax^3\\-6160Bx^4+1155A_0a^3x+4158A_0a^2x^2+5940A_0ax^3+3080A_0x^4+3465Ca^3+6930Ca^2x\\8316Cax^2+3960Cx^3\end{array}\right)}{3465C(6x+a)(a+2x)^2} \quad (15)$$

Z(x) allows generate the following analytical model:

$$e^{2\nu}=A^2(a+2x)^4 \quad (16)$$

$$e^{2\lambda}=\frac{3465C(6x+a)(a+2x)^2}{\left(\begin{array}{l}693A_1a^3x^2+2970A_1a^2x^3+4620A_1ax^4+2520A_1x^5-2310Ba^3x-8316Ba^2x^2-11880Bax^3\\-6160Bx^4+1155A_0a^3x+4158A_0a^2x^2+5940A_0ax^3+3080A_0x^4+3465Ca^3+6930Ca^2x+8316Cax^2\\3960Cx^3\end{array}\right)} \quad (17)$$

$$p_r=\frac{\left(\begin{array}{l}2772A_1a^3x^2+11880A_1a^2x^3+18480A_1ax^4+10080A_1x^5-9240Ba^3x-33264Ba^2x^2\\-47520Bax^3-24640Bx^4+4620A_0a^3x+16632A_0a^2x^2+23760A_0ax^3+12320A_0x^4\\+13860Ca^3+27720Ca^2x+33264Cax^2+15840Cx^3+9240A_0ax^4+5040A_0x^5\\+1386A_1a^3x^2+5940A_1a^2x^3-4620Ba^3x-16632Ba^2x^2-23760Bax^3-12320Bx^4\\+2130A_0a^3x+8316A_0a^2x^2+11880A_0ax^3+6160A_0x^4+6930Ca^3+13860Ca^2x\\+16632Cax^2+7920Cx^3-3465BCa^4-41580BCa^3x-166320BCa^2x^2-277200BCax^3\\-166320BCx^4\end{array}\right)}{3465(6x+a)(a+2x)^3}$$

$$-\frac{\left(\begin{array}{l}18480A_0ax^3+12600A_0x^4+1386A_1a^3x+8910A_1a^2x^2\\-2310Ba^3-16632Ba^2x-35640Bax^2-24640Bx^3+1155A_0a^3+8316A_0a^2x+17820A_0ax^2\\+12320A_0x^3+6930Ca^2+16632Cax+11880Cx^2\end{array}\right)}{6930(6x+a)(a+2x)^2} \quad (18)$$

$$+\frac{\left(\begin{array}{l}4620A_0ax^4+2520A_0x^5+693A_1a^3x^2+2970A_1a^2x^3-2310Ba^3x-8316Ba^2x^2-11880Bax^3\\-6160Bx^4+1155A_0a^3x+4158A_0a^2x^2+5940A_0ax^3+3080A_0x^4+3465Ca^3+6930Ca^2x\\+8316Cax^2+3960Cx^3\end{array}\right)}{1155(6x+a)^2(a+2x)^2}$$

$$p_t = \frac{\left(\begin{array}{l} 2772A_1a^3x^2 + 11880A_1a^2x^3 + 18480A_1ax^4 + 10080A_1x^5 - 9240Ba^3x - 33264Ba^2x^2 \\ -47520Bax^3 - 24640Bx^4 + 4620A_0a^3x + 16632A_0a^2x^2 + 23760A_0ax^3 + 12320A_0x^4 \\ +13860Ca^3 + 27720Ca^2x + 33264Cax^2 + 15840Cx^3 + 9240A_0ax^4 + 5040A_0x^5 \\ +1386A_1a^3x^2 + 5940A_1a^2x^3 - 4620Ba^3x - 16632Ba^2x^2 - 23760Bax^3 - 12320Bx^4 \\ +2130A_0a^3x + 8316A_0a^2x^2 + 11880A_0ax^3 + 6160A_0x^4 + 6930Ca^3 + 13860Ca^2x \\ +16632Cax^2 + 7920Cx^3 - 3465BCa^4 - 41580BCa^3x - 166320BCa^2x^2 - 277200BCax^3 \\ -166320BCx^4 \end{array}\right)}{3465(6x+a)(a+2x)^3}$$

$$- \frac{\left(\begin{array}{l} 18480A_0ax^3 + 12600A_0x^4 + 1386A_1a^3x + 8910A_1a^2x^2 \\ -2310Ba^3 - 16632Ba^2x - 35640Bax^2 - 24640Bx^3 + 1155A_0a^3 + 8316A_0a^2x + 17820A_0ax^2 \\ +12320A_0x^3 + 6930Ca^2 + 16632Cax + 11880Cx^2 \end{array}\right)}{6930(6x+a)(a+2x)^2}$$

$$+ \frac{\left(\begin{array}{l} 4620A_0ax^4 + 2520A_0x^5 + 693A_1a^3x^2 + 2970A_1a^2x^3 - 2310Ba^3x - 8316Ba^2x^2 - 11880Bax^3 \\ -6160Bx^4 + 1155A_0a^3x + 4158A_0a^2x^2 + 5940A_0ax^3 + 3080A_0x^4 + 3465Ca^3 + 6930Ca^2x \\ +8316Cax^2 + 3960Cx^3 + 1155A_1a^4x + 18480A_1a^3x^2 + 101640A_1a^2x^3 + 221760A_1ax^4 \\ +166320A_1x^5 + 1155A_0a^4 + 18480A_0a^3x + 101640A_0a^2x^2 + 221760A_0ax^3 + 166320A_0x^4 \end{array}\right)}{1155(6x+a)^2(a+2x)^2} \qquad (19)$$

$$\rho = \frac{\left(\begin{array}{l} 2772A_1a^3x^2 + 11880A_1a^2x^3 + 18480A_1ax^4 + 10080A_1x^5 - 9240Ba^3x - 33264Ba^2x^2 \\ -47520Bax^3 - 24640Bx^4 + 4620A_0a^3x + 16632A_0a^2x^2 + 23760A_0ax^3 + 12320A_0x^4 \\ +13860Ca^3 + 27720Ca^2x + 33264Cax^2 + 15840Cx^3 + 9240A_0ax^4 + 5040A_0x^5 \\ +1386A_1a^3x^2 + 5940A_1a^2x^3 - 4620Ba^3x - 16632Ba^2x^2 - 23760Bax^3 - 12320Bx^4 \\ +2130A_0a^3x + 8316A_0a^2x^2 + 11880A_0ax^3 + 6160A_0x^4 + 6930Ca^3 + 13860Ca^2x \\ +16632Cax^2 + 7920Cx^3 - 3465BCa^4 - 41580BCa^3x - 166320BCa^2x^2 - 277200BCax^3 \\ -166320BCx^4 \end{array}\right)}{1155(6x+a)(a+2x)^3}$$

$$- \frac{\left(\begin{array}{l} 18480A_0ax^3 + 12600A_0x^4 + 1386A_1a^3x + 8910A_1a^2x^2 \\ -2310Ba^3 - 16632Ba^2x - 35640Bax^2 - 24640Bx^3 + 1155A_0a^3 + 8316A_0a^2x + 17820A_0ax^2 \\ +12320A_0x^3 + 6930Ca^2 + 16632Cax + 11880Cx^2 \end{array}\right)}{2310(6x+a)(a+2x)^2}$$

$$+ \frac{\left(\begin{array}{l} 4620A_0ax^4 + 2520A_0x^5 + 693A_1a^3x^2 + 2970A_1a^2x^3 - 2310Ba^3x - 8316Ba^2x^2 - 11880Bax^3 \\ -6160Bx^4 + 1155A_0a^3x + 4158A_0a^2x^2 + 5940A_0ax^3 + 3080A_0x^4 + 3465Ca^3 + 6930Ca^2x \\ +8316Cax^2 + 3960Cx^3 + 1540Ba^4 + 24640Ba^3x + 135520Ba^2x^2 + 295680Bax^3 + 221760Bx^4 \end{array}\right)}{385(6x+a)^2(a+2x)^2} \qquad (20)$$

$$E^2 = 2\frac{\begin{pmatrix} 34650Ca^2 + 97020Cax + 83160Cx^2 - 693A_1a^3x - 2970A_1a^2x^2 - 4620A_1ax^3 - 2520A_1x^4 \\ + 2310Ba^3 + 8316Ba^2x + 11880Bax^2 + 6160Bx^3 - 1155A_0a^3 - 4158A_0a^2x - 5940A_0ax^2 - 3080A_0x^3 \\ - 6930Ca^2 - 8316Cax - 3960Cx^2 \end{pmatrix}}{3465(6x+a)(a+2x)^2}$$
$$-2\frac{\begin{pmatrix} 2772A_1a^3x^2 + 11880A_1a^2x^3 + 18480A_1ax^4 + 10080A_1x^5 - 9240Ba^3x - 33264Ba^2x^2 - 47520Bax^3 - 24640Bx^4 \\ + 4620A_0a^3x + 16632A_0a^2x^2 + 23760A_0ax^3 + 12320A_0x^4 + 13860Ca^3 + 27720Ca^2x + 33264Cax^2 + 15840Cx^3 \end{pmatrix}}{1155(6x+a)(a+2x)^3}$$
$$-\frac{\begin{pmatrix} 18480A_0ax^3 + 12600A_0x^4 + 1386A_1a^3x + 8910A_1a^2x^2 - 2310Ba^3 - 16632Ba^2x - 35640Bax^2 - 24640Bx^3 \\ + 1155A_0a^3 + 8316A_0a^2x + 17820A_0ax^2 + 12320A_0x^3 + 6930Ca^2 + 16632Cax + 11880Cx^2 + 6930Ba^3 \\ + 69300Ba^2x + 194040Bax^2 + 166320Bx^3 \end{pmatrix}}{3465(6x+a)(a+2x)^2}$$
$$+2\frac{\begin{pmatrix} 4620A_0ax^4 + 2520A_0x^5 + 693A_1a^3x^2 + 2970A_1a^2x^3 - 2310Ba^3x - 8316Ba^2x^2 - 11880Bax^3 - 6160Bx^4 \\ + 1155A_0a^3x + 4158A_0a^2x^2 + 5940A_0ax^3 + 3080A_0x^4 + 3465Ca^3 + 6930Ca^2x + 8316Cax^2 + 3960Cx^3 \end{pmatrix}}{1155(6x+a)^2(a+2x)^2}$$
$$+4\frac{\begin{pmatrix} 4620A_0ax^4 + 2520A_0x^5 + 693A_1a^3x^2 + 2970A_1a^2x^3 - 2310Ba^3x - 8316Ba^2x^2 - 11880Bax^3 - 6160Bx^4 \\ + 1155A_0a^3x + 4158A_0a^2x^2 + 5940A_0ax^3 + 3080A_0x^4 + 3465Ca^3 + 6930Ca^2x + 8316Cax^2 + 3960Cx^3 \end{pmatrix}}{3465(6x+a)(a+2x)^3} \tag{21}$$

With $\alpha = 4$, the eq.(12) becomes

$$\dot{Z} + \frac{\left[128x^2 + 48x(a+4x) + (a+4x)^2\right]}{2x\left[8x(a+4x) + (a+4x)^2\right]}Z = \frac{\left(\frac{x(A_0 + A_1x)}{C} + 1 - \frac{2xB}{C}\right)(a+4x)^2}{2x\left[8x(a+4x) + (a+4x)^2\right]} \tag{22}$$

Integrating (22), we obtain

$$Z(x) = \frac{\begin{pmatrix} 693A_1a^3x^2 + 5940A_1a^2x^3 + 18480A_1ax^4 + 20160A_1x^5 - 2310Ba^3x - 16632Ba^2x^2 \\ - 47520Bax^3 - 49280Bx^4 + 1155A_0a^3x + 8316A_0a^2x^2 + 23760A_0ax^3 + 24640A_0x^4 \\ + 3465Ca^3 + 13860Ca^2x + 33264Cax^2 + 31680Cx^3 \end{pmatrix}}{3465C(12x+a)(a+4x)^2} \tag{23}$$

With the equation (23), we can generate the exact analytical model

$$e^{2\nu} = A^2(a+4x)^4 \tag{24}$$

$$e^{2\lambda} = \frac{3465C(12x+a)(a+4x)^2}{\left(\begin{aligned}&693A_1a^3x^2+5940A_1a^2x^3+18480A_1ax^4+20160A_1x^5-2310Ba^3x-16632Ba^2x^2-47520Bax^3\\&-49280Bx^4+1155A_0a^3x+8316A_0a^2x^2+23760A_0ax^3+24640A_0x^4+3465Ca^3+13860Ca^2x+33264Cax^2\\&31680Cx^3\end{aligned}\right)} \tag{25}$$

$$\begin{aligned}p_r = {}&\frac{\left(\begin{aligned}&5544A_1a^3x^2+47520A_1a^2x^3+147840A_1ax^4+161280A_1x^5-18480Ba^3x-133056Ba^2x^2\\&-380160Bax^3-394240Bx^4+9240A_0a^3x+66528A_0a^2x^2+190080A_0ax^3+197120A_0x^4\\&+27720Ca^3+110880Ca^2x+266112Cax^2+253440Cx^3+2772A_1a^3x^2+23760A_1a^2x^3\\&+73920A_1ax^4+80640A_1x^5+9240Ba^3x+66528Ba^2x^2+190080Bax^3+197120Bx^4\\&+4620A_0a^3x+33264A_0a^2x^2+95040A_0ax^3+98560A_0x^4+13860Ca^3+55440Ca^2x\\&+133056Cax^2+126720Cx^3-3465BCa^4-83160BCa^3x-665280BCa^2x^2-2217600BCax^3\\&-2661120BCx^4\end{aligned}\right)}{3465(12x+a)(a+4x)^3}\\&+\frac{\left(\begin{aligned}&1386A_1a^3x^2+11880A_1a^2x^3+36960A_1ax^4+40320A_1x^5+4620Ba^3x+33264Ba^2x^2\\&+95040Bax^3+98560Bx^4+2310A_0a^3x+16632A_0a^2x^2+47520A_0ax^3+49280A_0x^4\\&+6930Ca^3+27720Ca^2x+66528Cax^2+63360Cx^3\end{aligned}\right)}{1155(12x+a)^2(a+4x)^2}\\&-\frac{\left(\begin{aligned}&2772A_1a^3x+35640A_1a^2x^2+147840A_1ax^3+201600A_1x^4+4620Ba^3+66528Ba^2x\\&+285120Bax^2+394240Bx^3+2310A_0a^3+33264A_0a^2x+142560A_0ax^2+197120A_0x^3\\&+27720Ca^2+133056Cax+190080Cx^2\end{aligned}\right)}{3465(12x+a)(a+4x)^2}\end{aligned} \tag{26}$$

$$p_t=\frac{\left(\begin{aligned}&5544A_1a^3x^2+47520A_1a^2x^3+147840A_1ax^4+161280A_1x^5-18480Ba^3x-133056Ba^2x^2\\&-380160Bax^3-394240Bx^4+9240A_0a^3x+66528A_0a^2x^2+190080A_0ax^3+197120A_0x^4\\&+27720Ca^3+110880Ca^2x+266112Cax^2+253440Cx^3+2772A_1a^3x^2+23760A_1a^2x^3\\&+73920A_1ax^4+80640A_1x^5+9240Ba^3x+66528Ba^2x^2+190080Bax^3+197120Bx^4\\&+4620A_0a^3x+33264A_0a^2x^2+95040A_0ax^3+98560A_0x^4+13860Ca^3+55440Ca^2x\\&+133056Cax^2+126720Cx^3-3465BCa^4-83160BCa^3x-665280BCa^2x^2-2217600BCax^3\\&-2661120BCx^4\end{aligned}\right)}{3465(12x+a)(a+4x)^3}$$

$$+\frac{\left(\begin{aligned}&1386A_1a^3x^2+11880A_1a^2x^3+36960A_1ax^4+40320A_1x^5+4620Ba^3x+33264Ba^2x^2\\&+95040Bax^3+98560Bx^4+2310A_0a^3x+16632A_0a^2x^2+47520A_0ax^3+49280A_0x^4\\&+6930Ca^3+27720Ca^2x+66528Cax^2+63360Cx^3+1155A_1a^4x+36960A_1a^3x^2\\&+406560A_1a^2x^3+1774080A_1ax^4+2661120A_1x^5+1155A_0a^4+36960A_1a^3x\\&+406560A_0a^2x^2+1774080A_0ax^3+2661120A_0x^4\end{aligned}\right)}{1155(12x+a)^2(a+4x)^2}$$

$$-\frac{\left(\begin{aligned}&2772A_1a^3x+35640A_1a^2x^2+147840A_1ax^3+201600A_1x^4+4620Ba^3+66528Ba^2x\\&+285120Bax^2+394240Bx^3+2310A_0a^3+33264A_0a^2x+142560A_0ax^2+197120A_0x^3\\&+27720Ca^2+133056Cax+190080Cx^2\end{aligned}\right)}{3465(12x+a)(a+4x)^2} \tag{27}$$

$$\rho=\frac{\left(\begin{aligned}&5544A_1a^3x^2+47520A_1a^2x^3+147840A_1ax^4+161280A_1x^5-18480Ba^3x-133056Ba^2x^2\\&-380160Bax^3-394240Bx^4+9240A_0a^3x+66528A_0a^2x^2+190080A_0ax^3+197120A_0x^4\\&+27720Ca^3+110880Ca^2x+266112Cax^2+253440Cx^3+2772A_1a^3x^2+23760A_1a^2x^3\\&+73920A_1ax^4+80640A_1x^5+9240Ba^3x+66528Ba^2x^2+190080Bax^3+197120Bx^4\\&+4620A_0a^3x+33264A_0a^2x^2+95040A_0ax^3+98560A_0x^4+13860Ca^3+55440Ca^2x\\&+133056Cax^2+126720Cx^3-3465BCa^4-83160BCa^3x-665280BCa^2x^2-2217600BCax^3\\&-2661120BCx^4\end{aligned}\right)}{1155(12x+a)(a+4x)^3}$$

$$+\frac{\left(\begin{aligned}&1386A_1a^3x^2+11880A_1a^2x^3+36960A_1ax^4+40320A_1x^5+4620Ba^3x+33264Ba^2x^2\\&+95040Bax^3+98560Bx^4+2310A_0a^3x+16632A_0a^2x^2+47520A_0ax^3+49280A_0x^4\\&+6930Ca^3+27720Ca^2x+66528Cax^2+63360Cx^3+1540Ba^4+49280Ba^3x\\&+542080Ba^2x^2+2365440Bax^3+3548160Bx^4\end{aligned}\right)}{385(12x+a)^2(a+4x)^2} \tag{28}$$

$$-\frac{\left(\begin{aligned}&2772A_1a^3x+35640A_1a^2x^2+147840A_1ax^3+201600A_1x^4+4620Ba^3+66528Ba^2x\\&+285120Bax^2+394240Bx^3+2310A_0a^3+33264A_0a^2x+142560A_0ax^2+197120A_0x^3\\&+27720Ca^2+133056Cax+190080Cx^2\end{aligned}\right)}{1155(12x+a)(a+4x)^2}$$

$$E^2 = 2\frac{\begin{pmatrix} 69300Ca^2 + 388080Cax + 665280Cx^2 - 693A_1a^3x - 5940A_1a^2x^2 - 18480A_1ax^3 \\ - 20160A_1x^4 + 2310Ba^3 + 16632Ba^2x + 47520Bax^2 + 49280Bx^3 - 1155A_0a^3 - 8316A_0a^2x \\ - 23760A_1ax^2 - 24640A_0x^3 - 13860Ca^2 - 33264Cax - 31680Cx^2 \end{pmatrix}}{3465(12x+a)(a+4x)^2}$$
$$-16\frac{\begin{pmatrix} 693A_1a^3x^2 + 5940A_1a^2x^3 + 18480A_1ax^4 + 20160A_1x^5 - 2310Ba^3x - 16632Ba^2x^2 - 47520Bax^3 \\ - 49280Bx^4 + 1155A_0a^3x + 8316A_0a^2x^2 + 23760A_0ax^3 + 24640A_0x^4 + 3465Ca^3 + 13860Ca^2x \\ + 33264Cax^2 + 31680Cx^3 \end{pmatrix}}{1155(12x+a)(a+4x)^3}$$
$$-\frac{\begin{pmatrix} 1386A_1a^3x + 17820A_1a^2x^2 + 73920A_1ax^3 + 100800A_1x^4 - 2310Ba^3 + 33264Ba^2x + 142560Bax^2 \\ + 197120Bx^3 + 1155A_0a^3 + 16632A_0a^2x + 71280A_0ax^2 + 98560A_0x^3 + 13860Ca^2 + 66528Cax \\ + 95040Cx^2 + 6930Ba^3 + 138600Ba^2x + 776160Bax^2 + 1330560Bx^3 \end{pmatrix}}{3465(12x+a)(a+4x)^2}$$
$$+4\frac{\begin{pmatrix} 693A_1a^3x^2 + 5940A_1a^2x^3 + 18480A_1ax^4 + 20160A_1x^5 + 2310Ba^3x + 16632Ba^2x^2 + 47520Bax^3 \\ + 49280Bx^4 + 1155A_0a^3x + 8316A_0a^2x^2 + 23760A_0ax^3 + 24640A_0x^4 + 3465Ca^3 + 13860Ca^2x \\ + 33264Cax^2 + 31860Cx^3 \end{pmatrix}}{1155(12x+a)^2(a+4x)^2}$$
$$+8\frac{\begin{pmatrix} 693A_1a^3x^2 + 5940A_1a^2x^3 + 18480A_1ax^4 + 20160A_1x^5 + 2310Ba^3x + 16632Ba^2x^2 + 47520Bax^3 \\ + 49280Bx^4 + 1155A_0a^3x + 8316A_0a^2x^2 + 23760A_0ax^3 + 24640A_0x^4 + 3465Ca^3 + 13860Ca^2x \\ + 33264Cax^2 + 31680Cx^3 \end{pmatrix}}{3465(12x+a)(a+4x)^3} \tag{29}$$

4. PHYSICAL ANALYSIS

Any physically acceptable solutions must satisfy the following conditions [27]:

i. Regularity of the gravitational potentials in the origin.

ii. Radial pressure must be finite at the centre.

iii. $p_r > 0$ and $\rho > 0$ in the origin.

iv. Monotonic decrease of the energy density and the radial pressure with increasing radius

The presented models constitute another new family of solutions for a charged quark star with anisotropic pressure. The metric functions can be written in terms of polynomial and elementary functions, and the variables energy density, pressure and charge density also are represented analytical. For the case α=2 , the metric functions $e^{2\nu(r)}$ and $e^{2\lambda(r)}$ behaves well inside the star and have the finite value $e^{2\nu(0)} = A^2 a^4$, $e^{2\lambda(0)} = 1$ and $\left(e^{2\lambda(r)}\right)'_{r=0} = \left(e^{2\nu(r)}\right)'_{r=0} = 0$.

This demonstrates that the gravitational potential is regular in the origin. At the center *r=0* the radial pressure, the tangential pressure, the energy density and electric field take the values $p_r = 2\frac{C}{a} + \frac{B}{3} - BC - \frac{A_0}{6}$, $p_t = 8\frac{C}{a} - BC + \frac{B}{3} + 5\frac{A_0}{6}$, $\rho = 24\frac{C}{a} - 3BC + 5B - \frac{A_0}{2}$, $E^2 = -A_0$, respectively. If $A_0 = 0$ we have that $E^2 = 0$ at the stellar centre. When $\Delta = 0$ we obtain the Malaver model [37] for isotropic pressure as special case.

For the solution with α=4, $e^{2\lambda(0)} = 1$, $e^{2\nu(0)} = A^2 a^4$ in the origin and $\left(e^{2\lambda(r)}\right)'_{r=0} = \left(e^{2\nu(r)}\right)'_{r=0} = 0$. The values of p_r, p_t, ρ and E^2 in *r=0* are given by for $p_r = 10\frac{C}{a} - BC - \frac{4}{3}B - \frac{2A_0}{3}$, $p_t = 10\frac{C}{a} - BC - \frac{4}{3}B + \frac{A_0}{3}$, $\rho = 30\frac{C}{a} - 3BC - 2A_0$ and $E^2 = -A_0$.

The new obtained solutions not admit singularities in the center *r=0* and in the stellar interior well behaved as in the quark star model of Maharaj et al. [17].

5. CONCLUSION

We have generated a new class of exact solutions for the Einstein-Maxwell system within the framework of MIT Bag Model with a particular form for the measure of anisotropy which is very useful for realistic models of stars. We have studied two new types of analytical solutions specifying the form of the metric potential $y(x)$ which depends on an adjustable parameter α.

The metric functions $e^{2\nu(r)}$ and $e^{2\lambda(r)}$ are well defined and can be written in terms of elementary functions and the matter variables also are represented analytical. The obtained solutions correspond to models which have finite values for the energy density, the pressure and the electric field at the center of the star. The method to generate analytical exact solutions depends on the choice of the form of *y(x)* with that there is obtained the function *Z(x)*, necessary to determine physically acceptable solutions. The new models may be used to study the physical features of relativistic charged compact objects in different astrophysical scenes.

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