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Algebraic division by zero implemented as quasigeometric multiplication by infinity in real and complex multispatial hyperspaces

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ABSTRACT

An unrestricted division by zero implemented as an algebraic multiplication by infinity is feasible within a multispatial hyperspace comprising several quasigeometric spaces.

Keywords: Division by zero, infinity, multispatiality, multispatial uncertainty principle

1. INTRODUCTION

Numbers used to be identified with their values. Yet complex numbers have two distinct single-number values: modulus/length and angle/phase, which can vary independently of each other. Since values are attributes of the algebraic entities called numbers, we need yet another way to define these entities and establish a basis that specifies their attributes. In an operational sense a number can be defined as the outcome of an algebraic operation. We must know the space where the numbers reside and the basis in which they are represented. Since division is inverse of multiplication, then reciprocal/contragradient basis can be used to represent inverse numbers for division [1]. Note that dual space, as conjugate space [2] is a space of functionals defined on elements of the primary space [3-5]. Although dual geometries are identical as sets, their geometrical structures are different [6] for duality can

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form anti-isomorphism or inverse isomorphism [7]. Moreover, since projective duality does not hold in non-Euclidean spaces [8], the concept of infinity should be drastically redefined.

Although the concept of vectorial basis is well understood and the notion of basis is used for complex and hypercomplex numbers, no special emphasis was ever placed on explaining the bases that provide implicit context for real numbers. Yet when one wants to distinguish reciprocal magnitudes, the issue of basis for numbers becomes of paramount importance. When a number designates an interval (such as distance between two points in a 3D Euclidean space), which is denominated in meters, then the reciprocal designating a single point is usually denominated in inverse meters [1/m], which is the reciprocal/inverse basis for single-value/point numbers representing spatial frequency. Hence inverses of intervals whose number basis is denominated in [m] must be denominated in [1/m]. Similarly, also an elapsing time interval is denominated in seconds [s], whereas the inverse basis for temporal frequencies is denominated in [1/s]. These two bases (primary and reciprocal) can qualify both the real numbers and the respective spaces in which the numbers reside. Notice that the radius **r** can be regarded as an interval counted from the origin of the local coordinate frame.

2. ALGEBRAIC DIVISION BY ZERO WAS PROHIBITED

When it comes to real numbers some authors claim that because $n\times 0=0$ is admissible for any real number n, so also $\infty\times 0=0$ should be admissible [9]. But the latter operation implies that $\infty=1$. If $(0/0)=1$, which seems natural to assume, for anything divided by itself should always yield 1, whereas the former equation seems to imply that $n=1$ if $(0/0)=1$, despite the original assumption that n was supposed to stand for any real number. The *ad hoc* presumed abstract operation $\infty\times 0=0$ is a conceptual nonsense elevating zero to an omnipotent nullifier.

Presumably it was because of such confusing results (or discrepant evaluations) of even simple algebraic operations involving zero and infinity, that the prohibition on division by zero was enacted and the notion of infinity is avoided whenever it is possible. The confusion plaguing algebraic operations on zero and infinity is just lame excuse for the prohibition, but it does not justify its enactment. Even abstract mathematics cannot be ruled by decree.

Euler explained the nuances of operating on the infinity as the reciprocal of zero so that $(1/0) = \infty$ and consequently $(0/0)=(\infty/\infty)=1$ [10] and $\infty\times 0=1$ [11]. But Euler did not resolve all the problems related to operations on the two reciprocal entities (zero and infinity).

Yet even in the Eulerian context certain other – allegedly self-evident – conventional assumptions of traditional mathematics can also lead to conceptual paradoxes. Couturat fittingly called the traditional stipulation that $0\times 0=0$ the source of indetermination of both zero and infinity [12, p. 99]. Although this particular case could be resolved without any significant modification of the traditional mathematics [11], the issue at stake is evidently deeper than could be successfully solved without changing some unspoken paradigms underlying algebraic operations. It is because numbers are identified with points.

3. SOME CONVENTIONAL INTERPRETATIONS OF ZERO AND INFINITY

Besides L'Hôpital, whose formula is taught in every introductory course on differential calculus, Euler also connected zero with inverse infinity, which is the natural reciprocal of zero. He treated infinity as number on equal footing with all other numbers [10]; compare also [11,12]. Zero and infinity come as a pair also in Hebrew tradition [13]. Yet because of conceptual difficulties with interpreting the infinity many authors tried to always avoid infinity, which can easily be circumvented by Laurent series [14]. When the ideal point at infinity is ignored, one arrives at more general class of meromorphic functions, which are characterized by the property of having no singularities other than just poles in the entire plane without regard to their behavior at infinity [15]; see also explanations of multivalued complex functions [16]. Yet operational (and structural) infinity is often unavoidable.

Notice that even using the classical notion of 'point charge' – with the understanding that physical point is a zero-dimensional abstract mathematical object – leads immediately to the difficulty of infinite energy [17] that could be ascribed to the point within a physical field. Since abstract notions are deployed in existence theorems, which in the mathematical analysis deal with functional transformations [18], even pure mathematics should respect physics. Recall that pure mathematics means investigating systems for their own sake [19].

4. NONTRADITIONAL OPERATIONS AKIN TO DIVISION BY ZERO

Without explicitly repealing the prohibition on division by zero, Jean Dieudonné offered an evasive proposal for division by zero by stipulating that xy=0 whenever one of these two factors x,y is 0 and the other ±∞ [20]. His proposal can be viewed as an alternative to division by zero. He fairly pointed out that the extension of multiplication makes the so-extended real field ℝ discontinuous yet he avoided to mention explicitly that it is just tacitly stipulated evasion of division by zero in the field ℝ of real numbers. He was able to operate under this assumption on certain abstract spaces, which are merely sets, however. For in order to treat an abstract set (which could always be defined by just specifying a selection rule) as a realistic geometric space, one would have to propose also at least the scalar real potential V=1/r (as inverse of radial distance interval r) so that the distribution and exchange of potential energy within the space could be ascertained. Ideally, the latter condition may demand not only unrestricted (i.e. without any conceptually unwarranted prohibitions) algebraic operations on all numbers, but on 3D vectors too. Note that within the abstract framework of constructive mathematics the term 'set' is employed as a synonym for the phrase "condition with one parameter" [21], thus as a certain selection rule.

Recall that while quaternions are regarded as 4D algebraic objects encompassing the 3D vectors, complex numbers (as ordered pairs of orthogonal real numbers) can also be viewed as a sort of algebraic vectors, which are restricted to the complex plane. Nonetheless, just by combining algebraic and differential-geometric relationships, one could also derive realistic (i.e. physically meaningful) geometric 3D vectors that are perpendicular to the 2D complex plane from differential equations originally comprising only the 2D complex numbers [22].

It is not what one defines that ultimately matters, but what could be derived from least restrictive definitions that eventually counts. When mathematicians start with some formal definition of an abstract object and then investigate it, they often quite inadvertently explore their mind (i.e. their thoughts about the so-defined object) rather than the object iself. They all too often mix what has been implicitly assumed with some consequences derived from operations on the objects or from some other objective findings [23]. Therefore, instead of making arbitrary definitions which may comprise unintented preconceived ideas distorting the so-defined objects, I want to synthesize the attributes of objects from experimental hints. Mathematical proofs might be viewed as thought experiments, of course, provided they are universally valid. Hence conjectures are fine. Existential postulates are not. Stipulations are admissible, provided they are supplied with restrictions on their expected validity. Without such restrictions some operations may result in discrepant or ambiguous evaluations.

Several papers have been written on a new simplified practical formula for (presumably restricted) division by zero, which has been revived from ancient Hindu writings, namely:

$$(0/0):=(1/0):=\{f(z)/0\}:=0 \quad (1)$$

which was discussed with many examples of its utilitarian usefulness in [24,25].

By effectively eliminating the notion of infinity, the formula (1) appears as if virtually folding the complex plane by elevating zero to the – somewhat utilitarian – role of both the zero and the absent infinity in a single superentity, whatever the latter notion may mean.

Although folding of geometric structures is not uncommon and sometimes might even simplify some practical applications, as the authors extensively exemplified, the price paid for the – implicitly suggested – amalgamation of the roles traditionally played by zero and infinity calls for placing restrictions on where the formula (1) can be safely applied. The authors themselves identified several issues that could not be explained at present without specifying further conceptual and/or operational restrictions on the usage of formula (1).
For one can see that multiplying the chain of substitutions in the formula (1) by zero

$$0:=1:=f(z):=0^2 \quad (2)$$

could be interpreted as equating zero to anything if no further restrictions are provided.

From group-theoretical point of view the formula (1) equates additive identity 0 with multiplicative identity 1. This fact indicates that the formula can be safely used only in the cases in which only one of the two groups is being considered as pertaining to the set that underlies the domain of validity of the investigation. In the sense, the formula (1) does not have unrestricted universal validity. Thus, its domains of validity need restrictions.

I shall leave it to the authors to come up with more specifics on the proposal offered in the formula (1). The implicit suggestion of the two nontraditional proposals, as well as of the prohibition on division by zero, is that there is no problem with performing a division by zero, but we just cannot fit both zero and infinity into the same abstract algebraic space, no matter how we try to do it. If so then why don't we just create yet another space, in which the infinity could reside apart from zero and other numbers, and instead of dividing by zero, let us devise an interspatial multiplication by infinity from the extra space into the primary space in which the zero resides, for every algebraic operation of division could indeed be efficiently implemented via the operation of multiplication by reciprocal/inverse magnitude.

5. INVERSES AS DUAL RECIPROCALS DEMAND AN EXTRA SPACE

Inverse functions/maps interchange domain and range [26-28]. A differentiable mapping F has a differentiable inverse in some neighborhood of a point if and only if the Jacobian of F at the point is different from zero [29]. Invertible operators are eloquently discussed in [30] and generalized inverses of operators in [31]. One should distinguish the left and right inverse [32] because their impact is not equal, in general [33].

The effect of a geometric inversion is to change a cross ratio into its conjugate [34, p. 44]; the inversions map $0 \rightarrow 1$, $1 \rightarrow 0$, $\infty \rightarrow \infty$ [34, p. 39]. The inversion of the plane with central point O and radius r is the transformation determined by the following rule: A point X different from the points O and O_∞ is carried into the point X' on the ray OX which satisfies the equation $OX'=r^2/OX \Leftrightarrow OX'/r=r/OX$ [35]. One can see that the equivalence also equates the operation of geometric inversion with algebraic reciprocation.

Every holomorphic function, which is not a constant, has invertible elements [36, p. 254]. The inversion transformation of the complex function $w=1/z$ is composed of two reflections, one in the real axis and one in the unit circle, neither of these reflections being a Möbius transformation; here the points ± 1 are left invariant and all other points are displaced [37]. Formal attributes of an inverse function/mapping resemble those of duals [36, p. 14ff]. Also, the bundle E' is called the inverse image of the vector bundle E by the mapping f and is usually denoted by $f^*(E)$ [38]. There is also the isomorphism of Banach space L^p onto the dual space L^q where $1/p+1/q=1$ and $p,q \in [1, \infty)$ for σ-finite measure spaces and there is a corresponding bounded linear functional [39]. In set-theoretical setting too, dual of the union of two sets is equal to the intersection of their duals, and the dual of the intersection of any two sets is equal to the union of their duals [40]. Hence multiplicative inversion, duality and reciprocation are operationally equivalent. Therefore, the operations on zero and infinity can be viewed as being interchangeable with respect to multiplication and division.

The above examples show that inversion and duality are related. Nevertheless, all these notions can be implemented within a single space as alternative abstract representations of the objects involved. Obviously traditional mathematics operated within the confines of the unspoken SSR paradigm and thus no need for multispatiality was ever suggested. However, some authors realized that there is something disconcerting in the previous mathematics. The dissonance was well discussed in [41] and thus we can follow that concise discussion.

Let L be a finite-dimensional real vector space, while its dual (or conjugate) D(L) is the vector space of all real-valued functions f() defined on L. Since D(L) is in its turn also a real vector space with the same dimension as L, it is clear that L and D(L) are isomorphic. But such an isomorphism cannot be exhibited until one chooses a definite set of basis vectors for L, and furthermore, the isomorphism which results will differ for different choices of this basis. For the iterated dual D(D(L)), on the other hand, it is well known that one can exhibit an isomorphism between L and D(D(L)) without using any special [vector] basis in L.

This exhibition of the isomorphism $L \cong D(D(L))$ is "natural" in that it is given simultaneously for all finite-dimensional vector spaces L. A discussion of the "simultaneous" or just "natural" character of the isomorphism $L \cong D(D(L))$ clearly involves a simultaneous consideration of all spaces L and all transformations connecting them [41]. Hence the notion of duality can appear as being both independent of the basis of the vector space and yet demanding it for the isomorphism between the dual and primary vector spaces to be exhibited. The latter case is absolutely necessary only under the MSR paradigm, however. Although the discussion in [41] did not challenge the SSR paradigm it indicated that the very concept of isomorphism, which is the engine that drives comparative reasonings in mathematics, apparently can reach far beyond the realm outlined by the confines of the SSR paradigm. Recall that spaces in the former mathematics are just sets determined by selection rules often disguised as functions.

The MSR paradigm introduces spatial and quasispatial structures. The apparent vagueness of duality exposed in [41] plainly suggests multispatial character of mathematical reality.

6. UNCOUNTABLY INFINITE RECIPROCAL SPACE IS ACTUAL INFINITY

When two entities cannot coexist together in a single space, we should separate them. Although infinity is here to stay, it cannot be conceptually accommodated within the same space as zero because the two entities are mutually reciprocal and thus structurally opposite. This view is reinforced by the rules of L'Hopital [42]. I am not divorcing zero from infinity, but am proposing to separate them by putting the two into distinct set-theoretical spaces, each of which could be regarded as a different view on the same entity, for the reciprocal relationship between zero and infinity actually means that infinity is just another face of the very same algebraic coin usually called zero. Since the operational reciprocity is defined in terms of algebraic operations, we can also consider these set-theoretical spaces as abstract algebraic spaces, i.e. views focused on the same entity from two distinct operational points of view. Weyl insisted that mathematics is the science of infinity [43,44].

This seems to me like a win-win proposal, for division by zero (if zero is understood as nothing) could never exhaust the amount to be divided and thus such an operation can never end [10]. It is presumably the neverending (i.e. infinite by definition) dividing by zero that makes the particular division problematic. Hence, we have operational infinity yet without the corresponding to it structural/geometric infinity, which would need an extra space for the infinity to reside. We should synthesize the extra space from the outlined operational procedure to be performed on it. The apparently neverending operation of division by zero is not doomed to be inherently infinite. It is inexhaustible only if the procedure is launched within a single space, but in two mutually fitting reciprocal spaces it is not really infinite.

This conclusion seems unproblematic to me, at least from abstract philosophical point of view. In philosophy of mathematics, the procedural/operational infinity was often called potential infinity whereas the structural/geometric infinity was called actual infinity, which was assumed as impossible to exist. The structural infinity was homeless and thus nobody could pinpoint its residence, from which fact they concluded its alleged nonexistence. Thus we need to construct a separate space for the infinity to be reside within it and to cease the pointless discussions about potential vs. actual infinity. We should tie structural/geometric objects to the corresponding to them procedures described in actionable operational terms.

We should try to accommodate these two entities (zero and infinity) in separate spaces, each of which would define the same abstract object from a different standpoint. Since it is obvious – judging from the L'Hopital rule – that zero and infinity seemingly determine the same mathematical entity, in the sense that the infinity is reciprocal to zero, let us shift the unspoken mathematical paradigm of single-space reality (SSR) to a certain multispatial reality (MSR) paradigm and explore consequences of the paradigm shift. Hence an extra space for reciprocals would be uncountable substitute for the actual/structural infinity.

Traditional mathematics used to identify realistic spaces with mere sets. By 'realistic' I mean geometric or quasigeometric spaces that are constructible, which means that elements of such spaces must be suitable for unrestricted operations. By the same token, if operations are artificially restricted by prohibition on division by zero or otherwise, the spaces that would correspond to such unreasonably restricted operations are not really constructible geometrically. In other words, my synthetic approach to mathematics requires that to each structure should correspond an unrestricted operational procedure and vice versa. If it does not, then either the structure cannot be reasonably constructed, or the operational procedure cannot be executed or both. In either case construction of a structure without corresponding to it operational procedure leads to nonsenses. Postulating the existence of an operational procedure without a corresponding to it geometric or quasigeometric structure is thus moot.

7. IDEAL POINT (AND LINE) AT INFINITY IN MULTISPATIAL CONTEXT

There are three kinds of geometry: metric, projective (based on the idea of [infinitely extendable] straight line) and totally qualitative topology [45]. Poncelet derived projective space from the ordinary space by postulating existence of a common "line at infinity" for all the planes parallel to a given plane [46]. By introducing these new elements: i.e. the ideal point at infinity – as the projected image of a point on the vanishing line – and the ideal line at infinity, he enlarged thus the affine plane (in which both the affine and the Euclidean geometries operate) so as to obtain the projective plane, which has simpler properties of incidence [47]. An expose on projective and Riemannian geometry is presented in [48].

Projective geometry is independent of the theory of parallels and it can be developed without the construction of measure [49]. Moreover, the three noneuclidean geometries (elliptic, hyperbolic and parabolic) can be derived from the projective geometry [49]. Note that every ideal point is considered to be infinitely removed from every other (ordinary or ideal) point [50], which fact also supports the request for a separate reciprocal space.

This feature provides perspective that supports abstract duality including reciprocity. An element A is said to be perspective to an element B (A~B) in case A and B have a common inverse [51] p.16, hence reciprocal; note that perspectivity and projectivity are equivalent [51, p. 40]. Note that projective duality principle was called Poncelet-Gergonne principle of reciprocity and interpreted as imaging/transforming points of one plane into lines of another plane [52].

Point at infinity can be formally introduced [53]. Cantor called the geometric infinity proper infinity whereas the one related to varying he considered as an improper infinity [54]. However, the ideal point at infinity is a cluster-point [or set-valued point/number], a totality of infinitely many points [55,56]. Hence an extra space is needed to accommodate such a geometric infinity if we want to avoid the operational contradictions. Since the extra space is not merely yet another set defined by a separate selection rule under the former single-space reality paradigm (SSR) but a distinct quasigeometric space, within the same abstract mathematical realm though, we should consider shifting the SSR to the multispatial reality (MSR) paradigm. The paradigm shift shall be further elaborated elsewhere.

In order to fit the operational algebraic procedure with unrestricted division by zero into the corresponding to it quasigeometric structure of 4D spacetime, the multispatiality of the abstract mathematical realm is unavoidable for several reasons, namely: logical, conceptual, procedural, operational, and structural (including geometric and quasigeometric).

8. VARIOUS CONCEPTIONS OF INFINITY

Transferring the abstract concept of infinity from the field of spatial extension to that of a temporal extension, Aristotle said that infinity [conceived] by addition cannot be traversed in a finite time, but infinity by division can [57]. Since the division can be thought of as an inverted multiplication, one can see that – in modern parlance – he effectively distinguished between additive and multiplicative groups and consequently thus between the 0, which is known as the additive identity, and 1, which is known as the multiplicative identity. Were it not for the fact that these now common group-theoretical notions had to wait about two millenia before they were formulated, one can say that he clearly understood the difference between reversible and invertible operations or at least delineated this distinction in terms that were available in his time. The reversal can be accomplished within the SSR paradigm. Inverse operations, however, including the fearsome division by zero, apparently require the shift to the MSR paradigm. If one would insist on maintaining the SSR paradigm anyway, then the idea of compactification would have to be maintained and supported, of course.

Although the hyperspatial compactification can be introduced, it cannot be maintained; neither in a conceptual nor in an operational sense. Besides the requirement that the extra compactified dimensions would have to be somehow hidden under the old SSR paradigm, which is true in the sense that they apparently need an extra spatial structure whose presence would challenge the single-space reality that is endorsed in the SSR, the extra dimensions should also be somehow available for operations, which is a conceptual dilemma under the SSR paradigm but nonissue under the MSR paradigm. The distinction made by Aristotle indicates that perhaps we should be dealing with at least two different kinds of infinities (distinguished by operations, not by some properties of the numbers' continuum) and thus also with at least two distinct and quite different spatial and/or quasispatial structures.

Compactification at infinity is discussed in [58]. Visnievsky also pointed out several difficulties with compactification in Kaluza-Klein theories, for spacetime transformations act on external coordinates whereas gauge transformations act on internal ones and thus he concluded that because the (external) spacetime symmetries and the internal [local] symmetries are totally independent and act on separate spaces, then some mechanism must be created to account for the breaking of symmetry into two separate groups [59].

This topic shall be entertained further elsewhere because it would require overwhelming digression into Kaluza-Klein theory or at least review of some abstract obstacles that string theories stumbled upon. Having said that I should clarify that the obstacles are not inherent to the string theories but rather to the SSR paradigm, even though I have never found this aspect mentioned (but I may be mistaken), presumably just because the SSR paradigm is an implicit/unspoken one, for it is allegedly self-evident presumption, not to be challenged.

D'Alembert asserted – contrary to the view of Fontenelle – that the notion of infinity is really that of infinitely large and is only a convenient abridgment for the interpretation in terms of the doctrine of limits. With this understanding he pointed out that one can have orders of infinitely large quantities analogous to those of infinitesimals. Euler had this very interpretation in mind when he said that the logarithm of an infinite number is an infinite number of lower order than that of any root of an infinite number [60]. Note that finite is not the opposite to infinite but to infinitesimal [61]. Indefinite iteration is perceived as infinity [62]. The two infinities, of space and of time cause anxiety [63]. In differential equations of Fuchsian class the point at infinity is placed on exactly the same footing as any other point

[64]. Construction of general form of multiplicative geometrization of numbers permits the establishment of the Euclid's postulate $d/0 = \infty$ where d is the distance of a point from a straight line that was not the geometric form of indetermination [65]. In algebra, however, the infinity usually represents some sort of impossibility [66].

Europeans tend to think that the notion of infinity was not established from experience, but from inexhaustibility of the process of counting [67]. Although the concept of an actual infinity is rather inconceivable for pragmatist [68], actual infinity (exemplified by arithmetic continua [69]) was frequently used by Cantor, who stated that cardinal numbers represent actual infinite magnitudes [70]. The concept was already known in middle ages [71,72].

Since an infinite set is comparable with its own part, it can be defined by one-to-one correspondence with their proper subsets [73-78] for the buildup of new numbers is endless or inexhaustible [79-83]. But such set-theoretical assertions of infinity are mainly arithmetical [84] for a set that is denumerable or includes denumerable set is infinite – because it has no maximal number [85]. Note that one-to-one correspondence is reciprocally unique [86] when it is restricted to only numbers. But the one-to-one correspondence can lead to theology of mathematics and thus to the belief in the possibility of existence of an actual infinity [87], which some mathematicians rejected.

Dedekind claimed that he "proved" the existence of actually infinite systems based on the idea of inexhaustibility by saying that his thought world is endless [88,89]. However, it is utterly impossible to prove in elementary logic the necessary and sufficient conditions for an infinite number of elements some claim [90]. Yet an actual infinity becomes fairly comprehensible in [structural] geometric setting [91] where the whole idea of infinity is fundamental [92] as even Euclidean geometry abounds in [unavoidable] infinities [93].

Many believed that lines consist of [an infinite number] of points [94]. Borel stated that there is an infinity of points on projective plane and on straight line [95]. Peano wrote that we can define point set (figure) as any set of points, whether limited in number or not [96]. Fontenelle distinguished geometric infinity from a metaphysical one [97, p. 53] saying that some algebraic operations on infinity can be executed exactly: $1 + 1/\infty = 1$, $1 + \infty/\infty = 2$ [97, p. 211], which is virtually written in multispatial rendition! But to make his geometric system [of infinity] coherent Fontenelle introduced also [the notion of] finite indeterminables [98].

One can see that his ingenious assumptions are indeed in sync with those of Euler [10]. Finsler noticed that speaking about arbitrarily [large] finite number of [any] things with always one more is the same as speaking about infinity, but in other words [99]. Similarly, Cusanus did not say that infinite numbers can actually exist, or that geometric figures can extend to infinity, but he extrapolates their properties to infinity, which he viewed as a limit [100]; he also observed that the actual infinity is [also] unity [101], which is remarkable statement indeed, for the abstract mathematical infinity is not absolute, but relative [102] and thus the modern notion of infinity is likewise the ghost to haunt the fact of unity which it attempts to replace [103]. One can introduce axioms according to which every set can be either well-ordered or infinite [104] because there is no difference between nondenumerable infinity and denumerable infinity [105]. Even the heart of the [Achilles] paradox lies in misunderstandings about logical relationships between concepts and infinity [106].

If one considers material substance it is credible to say that, in the real physical world, physical quantities do not become infinite and thus the prediction of such an infinity is just a signal that the theory is being applied beyond its proper domain of validity [107]. But from procedural standpoint that relies on the idea that the prototype of all infinite processes is iteration [108] one may say that the abstract mathematical concepts of infinity are tied to [infiniteness of matter in space and time] [109]. From philosophical standpoint the infinite remains the first and ultimate term in thought, the fundamental dimension common to all things, which remains the eternal background against which they all are defined and into which they soon disappear [110]. For infinity is the openness of the future [111].

We know nothing intuitively about infinity [112]. The set-theoretic Zermelo-Fraenkel axioms are insufficient for handling large powers/cardinals and the axiom of infinity needs to be postulated [113]. Thus we should take a closer look at the MSR paradigm, under which what appears as the unspecific infinity in primary space is just a gateway into the secondary space associated with the primary space, in which zero appears as reciprocal counterpart of infinity. The MSR paradigm makes the flimsy concept infinity realistic.

9. ALGEBRAIC DIVISION BY ZERO CAN BE IMPLEMENTED AS GEOMETRIC MULTIPLICATION BY INFINITY

Let $\Re$ be a representation cast in a 3D Euclidean set-theoretical space. Let us represent the inverse mapping $r \rightarrow (1/r)$ of a scalar interval r, which is length-based (i.e. a distance), in a set-theoretical space $\mathbb{R}$ that is equipped with real-valued basis $[[\mathbb{R}]]$ onto point 1/r residing in a set-theoretical space $\mathbb{R}^{-1}$ equipped with real-valued reciprocal basis [[ꓤ]] whose value is just the inverse interval. Since spaces are identified by their bases, the representations are:

$$\Re[\![\mathrm{R}]\!]\mathrm{r} \mathrel{::=} \Re[\![\text{ꓤ} = \frac{1}{\mathrm{R}}]\!]\frac{1}{\mathrm{r}} \qquad (3)$$

where the double substitution symbol ::= means here both: substitution of value of the variable r as well as transition to the basis associated with the reciprocal space on the right-hand side (RHS). Notice that although the representations are different, because they depend on the respective basis in each space, they point to the same object, again because both the respective bases are inverted and so are the variables in their spaces.

The upside-down symbol ꓤ emphasizes the fact that the basis (and the space) standing on the RHS of the formula (3) is reciprocal to the primary space R (and its proper basis) that stands on the LHS of (3). Notice that the intervals (or line segments) on the LHS of (3) are different entities than the points on the RHS. It is a stark reminder that even single-valued numbers cannot always be identified with their values, which are just their attributes. Cantor identified numbers with their values and with points viewed as elements of numbers sets.

In a sense, the representations are neither functions nor mappings because they point to (or represent) the very same geometrical object as it is viewed from inside of each of their respective spaces. In traditional mathematics the inverse mapping $r \rightarrow (1/r)$ is considered as a function that maps r onto its reciprocal, which is not entirely wrong, but it misses the fact that the interval r, which is denominated in meters [m], does not belong in the same space as its inverse 1/r, whose basis is denominated in inversed meters [1/m]. This infantile mixup created confusion. The synthetic approach to doing mathematics must avoid such mixups.

Let us maintain the traditional intuitive reciprocal relationship between zero and infinity

$$\Re[\![\mathrm{R}]\!]0 \coloneqq \Re[\![\mathrm{Я}]\!]\frac{1}{\infty} \tag{4}$$

where the double substitution symbol ⩴ means that the infinity does not belong in the same space as zero. Yet according to the formula (3) zero and infinity in the formula (4) depict the same object but viewed from within two different mutually reciprocal spaces. In other words: in order to move zero from the LHS of the formula (4) to the RHS, the zero must be converted into 1/∞ and vice versa: in order to move/operate on infinity on the LHS of (4) it must be converted to 1/0. We just cannot use operands that do not really belong to the given operational context of the space that is native to the operands. In other words: we should not use the operands outside of the space in which they reside and if we would want to use them outside of their native space anyway then we should convert them into the other basis that is native to the foreign/nonnative space. Obviously, there would be no point in launching any operations on operands that are not really represented in the – foreign (to them) – space. By the way, an assertion "let something be" without specifying the native basis of its holding space is not really representation but a veiled existential postulate disguised as declaration.

One can see that proper implementation of the formula (4) can mean call for a certain multispatial structure. Within such a quasigeometric multispatial hyperspace that comprises paired mutually reciprocal spaces the once feared division by zero becomes nonissue and so are the associated operations involving infinity, provided one operates on them either in their native/home spaces, in which they reside, or in any foreign spaces, in which they could be properly represented, i.e. after being converted to the foreign basis. With the restrictions on operations on the reciprocal numbers in mind we can operate on both zero and infinity.

With the formulas (3) and (4) as preconditions for algebraic operations in multispatial structures in mind we can now expand and explain the foundational reciprocity principle to comply with common sense yet without generating the former paradoxes involving infinity

$$0 \coloneqq \frac{1}{\infty} \;\Rightarrow\; \frac{0}{0} = 1 = \frac{\infty}{\infty} \qquad 5)$$

unsurprisingly expanding thus the realm of former mathematics into an abstract multispatial hyperspace. Although this implication was derived from the assumption that real zero and infinity are reciprocal, it could also be interpreted as equality of zero and infinity. That the point at infinity might be represented by zero has already been pointed out in [114], which can be true under the MSR paradigm. However, the zero identified with an ideal point at infinity would have to be multivalued just as the infinity is, and therefore it should live in the same reciprocal space wherein the infinity lives. I am not saying that it is impossible for zero to represent infinity, but merely that we need a different zero for that. In other words: we need both the single-valued pointlike zero as well as a distinct intervallike/clusterlike zero whose value can be set-number. Hence the valuations of each of these zeros and of the corresponding to them reciprocal infinities should depend on the native basis in each of the two mutually reciprocal spaces. We need thus a multispatial hyperspace to accomplish that.

Nonetheless, we should also repudiate few other conventionally decreed nonsenses from the – utterly confused, to say the least –traditional mathematics in order for the multispatial structure of (both the mathematical and physical) reality to divulge previously inconsistent derivations as well as few unwarranted conclusions drawn with only lip service to logic. No concession to tacitly enacted traditional conventions should be made if these are illogical.

Couturat recognized the traditional formula 0×0=0 as the source of indetermination of both zero and infinity [12] p.99. For the zero (marked with dot as 0.) that is deeper than the regular zero, is only approximately equal to the regular zero, but it is conceptually different a number (i.e. yet another algebraic entity), for simple symbolic calculation shows that [11]:

$$0 ≔ \frac{1}{\infty} \quad \Rightarrow \quad 0 \cdot 0 = \frac{1}{\infty} \cdot \frac{1}{\infty} = \frac{1}{\infty^2} = 0. \quad \Rightarrow \quad |0.| \approx 0 \tag{6}$$

if we want to be operationally precise and conceptually correct as well. Yet as far as I know almost nobody (besides Couturat) ever objected to the former conventional multiplication of zero by zero, namely that: 0×0=0 even though it is conceptually illogical. Despite the easily verifiable fact that ∞×0=1 [11] and explanations of Euler [10], some authors routinely claim that just because n×0=0 is acceptable so also ∞×0= should be 0 [9], but it is not. I am using dot for the new rule for multiplication of scalar values of numbers just to distinguish it from the traditional rule. We should not sacrifice operational truth for thoughtless conventions.

The other revision, which has already been proposed in [11], is that one can obtain

$$0 ≔ \frac{1}{\infty} \quad \Leftrightarrow \quad \frac{0}{0} = 1 \quad \Rightarrow \quad 0 \cdot \infty = 1 \tag{7}$$

in the old conventional notation. The conclusion that 0·∞=1 further indicates that infinity is not a single-valued but a set-valued cluster number deserving thus its own separate space that is reciprocal to the primary space in which the regular zero resides. Notice that I am not making *ad hoc* rules. I derive the rules from the presumed reciprocity of zero and infinity. It does not mean making existential postulates, but I synthesize the rules and properties yet to be formulated contingent on the presumption. In other words: If the presumed principle of reciprocity of zero and infinity is true then these rules may be applied in abstract operations performed on them. Therefore, I shall leave it up to experiments to either confirm or reject the particular reciprocity principle along with all, or maybe just some, consequences thereof.

As I mentioned it above, some formerly unexplained (and even deemed as unexplainable by accepted theories of contemporary physics) experiments have been reconciled [115] and explained [116] in defiance of the traditional mathematics that explicitly prohibited division by zero while claiming that such an operation is impossible to perform. The only valid truth the prohibition conveyed – though without openly acknowledging it – is that the prohibition (and some of its allegedly disastrous consequences) was impossible to interpret, indeed. Yet because interpretations depend on the paradigms being espoused, the actual culprit in the past fiascos with division by zero was the unspoken SSR paradigm, not the operation itself. If we would follow this line of reasoning, the insane prohibition could have been lifted long ago, and countless theoretical failures to explain some thought- and/or actual experiments might have been avoided. Mathematics itself could be in better shape than it looks today.

10. MULTISPATIAL REPRESENTATION OF COMPLEX ZERO

Since complex zero actually comprises two distinct parts, it could be evaluated as:

$$[\![\mathrm{R}]\!](0{,}0) ::= [\![\mathrm{R}]\!]0 + [\![\mathrm{R}]\!]i0 \Leftrightarrow [\![\mathrm{ʁ}]\!]\frac{1}{\infty} + [\![\mathrm{ʁ}]\!]\frac{1}{i\infty} \Rightarrow [\![\mathrm{ʁ}]\!]\frac{1}{\infty} - [\![\mathrm{ʁ}]\!]\frac{1}{\mathrm{i}\infty} =:: [\![\mathrm{R}]\!]\overline{(0{,}0)} \qquad (8)$$

where ‘i’ is the imginary unit presumed to be an operator here. Henceforth I am skipping the repetitive representation symbol $\Re$ from the bases $\Re$[[R]] and $\Re$[[ʁ]] for the sake of visibility. The value of these real bases is one: [[R]]=[1] and [[ʁ]]=[1/1=1], which suffice for abstract real numbers. On the far RHS of the formula (8) stands the conjugate complex zero, of course. Just as every other complex number, the complex zero has its conjugate counterpart too.

If written explicitly in its algebraic operational form, the complex zero reveals (or hints at, if you will) its hidden underlying abstract structure. Since the operational procedure is supposed to be universally valid for all complex numbers including the complex zero, then it must fit the inadvertently hidden compositional structure of the complex numbers that is unveiled in (8). Although avoidance of handling difficult issues pervades mathematics, at some point even the pure mathematics has to speak the whole truth rather than evade most inconvenient issues. Unresolved mathematical issues do not really disappear but can leave some curious experimental results unexplained, which could otherwise be easily explained.

Case in point: two curious results of Sadeh experiments have been reconciled [115] and explained [116] even though they were deemed unexplainable [117] by any gravitational or electromagnetic theory, which is indeed correct an assertion because the underlying them both traditional mathematical theory of radial force fields had to be complemented by (then unknown) theory of nonradial interactions happening within the radial/center-bound fields.

I am not talking about the conventional complex plane on which the complex numbers can be depicted for most practical purposes, but about a certain prospective quasigeometric or geometric structural form. As long as division by zero (and by its reciprocal, the infinity) was prohibited, we had no compelling reason to ask the question about the operational form (procedure) or the corresponding to it structural form, i.e. the compositional structure. But the chain of formulas (8) begs for a serious explanation of what is really meant there. As being reciprocal to zero, the infinity must certainly be denominated in quite different basis, whether it is vectorial or just numerical. The right-hand side (RHS) of the formula (8) just demands a hetrogenous basis. We are talking about complex zero as an abstract object. Its value is just an attribute, not the object itself. The complex zero and all the other complex numbers are definite geometric objects. The synthetic approach thus asks questions which mathematics ignored for millenia. In other words: the formula (8) has both an operational and a structural meaning and these two distinct aspects must be compatible with each other.

Recall that extending functions of a real variable f(x) to that of a complex variable with zero imaginary part f(x+i0) is not uncommon in singular integral equations [118] which can be easier to solve in the complex domain while the effective value of the extended variable remains intact. Even complex number with zero imaginary part belongs in complex domain.

We know very well the value and the algebraic/operational form of complex zero, but what is its compositional structure? If our complex numbers are denominated in meters or seconds, for instance, then their reciprocals should be denominated in [1/m] or [1/s] as the spatial or temporal frequencies, respectively. Since both the real and complex numbers are single-valued numbers (even though complex ones comprise two parts) there is no realistic way to unambiguously denominate them in a single heterogenous basis (such as 4-vectors of energy-momentum, for instance) within a single space, then we should opt for creating such a heterogenous basis for a multispatial structure. In other words: instead of devising single heterogenous basis for each single space we can create two single heterogenous bases for a quasispatial structure that would comprise two distinct single spaces, which construction is not impossible. Indeed, it is feasible in the context of the MSR paradigm, but this requires a conceptual paradigm shift. Let me repeat it again: the prospective structure of the formula (8) demands multispatial approach if it has to be unambiguous. Each single space would be denominated in its own native basis and the complex (though not necessarily 2D) structure with unrestricted operations (including division by zero) would be denominated differently in every pair of spaces yet uniquely within each of the reciprocal spaces. Note that the very same object can be represented differently in each of the two mutually reciprocal spaces.

The point I am trying to emphasize is that – whether real or complex – zero and infinity is just a different local name for exactly the same quasigeometric structure, at least from the standpoint of the MSR paradigm. Zero is just a gateway to the extra space in which infinity can be treated as zero – in a distinct basis though – and vice versa. While a reciprocal pair of spaces is ostensibly multiplicative a structure, the complex pair of reciprocal spaces is both an additive and multiplicative quasigeometric structure, whose pairing is well, much more complex, in the common sense of the latter term. This aspect shall be elaborated elsewhere.

That zero and infinity can be viewed as two faces of the same abstract coin was alluded to by a team of Japanese researchers in [114] in term of values rather than spatial structures, though. From synthetic point of view, however, it is important to represent all mathematical concepts in both procedural (i.e. algebraic or operational, in general) as well as geometric or quasigeometric rendition. It can be accomplished in the context of multispatial hyperspace. This approach will be offered here by examples of real and complex division by zero in the multispatial context, because my idea of multispatiality was either ignored or suppressed.

11. MULTISPATIAL DIVISION OF COMPLEX ZERO BY COMPLEX ZERO

The formula (7): $0 \cdot \infty = 1$ implies that $0 \cdot \mathrm{i}\infty = \mathrm{i}$. Thus, the formula (8) implies

$$[\![\mathrm{R}]\!]\frac{(0,0)}{(0,0)} = [\![\mathrm{R}]\!](0 + i0) \cdot [\![\text{ᴚ}]\!]\left(\frac{1}{\infty} + \frac{1}{\mathrm{i}\infty}\right) = [\![\mathrm{R}]\!](1 + \mathbf{i} + \mathbf{i} - 1) = [\![\mathrm{R}]\!]2\mathrm{i} \qquad (9)$$

which is division of complex zero by complex zero. It is implemented via multiplication of complex zero in the primary set-theoretical space by the inverse complex zero represented in the associated/paired space that is reciprocal to the primary space. In a similar way we can calculate also the other possible combinations of divisions by zero in complex domain:

$$[\![\mathrm{R}]\!]\frac{(0,0)}{(0,0)} = [\![\mathrm{R}]\!](0 + i0) \cdot [\![\text{ᴚ}]\!]\left(\frac{1}{\infty} - \frac{1}{\mathrm{i}\infty}\right) = [\![\mathrm{R}]\!]2 \qquad (10)$$

$$[\![\mathrm{R}]\!]\frac{\overline{(0,0)}}{(0,0)} = [\![\mathrm{R}]\!](0 - i0) \cdot [\![\text{ᴚ}]\!]\left(\frac{1}{\infty} + \frac{1}{\mathrm{i}\infty}\right) = [\![\mathrm{R}]\!]2 \qquad (11)$$

$$[\![\mathrm{R}]\!]\frac{\overline{(0,0)}}{(0,0)} = [\![\mathrm{R}]\!](0 - i0) \cdot [\![\text{ᴚ}]\!]\left(\frac{1}{\infty} - \frac{1}{\mathrm{i}\infty}\right) = [\![\mathrm{R}]\!](2 - 2\mathrm{i}) = [\![\mathrm{R}]\!]2(1 - \mathrm{i}) \qquad (12)$$

which are also the results of complex division by zero implemented via multiplication by infinity. The values of the complex bases is: [[R]]= [1,1] and [[Ƀ]]=[1,1] too for 1/1=1. Notice that the result 2i in the eq. (9) might be interpreted (in the complex plane) as reversal (i.e. as taking two turns at right angle each). The imaginary operator i can operate also on spaces of higher dimensions. The full meaning of the above results shall be discussed elsewhere.

The reader can see that division of complex zero by real zero can also be performed as

$$[\![\mathrm{R}]\!]\frac{(0,0)}{0} = [\![\mathrm{R}]\!](0 + i0)\cdot[\![\mathrm{Ƀ}]\!]\left(\frac{1}{\infty}\right) = [\![\mathrm{R}]\!](1 + \mathrm{i}) = [\![\mathrm{R}]\!]\frac{0}{(0,0)} \qquad (13)$$

which gives us clue as to where the number 2 (that appears in the above complex equations) comes from; it is the additive double algebraic structure (or composition) of the complex numbers that causes the apparent doubling evident in the eqs. (9)-(12).

It is obvious – based on the above operations – that division by zero is possible, at least in the case when both the numerator and the denominator are zeros, whether real or complex or quaternionic or octonionic; for one could easily expand the operations also onto the latter two hypercomplex numbers. But one might ask what is the point in dividing just nothing by nothing, no matter what could be the result of such operations? All we have seen thus far is that it is indeed possible to link the allegedly indefinite infinity with definite zero in terms of algebraic operations. The next conceptually expected step shall be to unveil the prospective abstract structure in which the infinity – and whatever is hidden behind it – could reside. For the infinity obviously can play constructive role in multispatial context of the new synthetic approach to mathematics. Yet before extending the clearly feasible operation of division by real and complex zero on the other hypercomplex numbers, however, we should get better acquainted with the fearsome infinity in the prospective structural and the corresponding to it operational context of the (only vaguely outlined) superimposed multispatial hyperspace.

The vagueness was made on purpose for I want to deduce the structures from syntheses of both algebraic operations and quasigeometric structural relationships with the operations and with relevant conceptual input from physical sciences as well. If the structure would be declared *a priori* then I would end up investigating ideas concocted in my mind, because such declarations are existential postulates in disguise, which can create artificial a reality.

The Hegelian dialectics underlying syntheses, as emerging from proper confrontation of theses and their proper antitheses, should not be interpreted as acceptance of even illogical suppositions for the sake of contrasting them with the – rather irrational – expectation that something positive might eventually emerge even from bad ideas. Further algebraic topics (in quaternionic and octonionic domains) shall be discussed elsewhere. Now I shall show that the proposed here division by zero can resolve formerly unresolved issues in physics.

12. SPATIAL FREQUENCY 4-VECTOR IN MULTISPATIAL CONTEXT

The formula (9) is not really of an algebraic origin, however. I have deduced it from well-known and experimentally exemplified theories describing physical phenomena.

Writing the expression for wave/photon with the invariant phase ($\mathbf{k}\bullet\mathbf{r} - \omega t$) in the form

$$\exp(\mathbf{k}\bullet\mathbf{r} - \omega t) \;\Rightarrow\; \mathbf{K} = \left[\mathbf{k}, \frac{\omega}{c}\right] \;\Rightarrow\; \mathbf{P} = \left[\frac{E}{c^2}\mathbf{c}, \frac{E}{c}\right] = \hbar\mathbf{K} \qquad (14)$$

where the value of the wave vector/number |**k**|=1/λ is the inverse of the wavelength λ, and the elapsing time parameter t acts on the angular velocity ω and c is the speed (i.e. scalar velocity) of light in vacuum; **K** is the [spatial, i.e. length-based] frequency 4-vector and **P** is the corresponding to it momentum 4-vector, E=hν=ℏω is the energy of the wave/photon that is expressed in terms of temporal frequency ν, and the Planck constant $\hbar$=h/2π denotes the quantum of action [119, p. 422]. Note that in order to comply with the angular velocity that is specified as angle per elapsing time, the wave vector must thus be scalarly multiplied by the radius vector **r** that points to the wave curve. The other curiosity is the vectorial magnitude of velocity of light **c**, which makes the whole vectorial notation consistent, even though the speed of light is the same in all spatial directions. By the way, all the 4-vectors appear in the form of quaternions. But despite all these conceptual curiosities, the chain of formulas (14) is both operationally and conceptually correct in the context of the former SSR paradigm.

Physically thus, the meaning of 4-vectors is well-understood. Yet one might still ask the question: what kind of theoretical hint might these curiosities suggest? Or perhaps: in which other theoretical context would these curiosities disappear or perhaps become reconciled?

13. DUALITY OF MULTISPATIAL DIFFERENTIAL OPERATORS

A new heterogeneous 4D nabla operator $\boxed{\nabla}$ was defined symbolically (on Grassmannian terms) as an abstract complex (rather than real) primary extension of the regular 3D nabla operator ∇ via a certain new 1D furled (i.e dimensionally compressed) nabla operator $\forall$:

$$\boxed{\nabla} \coloneqq \nabla + i\forall \qquad (15)$$

where the furled nabla $\forall$ is an imaginary operator with respect to the regular nabla operator ∇, for the 1D $\forall$ cannot be directly represented in terms of the vector basis of the primary space on which the regular 3D nabla operator ∇ is defined. For no other (than those three already present in the regular 3D nabla operator) distinct fourth component could ever be directly orthogonal to all the three directional components of the 3D nabla operator ∇ [120].

The furled 1D nabla operator $\forall$ has thus the effect of reducing the formal dimensional depiction from 3D to 1D. It supports thus unfurling of coordinates in the 3D primary space and furling of coordinates in the 1D reciprocal space that is associated with the primary one.

One can see that when the speed of light c is used as the reciprocal/inverted interspatial conversion coefficient from the 3D space represented by the usual 3D nabla ∇ into (one of) its reciprocal/inverse 1D spaces represented by the furled/curly nabla operator $\forall$ (i.e. as the conversion between their bases) the prototype formula (15) can be reformulated as follows:

$$\boxed{\nabla}^2 \coloneqq \nabla^2 + (i\forall)^2 \;\Rightarrow\; (ds)^2 = (dx)^2 + (dy)^2 + (dz)^2 - (cdt)^2 \qquad (16)$$

which implies also the reformulated line element cast within the multispatial hyperspace. One can see that if the spatial differentials dx,dy,dy would be treated in the vectorial terms rather than values then the algebraic signature (+++-) of the eq. (16) would be turned into geometric signature (---+). Hence the above 4-vectors present in the formula (14) allude to their natual multispatial treatment. These topics shall be further discussed elsewhere.

The regular 3D nabla operator ∇ resembles thus the effect of unfurling of the primary coordinates by expanding the abstract formal representation of an object corresponding to a transition function from the furled 1D to the usual/unfurled 3D formal representation of the given object in a single 3D space. This is a direct consequence of the prototype formula (8), which could be deduced also from the dual representation of the 4D spacetime that is overlaid by its dual 4D timespace [121]. In the prototype formula (15) I have assumed that each operator carries its own native homogeneous basis [122]. Hence the 4D differential operator $\boxed{\nabla}$ carries its own heterogeneous basis that comprises the two homogeneous bases. These issues shall be further explained elsewhere. In the present paper they are brought in just in order to shed the (conceptual) light on the topics discussed in the next section.

14. STRING DUALITY AND MULTISPATIAL UNCERTAINTY PRINCIPLE

When the point particles are replaced by small loops, as it happens in string theory, the stringy constant ά representing string tension has the dimension of length squared if one sets $\hbar$=c=1 [123, p .25f]. Yet if one tries to squeeze the string beyond the smallest circle of radius ρ the space will reexpand in yet another direction; hence the conclusion that there is no way to go deeper down beyond the string scale of $\sqrt{ά} \approx 10^{-34}$ [m], which seems to be the smallest infinitesimal [123, p. 29], clearly agrees with the point of view underlying the MSR paradigm as it is exposed in this present paper. This topic shall be further discussed elsewhere.

The stringy radius ρ can be viewed thus as local approximation of a certain stringy zero in the primary 3D space and therefore we can expect that a certain stringy infinity – which would correspond to the zero –should also exist in the 1D reciprocal space that is associated with the primary one. Notice that I am not making any existential postulates here, but only reformulating the issue at stake in terms of the multispatial hyperspace, which supplies just two alternative views depicting the very same object (here: string) as 3D or curly 1D object.

Furthermore, it has been suggested that – from the standpoint of the stringy constant – it appears as if the uncertainty principle should have the following extended composite form:

$$\Delta\, x \geq \frac{\hbar}{\Delta \mathrm{p}} + ά\, \frac{\Delta \mathrm{p}}{\hbar} \qquad (17)$$

where p denotes here the value of linear momentum [123, p. 29]. However, in the multispatial hyperspace each primary space shall have its own uncertainty principle. But the conclusion that the - clearly reciprocal - constant $\sqrt{ά}$ should also have an uncertainty principle on its own is quite correct indeed. In the multispatial hyperspace the 1D reciprocal magnitudes gained their own primary spaces in which they could vary quite independently.

Since the formulas (14) are evidently consistent with both the special theory of relativity (STR) as well as with quantum mechanics, and the prototype formula (15) has been deduced from the 4D spacetime whose conception was also deduced from the STR, then perhaps the conventional quantum mechanical formulas should also comply with the formula (15). Thus the inequality (17) should also comply with the MSR and should be reformulated too.

Hence the far RHS term in the uncertainty formula (17) actually pertains to the spatially localized potential energy of the underlying stringy fields, i.e. the primary and its reciprocal:

$$(\Delta x + i\zeta\Delta\lambda) \geq \frac{\mathrm{h}}{4\pi\Delta \mathrm{p}} + i\zeta\frac{\mathrm{h}}{4\pi\Delta \mathrm{p}} \geq \frac{\hbar}{2\Delta \mathrm{p}} + i\zeta\frac{\hbar}{2\Delta \mathrm{p}} \quad \Rightarrow \quad \zeta = f(\tfrac{1}{\acute{\alpha}}) \qquad (18)$$

where the auxiliary variable ζ denotes the conversion factor from the reciprocal basis to the native basis of the primary length-based space (LBS), in which neither the wavelength λ nor the stringy constant ά cnould be directly represented. Since the uncertainty principle implies certain wave aspects of a particle [or string] then there must be a momentum associated with each wave packet (p=h/λ), which is the relationship that de Broglie discovered [124]. Since the uncertainty relation (18) is shown entirely in the (primary) LBS space, the reciprocal magnitudes λ and ά must be converted to the native basis of the primary spaceon the LHS.

The latter appears in the multidimensional uncertainty formula (18) as a certain function of the stringy constant for the sake of simplicity. This issue shall be discussed elsewhere.

Nevertheless, I can hear the question: Where did you get the damn 4π in (18)? If we move things from the RHS to the LHS of an equation or inequality – as we did thus far – we were just unfurling the 1D representation into a 3D representation according to prototype formula (15). In (18), however, we have moved terms from the LHS to RHS, i.e. from the unfurled 3D representation to the furled 1D representation. Hence we did virtually employ a funnel squeezing 3D volume into the 1D line of flow (of energy) that – after passing the ideal point at infinity – ends up in yet another space, wherein it can become unfurled again ino a 3D inverted representation in yet another basis, though, which process shall be fully explained elsewhere. The funnel is just a pseudosphere of constant negative curvature whose surface (because the line segment Δx still remains on the LHS) is equal to 2π [125,126].

This particular process is concisely explained by the Gauss-Bonnet theorem [127]. Notice that we are talking about two 2π funnels connected at its pinlike ends, i.e. at the gateway through the apparent infinity. Recall that pseudosphere is a folded hyperbolic triangle. Hence these two connected funnels amount to the surface of the 4π when counted together. However, as with every other truth, this particular result could be explained in several other ways, of course.

By the same token, the time-energy uncertainty relation could be formulated as follows

$$(\Delta t + i\xi\Delta\nu) \geq \frac{\hbar}{2\Delta \mathrm{E}} + i\xi\frac{\hbar}{2\Delta \mathrm{E}} \qquad (19)$$

where the auxiliary variable ξ denotes here the conversion factor from the reciprocal basis to the primary basis in which the wave frequency ν cannot be directly represented and E is the energy. The physical nature of the new converting variable ξ shall be explained elsewhere. Recall that the precision about [the determination of elapsing] time imposes also a certain vagueness about the frequency ν, and vice versa [128, p. 53], for signal [propagating in the physical space] just cannot be localized simultaneously in time and frequency [128, p. 209].

The previous uncertainty relations involved both zero and infinity as mutually reciprocal entities. For example: in the case of zero time delay, the uncertainty principle would require that there be essentially no energy dependence in the (nuclear) scattering amplitude ($\Delta E \to \infty$ since $\Delta t \to 0$) [129]. That is why unrestricted division by zero is instrumental for physics too.

As pertaining to regions of extreme energy and infinitesimal distances, the string theory made the abstract relationship between zero and infinity relevant to physics again. Although the former concept of infinity was not obliterated by the MSR paradigm, the quasigeometric structure of multispatial hyperspace removed its former, philosophical, as it were, mystery.

15. WHY WAS THE PROHIBITION ON DIVISION BY ZERO INSTITUTED?

If a course in mathematical pathology would ever be offered, a lot of topics now taught as legitimate abstract knowledge, and pretending to represent sophisticated pure mathematics, could end up in that course. Mathematicians certainly have healthy brains (understood as the engines designed and calibrated for logical abstract reasonings) but their minds (understood as the software that fuels and operates the brains) sometimes appear somewhat diseased, if judged by some of their most abstract yet frequently rather evasive creations. Inadvertent or not, the evasion of easily perceivable truths for the sake of intellectual glory, is astonishing.

The evasion of truth practiced in pure mathematics is widespread. Thus their "abstract diseases" spread fast and far beyond mathematics indeed, even threatening to engulf also the most vulnerable physical applications that must rely on the theoretical mathematics without ever doubting, not to mention questioning, its conceptual validity or operational correctness or the structural constructivity of its abstract ideas. No pun on étale cohomology intended.

Algebraic nonsenses create conceptual confusion, which could adversely affect mental health and sometimes be deadly even to such great minds as Georg Cantor. The prohibition on division by zero is fairly easy to recognize as clearly very illogical decree, if not obvious conceptual nonsense, but it was not allowed to be challenged. Why?

Geometric nonsenses are often much more difficult to identify and even more difficult to rectify conceptually than the algebraic ones, but not really impossible to remedy at least operationally, given the copious experimental evidence that seemingly contradict our most sophisticated axioms. The axioms worked well up to third dimensions, but fail (at least in some cases) for more than four-dimensional abstract quasigeometric structures. Yet the pure mathematics resisted the need to alter the – implicitly suggested by certain previously quite unexplained results of some formerly unanticipated experiments – assumptions. Why?

That is why it is important to ask the question "why?" in order to avoid repeating those past mistakes and blatant misrepresentations. If just a single individual, or perhaps a small fringe group, commits a mistake and sheepishly defends it despite its indefensibility, then it might be traced to undue influence of some too influential individuals. But nearly universal acceptance (or just tolerance) of obvious conceptual nonsenses, is very suspicious indeed.

As with other investigations, the inquiry into what caused the apparent suppression of truth that resulted in instituting the infamous prohibition on division by zero, should perhaps start with asking who could have benefited from the prohibition. Or perhaps against whom it could be deployed. There was only one person who has actually performed the prohibited yet unrestricted operation of division by zero: Jesus Christ, the son of God the Father, when He was incarnated as mere man, and though sinless, He volunteered to be crucified and thus has suffered an infinite penalty (which is the inversion 1/0 for His zero sins) through which He offered free redemption to as many as would accept His offer (i.e. to practically infinite number of repentant sinners who would desire to get everlasting life in heaven).

Technically, there is nothing special about performing division by zero. Yet because the devil, who is Christ's adversary/satan, tries to pervert and obscure everything that pertains to what Christ has done [130], it is evident that the devil (and his cohort of wicked spirits) is the one most interested in preventing the spread of understanding of the redemption offer. For the devil makes every effort to confuse otherwise intelligent scientists even when it comes to such trivial topics as simple algebraic operations and easily comprehensible ideas.

The prohibition on division by zero might have originated from a confusion of a single individual, but the universal acceptance of the deception caused by the prohibition suggests that it is supported by an influential individual (or a group) of not terrestrial origin, who can reward those seduced to become the unintended mercenaries of the otherworldly powers.

For – as Apostle Paul wrote – "we wrestle not against flesh and blood [i.e. not against people], but against principalities, against powers, against the rulers of the darkness of this world, against spiritual wickedness in high [places]" [131]. The wickedness is the result of a war [of words – Greek: polemos] that had erupted in heaven – see [130], which also answers the question why is the world so bad, since it was declared in the Bible to be very good at the time it was created. The former prohibition on division by zero – which is indeed the most spectacular mathematical nonsense taught from kindergarten up to doctoral seminars – had been instituted clearly in order to diminish the sacrifice of Christ on behalf of all of us, and to confuse even those who can understand mathematics, not to mention all others [132].

In the sense, the infamous prohibition on division by zero effectively suppressed thus not only the structural truths outlined above but also messed up logical reasonings in other sciences as well as in all other domains of human activities. Notice that either intentional exaggeration or suppression of truth, by which an injury may result to others, is a violation of the God's 9th commandment that forbids bearing of false witness [133].

By suppressing truths and thus instituting prohibition on division by zero, the so-called "pure" theoretical mathematics walks on crutches while breeding nonsenses, and the physics that is either unwilling or unable to question the nonsense, is staggering behind the confused (conceptually, operationally and structurally) mathematics.

16. CONCLUSIONS

It has been shown that algebraic division by zero is quite feasible and can be reasonably implemented via the inverse operation of multiplication by infinity treated as reciprocal of zero. The fact that the division was once prohibited has its roots in misunderstanding of the notion of infinity, which can be fairly well determined operationally and structurally if the unspoken former paradigm of single-space mathematical reality is replaced by a multispatial reality paradigm. The proposed here division by zero has been explained by examples in the real and complex domains. It was shown that the unrestricted division by zero can make some heretofore unexplained topics, even in advanced mathematical physics, nonissues.

The unrestricted division by zero also suggests that the multidimensional extension of the former quantum-mechanical uncertainty principle, which was conventionally restricted to the usual length-based single space, should be extended onto the multispatial hyperspace in agreement with the suggestion that had already been derived also from certain abstract mathematical investigations pursued in the string theory that is still restricted to the former single space reality though. It has been shown that the special theory of relativity, quantum mechanics, and also the string theory, which attempts to expand the previously developed quantum mechanics, all comply with the quite unrestricted division by zero. The former prohibition on division by zero virtually constricted thus the prospective development of several theories of physics. Unrestricted division by zero is also needed for the multispatial reality paradigm to be implemented in both mathematics and physical sciences.

If cast within the context of multispatial hyperspace, the potential infinity that was often considered as neverending counting, corresponds to an actual infinity that can be regarded as quasigeometric spread of numbers/points in the multispatial hyperspace. This conclusion is synthesized in the context of abstract multispatial paradigm under the presumption that to each algebraic or operational procedure a geometric (or quasigeometric) spatial (or perhaps a quasispatial) structure should correspond and vice versa. This assumption is made in order to ensure full constructability of the structures within the multispatial hyperspace as well as quite unrestricted operability of the operational procedure that corresponds to the structure.

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