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Shapley value as a measurer of shareholders decision power

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ABSTRACT

Making decisions in joint-stock company on stockholders meeting is an example of cooperative game. Cooperative game theory focuses on the coalition players may form. Voting at the general meeting of shareholders is a special kind of cooperative game. We assume each coalition may attain some payoffs, and then we try to predict which coalitions will form. To determine the solution and measure the ability of shareholders to create victorious coalitions we can use Shapley value. Among the shareholders it assigns a unique distribution of a total surplus generated by the coalition of all players.

***Keywords*:** Game theory, Shapley value, joint-stock company, decision power

1. INTRODUCTION

Von Neumann and Morgenstern introduced the cooperative game in the form of a co-operative characterization of the coalition [Copeland, 1945]. Consider the interaction between a potential seller and two potential buyers of some object that the seller who is the current owner values at ten euro, the first buyer values at twenty euro, and the second buyer values at thirty euro. If the players can freely transfer money among themselves, and if they are risk neutral, this situation can be modeled as the game with players N = {1,2,3} and v given by v(1) = 10, v(2) = v(3) = v(23) = 0, v(12) = 20, v(13) = v(123) = 30. This situation shows that only coalitions containing the seller (player 1) and at least one buyer can create transaction

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that gives profit. A coalition that contains player 1 is worth the maximum that the object in question is worth to any member of the coalition. The tools of cooperative game theory applied to this model reflect some of the important features of such an interaction. For example, the core of the game corresponds to the set of outcomes at which the seller sells to the buyer with the higher reservation price, at some price between twenty and thirty euro, and no other transfers are made. Von Neumann and Morgenstern proposed solution for such a game, which today is called a stable set or a von Neumann-Morgenstern solution [Roth, 1988].

Shapley proposed to summarize the complex possibilities facing each player in a game in characteristic function form by a single number representing the value of playing the game [Shapley, 1953]. Thus the value of a game with a set N = {1, . . . , n} of players would be a vector of n numbers representing the value of playing the game in each of its n positions. Formally defined, we say that for every set of players N = {1,2, ..., n}, we assign a certain value of v(K) to the union, which expresses the payoff that members of the coalition can work together. This game has a certain property, which we call superadditivity. It means that each of the two players can get more acting together than they would get together, but acting separately. Let us assume that K and L are disjoint coalitions of the set N. The characteristic function v will have the property of superadditivity if it fulfilled the following condition:

$$(K \cup L) \geq v(K) + v(L), \text{ if } K \cap L = \phi. \qquad (1)$$

and we have:

$$v(K \cup L) = v(K) + v(L), \qquad (2)$$

then it is not worthwhile to create coalitions.

Superadditivity is justified when coalitions can always work without interfering with one another. The value of two coalitions will be no less than the sum of their individual values. Superadditivity implies that the grand coalition has the highest payoff.

Consider a situation in which agents need to get connected to the public good in order to enjoy its benefit. The example from [Shoham, Leyton-Brown, 2008] is the problem of multicast cost sharing and based on characteristic function with the property of superadditivity. A group of customers must be connected to a critical service provided by some central facility, such as a power plant or an emergency switchboard. In order to be served, a customer must either be directly connected to the facility or be connected to some other connected customer. Let us model the customers and the facility as nodes on a graph, and the possible connections as edges with associated costs. This situation can be modeled as a coalitional game (N, v). N is the set of customers, and v(S) is the cost of connecting all customers in S directly to the facility minus the cost of the minimum spanning tree that spans both the customers in S and the facility.

Another example from [Shoham, Leyton-Brown, 2008] concerns sharing the cost of a public good, along the lines of the road-building referendum. A number of cities need airport capacity. If a new regional airport is built the cities will have to share its cost, which will depend on the largest aircraft that the runway can accommodate. Otherwise each city will have to build its own airport. This situation can be modeled as a coalitional game (N,v), where N is the set of cities, and v(S) is the sum of the costs of building runways for each city in S minus the cost of the largest runway required by any city in S. This airport game is also an example of superadditive game.

In the n-personal cooperative game, players have to divide the total utility of v(N). This value can be divided in any way, but it is obvious that no rational player will agree to a division in which he obtains less than he would have obtained by acting alone. The division called also the imputation in the n-personal co-operative game is the vector x = (x1, ..., xn) that meets the conditions:

$$\sum_{i \in N} x_i = v(N), \tag{3}$$

$$x_i \geq v(\{i\}) \text{ for every } i \in N. \tag{4}$$

Condition (4) means optimality in the sense of Pareto.

The essence of solving a given n-personal game is to indicate such a division or to define a set of divisions that will satisfy all the players. Considering the concepts of n-personal game solutions, Shapley in [Shapley, 1953] formulated three axioms that reflect the idea of a fair division and proved that in every n-personal game with the property of a superadditivity characteristic function, only one payoff system v(S) can be clearly identified and there is exactly one imputation defined for all axiomatic games:

$$\sum_{S} \varphi_i[v] = v(S). \tag{5}$$

For any permutation we have:

$$\varphi_i[\pi v] = \varphi_i[v]. \tag{6}$$

For any game we have:

$$\varphi_i[u+v] = \varphi_i[u] + \varphi_i[v]. \tag{7}$$

The Shapley value is calculated as follows:

$$\varphi i[v] = \sum_{\substack{T \in N \\ i \in T}} \frac{(t-1)!(n-t)!}{n!}[v(S) - v(S - \{i\})] \, . \tag{8}$$

The summation runs after all coalitions to which system S belongs. When we have a simple game, the value of Shapley is simplified because the equation gives always value 0 or 1:

$$\varphi_i[v] = \sum_{\substack{T \in N \\ i \in T}} \frac{(t-1)!(n-t)!}{n!} \, . \tag{9}$$

In the example of one seller and two buyers, the three players can create coalition in six possible orders. If they enter in order 1,2,3, their marginal contributions are (0,20,10), and their Shapley values are the average of these marginal contributions over all six orders:

$$\varphi_i[v] = (18.33, 3.33, 8.33).$$

An axiomatic characterization of the Shapley value show that there is a unique value satisfying the efficiency, symmetry, nullity, and additivity axioms, and this value is the Shapley value. The efficiency assumes that the value v(V) is the sum of separates values for every player which is consider to create coalition. We say that players are symmetric in game if they make the same marginal contribution to any coalition. Nullity means that a player is a dummy player if her marginal contribution to any coalition is zero. And additivity assumes that for given two games (N, v) and (N, w), we define the game (N, v + w), by (v + w)(S) = v(S) + w(S).

Consider a game with more players, for example modeled on the United Nations Security Council, which consists of fifteen members. Five of these are permanent members and have a veto, and ten are rotating members. The voting rule is that a motion is passed if it receives nine votes and no vetoes. Let's denote v(S) = 1 if S contains all five permanent members and four or more other members, and v(S) = 0 otherwise. Because 15! is a number on the order of 10^{12}, obviously it is time-consuming and arduous to calculate the Shapley value by enumerating all possible orderings of the players. But to calculate the value random-order property can be used, along with symmetry and efficiency. Note that by symmetry all rotating members have the same value $\varphi_r(v)$, all permanent members have the same value $\varphi_p(v)$ and efficiency requires that $10\ \varphi_r(v) + 5\ \varphi_p(v) = 1$. In order for a rotating member to make a positive marginal contribution in a random order, all five permanent members and exactly three of the other nine rotating members must precede him or her. There are 9!/3!6! such coalitions, corresponding to the different ways to choose three out of the nine other rotating members. Such coalition S (of size s = 9) occurs with probability (s – l)!(n – s)!/n!.

Corresponding to equation [9], the Shapley value of a rotating member is:

$$\varphi_r(v) = (9!/3!6!)(8!6!/15!) = 0.00186$$

and the Shapley value of a permanent member is:

$$\varphi_p(v) = (1 - 10\ \varphi_r(v))/5 = 0.196,$$

which is over 100 times greater [Roth, 1988]. The results of a similar calculation, using the then existing rules of the Security Council, are described in the 1954 paper of Shapley and Shubik, which was the first to propose applying the Shapley value to the class of simple games, which are natural models of voting rules. A simple game is a game represented by a characteristic function v that takes on only the values 0 and 1. Because simple games are essentially no more than lists of winning coalitions, they are often natural models of situations in which the full weight of the usual assumptions about characteristic function games may not be justified [Roth, 1988].

2. CREATING WINNING COALITION

Suppose that there are two players and payoff for them is v(1) = 10, v(2) =12 and v(1,2) = 23. There are two possible orders of arrival: (1) first 1 then 2, and (2) first 2 then 1. If 1 comes first and then 2, 1's contribution is v(1) = 10; when 2 arrives the surplus increases from 10 to v(1,2) = 23 and therefore 2's marginal contribution is v(1,2) – v(1) = 23 – 10 = 13. If 2 comes first and then 1, 2's contribution is v(2) = 12; when 1 arrives the surplus increases from 12 to v(1,2) = 23 and therefore 1's marginal contribution is v(1,2) – v(2) = 23 – 12 = 11. Thus 1's expected marginal contribution is: ½ * 10 + ½ * 11 = 10.5 and 2's expected marginal contribution is ½ * 13 + ½ * 12 = 12.5. Then the Shapley value is calculated as:

$$x_1 = 10.5 \text{ and } x_2 = 12.5.$$

In a given n-personal game players can join a given coalition on n! ways, so with the increase of the number of players, it becomes more and more complicated to list all possible variants of coalitions creation, and thus the calculation of the value. However, the calculation method can also be described intuitively. Each time a new player joins a coalition, we reward the coalition with a payoff of the value given the player brought in enlarged coalition. Calculating the average of the imputations obtained for each possible order, we get Shapley value. Let's analyze the following simple game called "Christmas promotion":

During Christmas, shopping malls encourage customers to buy, mainly parents with children, running a promotional campaign using characters like: Santa, Reindeer and Elves. The performance of these characters is supposed to introduce a festive atmosphere, while encouraging them to re-visit the store. Let's assume that the store wants to employ three characters: Santa, Reindeer and Elves, who appear together in the store for one day's work and receive 1000 euro. It is also known that if only Santa and Reindeer performed, they would earn 800 euro, while Santa with Elves earn 650 euro, and Elves with Reindeer only 500 euros. Santa himself would earn 300 euro, Reindeer 200 euro, while Elves appearing alone is not an attraction, so he would not be employed. Table 1 shows the withdrawals of individual characters acting separately and together.

Table 1. Payoffs of each team in Christmas Promotion game

Team	Payoff
Santa, Reindeer, Elves	1000
Santa, Elves	650
Santa, Reindeer	800
Reindeer, Elves	500
Santa	300
Reindeer	200
Elves	0

In this game, the superadditivity property is fulfilled. Each of the two characters acting together will earn more than they would earn together, however, acting separately. Let's assume that these 3 characters will not appear in the store in one time. Therefore, we have six possible sequences:

1) Santa, Elves, Reindeer
2) Santa, Reindeer, Elves
3) Elves, Santa, Reindeer
4) Elves, Reindeer, Santa
5) Reindeer, Santa, Elves
6) Reindeer, Elves, Santa

It is assumed that the results of the setting are equally likely. In addition, let's assume that each sequence of the arrival of each character is interpreted as a process of forming a team. We are interested in the value of the payout that a given player brings to an existing team. In the case of order # 1, the Reindeer finds a coalition {Santa, Elves}, worth according to Table 1, 650 euro. Upon its arrival, the value of the coalition rises to 1.000 euro. This is an increase of 350 euro. In the same order Elves finds a coalition {Santa}, worth 300 euro. His arrival brings the value up by 350 euro. We perform the same analysis for the remaining sequences. All possible sequences of team formation are as follows:

1: Santa, Elves, Reindeer
2: Santa, Reindeer, Elves
3: Elves, Santa, Reindeer
4: Elves, Reindeer, Santa
5: Reindeer, Santa, Elves
6: Reindeer, Elves, Santa

In Table 2 are calculated the amounts that each character brings after arriving at the scene.

Table 2. Input from joining a new character to existing team

Santa input	Elves input	Reindeer input
300	350	350
300	200	500
650	0	350
500	0	500
600	200	200
500	300	200

Then we calculate the average contribution of each character for all possible sequences of formation. In this way, we receive a distribution of earnings for three characters: Santa = 475 euro, Elves = 175 euro, Reindeer = 350 euro. This division is called Shapley's value.

Shapley's value is one of the ways to solve a cooperative games. Its advantage is mathematical simplicity. There is another very important point: Shapley's value always exists and there is always only one [Roth, 1988].

Consider three companies (A, B and C) which can cooperate to generate a measurable outcome. If nobody participates nothing is produced, and each participant alone can produce 4 units. The output of each possible grouping of the three participants is detailed in the Table 3.

Table 3. The output of possible teams created by producers with marginal contribution

Created Team	Team output	Marginal contribution of A	Marginal contribution of B	Marginal contribution of C
A	4	4	0	0
B	4	0	4	0
C	4	0	0	4
A, B	9	5	5	0
A, C	10	6	0	6
B, C	11	0	7	7
A, B, C	15	4	5	6
Shapley value	**---**	**4,5**	**5**	**5,5**

The marginal contribution of a player to a team is calculated as the output of the team minus the output of the same team excluding the individual participant. For instance, the marginal contribution of A to the output of the overall team (A, B, C) is equal to the difference between 15, which is the overall team's output, and 11, which is the output of B and C together. The Shapley value of each player is the average of its marginal contributions across all differently sized teams. For example, the value of B is equal to 5. It is calculated as the average of 4, which is its individual output, 6, which is the mean contribution it makes to team of size two, and 5, which is its marginal contribution to the overall. The calculation can also be motivated as the expected marginal contribution of an individual participant in teams that are formed randomly by sequentially selecting players.

3. MEASURING THE DECISION POWER

Voting at the general meeting of shareholders is a special kind of cooperative game [Farquharson, 1969]. Each participant has a certain number of votes depending on his or her shareholdings in the company. We have n players - the shareholders of the company. The participation of the player in the company is expressed as the percentage of shares (or votes at the general meeting) and shares of all players will amount to 1.

Let's analyze the simplest example, where we have four shareholders, whose shares are respectively: 35, 25, 20, 20. Let us assume that a majority of the shareholders is required to adopt the resolution, so it is needed at least 51 votes. Thus, a coalition that chooses more than half of the vote decides on each resolution. Such a coalition, which we call a winner, have payoff of 1. And the coalition which has no majority is assigned a payment equal to 0.

In the first step we determine how the individual players' settings affect the result of the vote. Table 3 contains coalitions of shareholders (for example, coalition {1.4} consists of shareholder No. 1 and shareholder No. 4) and assigned a payoff according to the division winning/losing. Payoff equal to 1 is given to a coalition which have at least 51 percent of votes.

Table 4. Acting together as winning or losing team

Coalition	Payoff (win/lose)
Shareholder no. 1 separately	0
Shareholder no. 2 separately	0
Shareholder no. 3 separately	0
Shareholder no. 4 separately	0
Shareholder no. 1 + Shareholder no. 2	1
Shareholder no. 1 + Shareholder no. 3	1
Shareholder no. 1 + Shareholder no. 4	1
Shareholder no. 2 + Shareholder no. 3	0
Shareholder no. 2 + Shareholder no. 4	0
Shareholder no. 3 + Shareholder no. 4	0
Shareholder no. 1 + Shareholder no. 2 + Shareholder no. 3	1
Shareholder no. 1 + Shareholder no. 2 + Shareholder no. 4	1
Shareholder no. 1 + Shareholder no. 3 + Shareholder no. 4	1
Shareholder no. 2 + Shareholder no. 3 + Shareholder no. 4	1
Shareholder no. 1 + Shareholder no. 2 + Shareholder no. 3 + Shareholder no. 4	1

In the second step, we check the players' settings where the new player has a crucial vote. When calculating the Shapley value, we take into account all possible player settings, in this case there will be 24 of them. They are as follows: 1234, 1243, 1324, 1342, 1423, 1432, 2134, 2143, 2314, 2341, 2413, 2431, 3124, 3142, 3214, 3241, 3412, 3421, 4123, 4132, 4213, 4231, 4312, 4321. As before, we interpret each such setting as the process of forming a coalition and assume that all settings are equally likely. Next, we check how the value of the coalition has changed after joining new shareholder. When a player has joined a losing coalition, which after his joining it is still a loser, or when the player has already found the winning coalition, the payoff change will be zero. In the case when the player found the losing coalition, which after joining it became the winning coalition, the change will be one, and the joining player will be determined as crucial. Analyzing all possible settings of joining to coalition mentioned earlier we can see that shareholder no. 1 is crucial with 12 coalitions, shareholder no. 2 is crucial with 4 settings, same shareholder no. 3 and no. 4. This means that the "average" contribution of each player, equal to the number of settings at which this player is crucial, divided by the number of all settings (24), is for shareholder no. 1 ½ and for the rest 1/6. In this way, we get the Shapley value, which expresses the ability of individual players to form a coalition capable of voting for a given resolution. As we can see although shareholder no. 2 has a larger percentage shares in the company than shareholders no. 3 and no. 4, his ability to influence on winning votes is the same. It can be said that the surplus of shares is in some sense "wasted" and is not properly translated into its decision-making power at the shareholders' general meetings.

4. ANALYSIS OF SHAREHOLDERS DECISION POWER

The shareholders decision power analysis was made for selected companies. Such an analysis makes sense only for companies in which any shareholder has more than 50 percent of shares. It is worth noting that shareholders who are individual persons, especially in the case of companies listed on the Stock Exchange, may own a small number of shares, however, the total value of these shares may account for nearly 50% of shares, or more than half of shares. In addition, small shareholders who use shares mainly to play on the stock market rarely appear at the general meeting of shareholders. These shares are recognized as free float and are not included to calculate votes. Therefore, in some cases, a more interesting analysis concerns the situation of decision power over the total number of votes at the company's general meeting. It happens quite often that only shareholders possessing a larger number of shares participate in the general meeting.

Table 5. List of shareholders for Orlen S.A.

Shareholder	Shares
State Treasury	27,52%
OFE Nationale-Nederlanden	8,32%
OFE Aviva BZ WBK	7,16%

PERN SA	4,90%
OFE PZU Złota Jesień	4,14%
OFE MetLife	2,49%
OFE AXA	2,36%
OFE Generali	1,65%
OFE Allianz Polska	1,53%
OFE PKO BP Bankowy	1,52%
OFE Nordea	1,51%
Others (each below 1%)	36,90%

Consider the shareholders of the polish company from the fuel industry – Orlen S. A. Shareholders holding at least 1% of the total shares accumulate 63,1% of total shares. Free float on market is estimated on 57%. Table 5 contains a list of shareholders holding more than 1% of shares.

On 30 June 2017 a shareholders' meeting took place on which weren't represented all shareholders. Table 6 contains a list of shareholders present on the meeting and involved in making decisions. Their shares were converted into votes.

Table 6. List of shareholders at general meeting on 30th June 2017

Shareholder	Shares	Votes
State Treasury	27,52%	41,46%
OFE Nationale-Nederlanden	8,32%	11,62%
OFE Aviva BZ WBK	7,16%	10,53%
PERN SA	4,90%	7,38%
OFE PZU Złota Jesień	4,14%	5,63%
Others	<1%	23,38%

Assigning small shareholders which separately have shares below 1% to one group, in total possible coalition configurations are 6!. By appointing winning coalitions and using the formula (9), we will determine Shapley's values for particular shareholders (see Table 7).

Table 7. Shapley value calculated for shareholders participating in the general meeting

Shareholder	Shares
State Treasury	0,450
OFE Nationale-Nederlanden	0,250
OFE Aviva BZ WBK	0,250
PERN SA	0,150
OFE PZU Złota Jesień	0,150
Others	0,250

Analysis of Shapley value of individual shareholders shows that a different number of shares, and therefore votes when adopting resolutions, does not mean different decision power. You can have fewer shares but at the same time you can have the same influence on making decisions as shareholders with larger shares. Looking at the other side, you can have a larger share in the company and the same influence on decision making as shareholders with fewer shares. This is due to the fact that in the case of making decisions, it is important to obtain the majority, not the value of the majority. Thus, Shapley value shows us the shareholders' ability to create winning coalitions, those that are able to pass resolutions irrespective of the opinion of other shareholders. It is also worth noting that small shareholders grouped in a one team may have significant influence at the general meeting.

5. CONCLUSIONS

Shapley's idea of a value proposition was to establish a fair distribution of the team's earned profit from a joint action versus total profit with a separate Action [Dubey, 1975]. As Young's game called "jazz orchestra" [Young, 1993] or the "Christmas promotion" game presented in this article, each of the two players earns more money together than they earn together, but acting separately. Similarly, a team of three players will earn more by appearing together than a combined team of two and a third player acting separately. This is a cooperative game solution, an example of which is the voting system at the general meeting of shareholders of a joint-stock company. Using Shapley value, we can measure the potential of each shareholder to form teams with the majority of votes. The ability to create such victorious coalitions is defined as the decision-making power of a given shareholder. But the analysis of Shapley value shows that the decision-making power does not reflect the number of owned shares or the number of votes attached to shares held. You may have a small number of shares and a decision-making power equal to shareholders with more shares (but not more than 50%). In addition, the analysis of decision-making power also shows the importance of smaller shareholders in the course of voting on resolutions. There are so-called the weight of the tongue at the general assembly, and their participation in the meetings can be crucial, because the absence can increase the decision-making power of the shareholders present at the meeting.

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