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Analytical models for quark stars with electric field

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ABSTRACT

We found new class of solutions to the Einstein-Maxwell system of equations for charged quark matter within the framework of MIT Bag Model considering a prescribed form of gravitational potential Z(x) used by Malaver (2016), which depends on an adjustable parameter n. Variables as the energy density, charge density, pressure and the metric functions are written in terms of elementary and polynomial functions. We show that the form chosen for the gravitational potential allows obtain physically acceptable solutions with any value of the adjustable parameter.

Keywords: gravitational potential, adjustable parameter, Einstein-Maxwell system, energy density, charged quark matter, MIT Bag Model

1. INTRODUCTION

From the development of Einstein´s theory of general relativity, the modelling of superdense mater configurations is an interesting research area [1,2]. In the last decades, such models allow explain the behavior of massive objects as neutron stars, quasars, pulsars, black holes and white dwarfs [3-5]. Malaver [3] studied the behavior of the thermal capacity C_v for Schwarzschild´s black hole when $T>>T_C$ and $T<<T_C$ where T_C is the characteristic temperature of the Schwarzschild black hole and found that the value for C_v if $T>>T_C$ is the same that would be obtained in an ideal diatomic gas if only are considered the degrees of freedom rotational.

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Komathiraj and Maharaj [4] find new classes exact solutions to the Einstein-Maxwell system of equations for a charged sphere with a particular choice of the electric field intensity and one of the gravitational potentials. Sharma et al. [5] have obtained a class of solutions to the Einstein-Maxwell system assuming a particular form for the hypersurface (t = constant) containing a parameter λ.

In theoretical works of realistic stellar models, is important include the pressure anisotropy [6-8]. The existence of anisotropy within a star can be explained by the presence of a solid core, phase transitions, a type III super fluid, a pion condensation [9] or another physical phenomenon as the presence of the electrical field [10]. The physics of ultrahigh densities is not well understood and many of the strange stars studies have been performed within the framework of the MIT-Bag model [11]. In this model, the strange matter equation of state has a simple linear form given by:

$$p = \frac{1}{3}(\rho - 4B)$$

where ρ is the energy density, p is the isotropic pressure and B is the bag constant.

Many researchers have used a great variety of mathematical techniques to try to obtain exact solutions for quark stars within the framework of MIT-Bag model; Komathiraj and Maharaj [11] found two new classes of exact solutions to the Einstein-Maxwell system of equations with a particular form of the gravitational potential and isotropic pressure. Malaver [12,13] also has obtained some models for quark stars considering a potential gravitational that depends on an adjustable parameter. Thirukkanesh and Maharaj [14] studied the behavior of compact relativistic objects with anisotropic pressure in the presence of the electromagnetic field. Maharaj et al. [15] generated new models for quark stars with charged anisotropic matter considering a linear equation of state. Thirukkanesh and Ragel [16] obtained new models for compact stars with quark matter. Sunzu et al. found new classes of solutions with specific forms for the measure of anisotropy [17].

With then use of Einstein´s field equations, important advances has been made to model the interior of a star. In particular, Feroze and Siddiqui [18,19] and Malaver [20-23] consider a quadratic equation of state for the matter distribution and specify particular forms for the gravitational potential and electric field intensity. Mafa Takisa and Maharaj [24] obtained new exact solutions to the Einstein-Maxwell system of equations with a polytropic equation of state. Thirukkanesh and Ragel [25] have obtained particular models of anisotropic fluids with polytropic equation of state which are consistent with the reported experimental observations. Malaver [26,27] generated new exact solutions to the Einstein-Maxwell system considering Van der Waals modified equation of state with and without polytropical exponent and [28] presented a anisotropic strange quark matter model by imposing a linear barotropic equation of state with Tolman IV form for the gravitational potential. Mak and Harko [29] found a relativistic model of strange quark star with the suppositions of spherical symmetry and conformal Killing vector. Bhar et al. [30,31] have studied extensively the behavior of static spherically symmetric relativistic objects with anisotropic matter distribution considering the Tolman VII space-time. Pant et al. [32] have found new exact solutions of the field equations for anisotropic neutral fluid in isotropic coordinates.

Our objective in this paper is to generate a new class for charged isotropic matter with the bag equation of state that presents a linear relation between the energy density and the radial pressure in static spherically symmetric spacetime using a prescribed for a gravitational potential $Z(x)$ which depends on an adjustable parameter n. We have obtained some new classes of static spherically symmetrical models of charged matter where the variation of the parameter n modifies the radial pressure, charge density, energy density and the metric functions of the compact objects. This article is organized as follows, in Section 2, we present Einstein´s field equations as an equivalent set of differential equations using a transformations due to Durgapal and Bannerji [33]. In Section 3, we make a particular choice of gravitational potential $Z(x)$ that allows solving the field equations and we have obtained new models for charged isotropic matter. In Section 4, a physical analysis of the new solutions is performed. Finally in Section 5, we conclude.

2. EINSTEIN FIELD EQUATIONS

We consider a spherically symmetric, static and homogeneous and anisotropic spacetime in Schwarzschild coordinates given by

$$ds^2 = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{1}$$

using the transformations, $x = Cr^2$, $Z(x) = e^{-2\lambda(r)}$ and $A^2y^2(x) = e^{2\nu(r)}$ with arbitrary constants A and c, suggested by Durgapal and Bannerji [33], the Einstein field equations as given in (1) are

$$\frac{1-Z}{x} - 2\dot{Z} = \frac{\rho}{C} + \frac{E^2}{2c} \tag{2}$$

$$4Z\frac{\dot{y}}{y} - \frac{1-Z}{x} = \frac{p_r}{C} - \frac{E^2}{2C} \tag{3}$$

$$\Delta = \frac{4xCZ\ddot{y}}{y} + C(2x\dot{Z} + 6Z)\frac{\dot{y}}{y} + C\left[2\left(\dot{Z} + \frac{B}{C}\right) + \frac{Z-1}{x}\right] \tag{4}$$

$$\sigma^2 = \frac{4CZ}{x}\left(x\dot{E} + E\right)^2 \tag{5}$$

where: ρ is the energy density, p_r is the radial pressure, E is electric field intensity, σ is the charge density, Δ is the anisotropy and dot denote differentiations with respect to x. The equation (4) is the measure of anisotropy. We can replace the system of field equations, including the bag equation of state by the system

$$\rho = 3p + 4B \tag{6}$$

$$\frac{p}{C} = Z\frac{\dot{y}}{y} - \frac{1}{2}\dot{Z} - \frac{B}{C} \tag{7}$$

$$\frac{E^2}{2C} = \frac{1-Z}{x} - 3Z\frac{\dot{y}}{y} - \frac{1}{2}\dot{Z} - \frac{B}{C} \tag{8}$$

$$\sigma = 2\sqrt{\frac{CZ}{x}}\left(x\dot{E} + E\right) \tag{9}$$

The equations (6), (7), (8) and (9) governs the gravitational behavior of a charged quark star.

3. GENERATING NEW SOLUTIONS

Using the procedure suggested by Komathiraj and Maharaj [11], it is possible to obtain a exact solution of the Einstein-Maxwell system. In this paper, motivated by Feroze and Siddiqui [18] and Malaver [34], we take the form of the gravitational potential $Z(x)$ as

$$Z(x) = \left(\frac{1-ax}{1+ax}\right)^n \tag{10}$$

where is a real constant and n is an adjustable parameter. This potential is regular at the origin and well behaved in the interior of the sphere. For the electric field we make the choice

$$E^2 = \frac{nx}{(1+ax)^2} \tag{11}$$

This electric field is finite at the centre of the star and remains continuous in the interior. We have considered the particular cases for *n* = 1, 2.

For the case *n* = 1, the substitution of $Z(x)$ and (11) in (8), it allows to obtain the differential equation of the first order

$$\frac{\dot{y}}{y} = \frac{D + Ex - 2Ba^2x^2}{2C(1-a^2x^2)} \tag{12}$$

where: $D = 6ac - 2B$ and $E = 4a^2C - 4aB - 1$

Integrating (12), we obtain

$$y(x)=C_1(1+ax)^{-\frac{2aB-aD+E}{4Ca^2}}(-1+ax)^{\frac{2aB-aD-E}{4Ca^2}}e^{\frac{Bx}{C}} \tag{13}$$

The equation (13) and $Z(x)$ allows generate the following analytical model:

$$e^{2\nu(r)}=C_1^{\ 2}A^2(1+ax)^{-\frac{2aB-aD+E}{2Ca^2}}(-1+ax)^{\frac{2aB-aD-E}{2Ca^2}}e^{\frac{2Bx}{C}} \tag{14}$$

$$e^{2\lambda(r)}=\frac{1+ax}{1-ax} \tag{15}$$

$$p=\frac{D+2aC-B+(E-4aB)x-4Ba^2x^2}{2(1+ax)^2} \tag{16}$$

$$\rho=\frac{3D+6aC+5B+(3E+4aB)x-4Ba^2x^2}{2(1+ax)^2} \tag{17}$$

$$\sigma^2=\frac{C(1-ax)(3+ax)^2}{(1+ax)^5} \tag{18}$$

$$\Delta=\frac{4xC(1-ax)\ddot{y}}{(1+ax)y}-2c\left[\frac{2ax-3(1-a^2x^2)}{(1+ax)^2}\right]\frac{\dot{y}}{y}+\frac{2a^2Bx^2+4aBx+2(B-2aC)}{(1+ax)^2}-\frac{2ac}{1+ax} \tag{19}$$

with n = 2, the eq. (8) becomes

$$\frac{\dot{y}}{y}=\frac{4aC-x-BC(1+ax)^2}{3C(1-ax)^2}+\frac{2a}{3(1-a^2x^2)} \tag{20}$$

Integrating (20), we have

$$y(x)=C_2(1+ax)^{-\frac{2}{3(ax-1)}}(-1+ax)^{-\frac{4aBC+1}{Ca^2}}e^{-\frac{(BCa^3x^2-BCa^2x-4aBC+4Ca^2-1)}{3(ax-1)Ca^2}} \tag{21}$$

Therefore with eq. (21) and $Z(x)$ we can generate the analytical model:

$$e^{2\nu(r)} = C_2^2 A^2 (1+ax)^{-\frac{4}{3(ax-1)}} (-1+ax)^{-\frac{8aBC+2}{Ca^2}} e^{-\frac{2\left(BCa^3x^2 - BCa^2x - 4aBC + 4Ca^2 - 1\right)}{3(ax-1)Ca^2}} \tag{22}$$

$$e^{2\lambda(r)} = \frac{(1+ax)^2}{(1-ax)^2} \tag{23}$$

$$p = \frac{4aC - BC - (1+2aBC)x - a^2BCx^2}{3(1+ax)^2} + \frac{8aC(1-ax)}{3(1+ax)^3} - B \tag{24}$$

$$\rho = \frac{4aC - BC - (1+2aBC)x - a^2BCx^2}{(1+ax)^2} + \frac{8aC(1-ax)}{(1+ax)^3} + B \tag{25}$$

$$\sigma^2 = \frac{2C(1-ax)^2(3+ax)^2}{(1+ax)^6} \tag{26}$$

$$\Delta = \frac{4xC(1-ax)^2\ddot{y}}{(1+ax)^2 y} + 2c\left[\frac{3a^3x^3 + a^2x^2 - 7ax + 3}{(1+ax)^3}\right]\frac{\dot{y}}{y} + \frac{2a^2Bx^2 + 4aBx + 2(B-2aC)}{(1+ax)^2} - \frac{8ac(1-ax)}{1+ax} \tag{27}$$

4. PHYSICAL FEATURES OF THE SOLUTIONS

The showed models satisfy the system of equations (6) - (9) and constitute another new family of solutions for a charged quark star with isotropic pressure. The metric functions $e^{2\nu(r)}$ and $e^{2\lambda(r)}$ can be written in terms of polynomial functions, and the variables energy density, pressure and charge density also are represented analytical. For the case n=1, $e^{2\lambda(0)}=1$, $e^{2\nu(r)} = C_1{}^2 A^2 (-1)^{\frac{2aB-aD-E}{2Ca^2}}$ and $\left(e^{2\lambda(r)}\right)'_{r=0} = \left(e^{2\nu(r)}\right)'_{r=0} = 0$

This demonstrates that the gravitational potential is regular in the origin. The energy density is positive in the interior and in the center of the star takes the value $\rho(0) = 12aC - \frac{B}{2}$. The pressure p_r is regular and in the center *r = 0* takes the value $p_r(0) = 4aC - \frac{3B}{2}$. The charge density is continuous in the interior of the star and it vanishes in the center. For the case *n=2* $e^{2\lambda(0)}=1$, $e^{2\nu(0)} = A^2 C_2{}^2 (-1)^{-\frac{8aBC+2}{Ca^2}} e^{\frac{2\left(4Ca^2-1\right)}{3Ca^2}}$ in the origin and $\left(e^{2\lambda(r)}\right)'_{r=0} = \left(e^{2\nu(r)}\right)'_{r=0} = 0$. Further the gravitational potential is regular in

$r=0$. In the center $r=0$ $\rho(0)=12aC-BC+B$ and $p_r(0)=4ac-\frac{BC}{3}-B$. In all the cases, the charge density is continuous and behaves well inside of the star.

The fact that the functions $e^{2\nu(r)}$, $e^{2\lambda(r)}$, p, ρ and σ have a finite value in *r = 0* implies that the solutions (13) - (27) for charged quarks stars are physically acceptable and do not present singularities in the origin, as it established Jotania and Tikekar [35].

5. CONCLUSION

We have generated a new class of exact solutions for the Einstein-Maxwell system which not present singularities at the origin. We have studied two new classes of analytical solutions specifying the form of the gravitational potential *Z(x)* and of the electrical field *E*. Three obtained solutions correspond to models which have finite values for the energy density, the pressure and the charge density at the center of the star. All the solutions present a behavior similar to the solution proposed by Komathiraj and Maharaj [11] and Malaver [12] con anisotropic pressure. The method to generate analytical exact solutions will depend on the form of *Z(x)* and the electric field intensity *E*, necessary to determine physically acceptable solutions.

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