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Effects of heat transfer on unsteady magnetohydrodynamics (MHD) boundary layer flow of an incompressible fluid a moving vertical plate

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ABSTRACT

This paper deals with the effects of heat transfer on unsteady MHD boundary layer flow of a chemically reading fluid past a vertical plate in the presence of thermal radiation and viscous dissipation. The governing equations of the problem under investigation are systems of partial differential equations. These equations are namely: continuity equation, linear momentum equation and energy equation. These systems of equations were non-dimentionalized by introducing non-dimensional variables. We resulted to dimensionless systems of partial differential equations. The fluid considered in this paper are optically thick, however we employ the Roseland model. Simulations were done numerically on all controlling parameters and the results generated were displayed in graphs and tables. Results review that thermal Grashof number intensifies the velocity profile and increase in Prandtl number leads to decrease in both the velocity and temperature field. Increasing thermal radiation intensifies the velocity and temperature profile. Thermal radiation speeds up the convection flow and thermal boundary layer of the fluid. Increase in magnetic parameter reduces the velocity profile as a result of the applied transverse magnetic field.

***Keywords*:** Boundary layer, Heat transfer, MHD, Thermal radiation, Viscous dissipation

1. INTRODUCTION

Matter is anything that has weight and occupies space. There are three states of matter namely: solid, liquid, and gas. Fluids can be in form of liquid and gas. It can be regarded as any substance that is capable of flowing. It can also be defined as a substance which deforms continuously under the application of a shear stress. Fluids can be classified as Newtonian and Non-Newtonian. Its flow behaviour can be in form of laminar, turbulence, compressible and incompressible. Over decades, many researchers have investigated the behaviour and nature of fluid flow. Mohammed (2015) studied the effect of radiation on MHD mixed convection flow from a vertical plate embedded in a saturated porous media with melting. In the analysis two-dimensional MHD mixed convection laminar boundary layer flow was considered. He concluded in the study that heat transfer coefficients are reduced with increasing melting parameter and grows with increasing radiation parameter. In 2014, Kishan and Karitha investigated MHD non-Newtonian power law fluid flow and heat transfer past a non-linear stretching surface with thermal radiation and viscous dissipation. Ghosh and Shit (2012) studied mixed convection MHD flow of viscoelastic fluid in a porous medium past a hot vertical plate. The type of fluid considered in their work has a viscous properties and elastic properties. The MHD boundary layer flow of a Non-Newtonian fluid over an exponentially permeable stretching sheet with radiation and heat source/sink was studied by Kishan et al. (2016).

Thermal radiation on fluid flow is of great importance when the environment is at higher temperature. Higher thermal radiation enhances convective flow and increases the thermal condition of fluid in the boundary layer. However, thermal radiation and heat transfer are also of great importance in the design of many advanced energy conversion system if and only if it operates at a very high temperature. It is of everyday experience for heat to flow from one object to another. When a hot body falls on cold body, heat will be transfered and the cold body becomes hot while the temperature of the hot body will drop. Researchers in the field of fluid mechanics have worked in this area. Sharma and Gupta (2016) worked on the analytical study of MHD boundary layer flow and heat transfer towards a porous electrically stretching sheet in presence of thermal radiation. Jimoh et al. (2015) investigated the numerical study of unsteady free convective heat transfer in Walters-B viscoelastic flow over an inclined stretching sheet with heat source and magnetic field. Again, Alao et al. (2016) studied effects of thermal radiation, soret and dufour on an unsteady heat and mass transfer flow of a chemically reacting fluid past a semi-infinite vertical plate with viscous dissipation. Manna et al. (2012) studied the effects of radiation on unsteady MHD free convective flow past an oscillating vertical porous plate embedded in a porous medium with oscillatory heat flux. The attention of some researchers was also drawn to the combine effect of heat and mass transfer flows.

Recently, Ajayi et al. (2017) studied viscous dissipation effects on the motion of casson fluid over an upper horizontal thermally stratified melting surface of a paraboloid revolution: Boundary Layer Analysis. Nandeppanavar and Siddajingappa (2013) studied the effects of viscous dissipation and thermal radiation on heat transfer over a nonlinearly stretching sheet through porous medium. Roga et al. (2013) investigated radiation and mass transfer effect on MHD free convective flow of a micro polar fluid past an infinite vertical porous plate embedded in a porous medium with viscous dissipation. Hazarika and Konch (2014) examined the effect of variable viscosity and thermal conductivity on MHD free convection flow along a vertical porous plate with viscous dissipation. Suresh et al. (2016) studied the numerical investigation of an unsteady mixed convective mass and heat transfer MHD flow with soret effect and viscous dissipation in the presence of thermal radiation and heat source/sink.

We mentioned in this paper, work considered by Mohammed and Bhashar; Sharma et al. (2012); Mohammed et al. (2014); Karthikeyan et al. (2016); Awad and Sibanda (2010); Madhuri et al. (2012); Mangathai et al. (2015); Jimoh et al. (2014).

The motivation of this paper is the fact that no studies have investigated heat transfer effect on unsteady MHD boundary layer flow of an incompressible fluid past a vertical plate in the presence of thermal radiation and viscous dissipation. Hence there is need for the present investigation. Our objectives in the present study are; (i) to account for the effects of all flow parameters under investigation (ii) a novel, accurate and efficient method called the Spectral Relaxation Method (SRM) will be used to solve the dimensionless systems of partial differential equations. The rest of the paper is organized as follows: Section Two is the mathematical formulation. Section Three presents numerical technique. In Section Four, we present discussion of results. Section Five concludes the paper.

2. MATHEMATICAL FORMULATION

An unsteady two-dimensional, viscous, incompressible laminar boundary layer flow in the presence of thermal radiation and viscous dissipation is considered. The plate considered is a vertical moving plate which moves towards the horizontal direction and gradually the x^* direction are negligible [see Figure 1 below]. The x^*- axis is considered along the vertical plate in an upward direction while y^* -axis is normal to it. All variables are function of t^* and y^* only. The radiative heat flux in y^*-direction is only considered. We consider an optically thick fluid and we thereby use Roseland model to describe the heat flux. All fluid properties are assumed constant and under the usual Boussinesq approximation the governing equations are defined below:

$$\frac{\partial v^*}{\partial y^*} = 0 \tag{1}$$

$$\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} = v \frac{\partial^2 u^*}{\partial y^{*2}} + g\beta_t (T - T_\infty) - \frac{\sigma B_0^2 u^*}{\rho} \tag{2}$$

$$\frac{\partial T}{\partial t^*} + v^* \frac{\partial T}{\partial y^*} = \alpha \frac{\partial^2 T}{\partial y^{*2}} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y^*} + \frac{\mu}{\rho c_p} \left(\frac{\partial u^*}{\partial y^*}\right)^2 \tag{3}$$

Subject to the initial and boundary conditions:

$$u^* = U_0; T = T_w + \epsilon (T_w - T_\infty) e^{n^* t^*} \text{ at } y^* = 0 \tag{4}$$

$$u^* \longrightarrow 0; T \longrightarrow T_\infty \quad \text{as } y^* \longrightarrow \infty \tag{5}$$

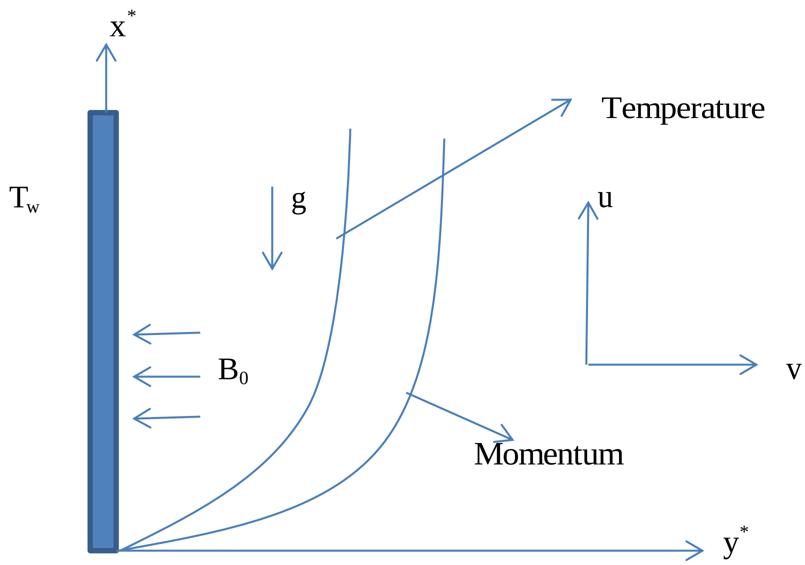

Figure 1. Physical Interpretation of the Problem

where u^* and v^* are velocity components in (x^*, y^*) coordinate respectively, t^* is the time, T is the fluid temperature, g is the acceleration due to gravity, ν is the kinematic viscosity, c_p is the specific heat at constant pressure, ρ is the density of the fluid, T_∞ is the free stream temperature, σ is the fluid electrical conductivity, α is the thermal diffusivity of the fluid, n^* is a constant, C_∞ is the free stream temperature, β_t is the thermal expansion coefficients.

The radiative heat flux in the present study is taking only in y^* direction and we have adopted the Roseland model as pointed out in Alao et al. (2016) given by:

$$q_r = -\frac{4\sigma_0}{3k_0}\frac{\partial T^4}{\partial y^*} \tag{6}$$

Using the Roseland model on equation (6) above, we assumed the temperature within flow regimes are sufficiently small in a way that T^4 can be expressed as a linear function of the free stream temperature T_∞ and expanding T^4 in Taylor series aboutT_∞. We neglect the higher order terms and simplify to obtain

$$T^4 = 4T_\infty^3 T - 3T_\infty^4 \tag{7}$$

From equation (6), σ_0 is the Stefan-Boltzman constant and k_0 is the mean absorption coefficient. Substituting equations (6) and (7) into the energy equation and simplify further to obtain:

$$\frac{\partial T}{\partial t^*} + v^*\frac{\partial T}{\partial y^*} = \left(\alpha + \frac{16\sigma_0 T_\infty^3}{3\rho c_p k_0}\right)\frac{\partial^2 T}{\partial y^{*2}} + \frac{\mu}{\rho c_p}\left(\frac{\partial u^*}{\partial y^*}\right)^2 \tag{8}$$

The suction velocity normal to the plate is a constant or assumed to be a function of time. This can be obtained from the continuity equation (1). The present investigation considered the suction velocity as the combination of constant and time. We define the suction velocity as the equation given by:

$$v^* = -V_0\left(1 + \epsilon A e^{n^* t^*}\right) \tag{9}$$

The following non-dimensional quantities are introduced on the governing equations and boundary conditions in other to write them in a dimensionless form of equations.

$$u = \frac{u^*}{U_0}, y = \frac{V_0 y^*}{v}, t = \frac{V_0^2 t^*}{v}, n = \frac{v n^*}{V_0^2} \tag{10}$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, P_r = \frac{v \rho c_p}{k} = \frac{v}{\propto}, G_r = \frac{g \beta v (T_w - T_\infty)}{u_0 v_0^2} \tag{11}$$

$$E_c = \frac{U_0^2}{c_p (T_w - T_\infty)}, M = \frac{\sigma B_0^2 v}{\rho V_0^2}, \ T_R = \frac{16 \sigma_0 T_\infty^3}{3 k_0 k} \tag{12}$$

Substituting equations (9)-(12) into equations (2)-(5) the dimensionless forms of equations yields:

$$\frac{\partial^2 u}{\partial y^2} + G_r \theta - M^2 u + (1 + \varepsilon A e^{nt}) \frac{\partial u}{\partial y} - \frac{\partial u}{\partial t} = 0 \tag{13}$$

$$\left(\frac{1 + T_R}{Pr}\right) \frac{\partial^2 \theta}{\partial y^2} + E_c \left(\frac{\partial u}{\partial y}\right)^2 + (1 + \varepsilon A e^{nt}) \frac{\partial \theta}{\partial y} - \frac{\partial \theta}{\partial t} = 0 \tag{14}$$

Subject to the following dimensionless boundary conditions:

$$u = 1, \theta = 1 + \varepsilon e^{nt} \qquad at\ y = 0 \tag{15}$$

$$u \longrightarrow 0, \theta \longrightarrow 0 \qquad at\ y \longrightarrow \infty \tag{16}$$

In our dimensionless equations above, Gr, M, T_R, Pr, and Ec are the thermal Grashof number, Magnetic parameter, thermal radiation parameter, Prandtl number and the viscous dissipation term respectively. The physical quantities of interest in this paper are the local skin friction coefficient and local Nusselt number due to their practical applications in engineering. These two physical quantities at the plate are given by:

$$C_f = \frac{\tau_w}{\rho U_0 V_0} \qquad and \qquad Nu = \frac{x q_w}{k (T_w - T_\infty)} \tag{17}$$

where

$$\left.\begin{aligned} \tau_w &= \mu\left(\frac{\partial u^*}{\partial y^*}\right)_{y^*=0} \\ q_w &= -k\left(\frac{\partial T}{\partial y^*}\right)_{y^*=0} \end{aligned}\right\} \tag{18}$$

Substituting the dimensional quantities defined in equations (10)-(12) and upon simplification the physical quantities becomes:

$$C_f Re_x^{-1} = -\left(\frac{\partial u}{\partial y}\right)_{y=0}$$

$$NuRe_x^{-1} = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0}$$

3. NUMERICAL TECHNIQUE: SPECTRAL RELAXATION METHOD (SRM)

The spectral relaxation method is an iterative method which employs the Gauss-Seidel type of relaxation approach to decouple and linearize system of nonlinear differential equations. In this approach, the linear terms are evaluated at the current iteration (given by $r + 1$) and the nonlinear terms are assumed from the previous (given by r). Details and basic idea of spectral relaxation method (SRM) can be found in Motsa (2012).

In other to apply the spectral methods, the domain in which the governing equation is defined is transformed to the interval $[-1,1]$. Following Motsa (2012), the transformation $\eta = \frac{(b-a)(\tau+1)}{2}$ is used to map the interval $[a, b]$ to $[-1,1]$. Using the idea of Spectral relaxation method (SRM) explained above and implementing the Chebyshev pseudo-spectral method on the dimensionless equations (13)-(16) yields:

$$u''_{r+1} + Gr\theta_r - M^2 u_{r+1} + (1 + \varepsilon A e^{nt})u'_{r+1} = \frac{\partial u_{r+1}}{\partial t} \tag{19}$$

$$(1 + T_R)\theta''_{r+1} + PrEc(u'_{r+1})^2 + Pr(1 + \varepsilon A e^{nt})\theta'_{r+1} = Pr\frac{\partial \theta_{r+1}}{\partial t} \tag{20}$$

with the boundary conditions:

$$u_{r+1}(0, t) = 1,\ u_{r+1}(\infty, t) = 0 \tag{21}$$

$$\theta_{r+1}(0, t) = 1 + \varepsilon e^{nt},\ \theta_{r+1}(\infty, t) = 0 \tag{22}$$

Setting:

$$d_{0,r} = Gr\theta_r,\ d_{1,r} = (1 + \varepsilon A e^{nt}) \tag{23}$$

$$e_{0,r} = (1 + T_R),\ \ e_{1,r} = PrEc(u'_{r+1})^2,\quad e_{2,r} = Pr(1 + \varepsilon A e^{nt}) \tag{24}$$

and defining the initial approximations $u_0(y,t)$ and $\theta_0(y,t)$ as:

$$u_0(y,t) = e^{-y}\ ,\ \ \theta_0(y,t) = e^{-y} + \varepsilon e^{nt} \tag{25}$$

Substituting equations (23)-(24) into equations (19)-(20) yields:

$$u''_{r+1} + d_{0,r} - M^2 u_{r+1} + d_{1,r} u'_{r+1} = \frac{du_{r+1}}{dt} \tag{26}$$

$$e_{0,r}\theta''_{r+1} + e_{1,r} + e_{2,r}\theta'_{r+1} = Pr\frac{d\theta_{r+1}}{dt} \tag{27}$$

subject to (21) and (22).

Now, starting from the initial approximation $u_0(y,t)$ and $\theta_0(y,t)$ defined in equation (25), we solve equations (19)-(22) iteratively. Using the Chebyshev collocation method in the t-direction, we discretize equations (19)-(22) and applying implicit finite difference method in the y-direction. Also, we employed finite difference scheme centering about a mid-point between $t^{n+\frac{1}{2}}$ to u and all derivatives defined as;

$$\left.\begin{aligned} & t^{n+\frac{1}{2}} = \frac{t^{n+1}+t^n}{2},\ \ u\left(y_J, t^{n+\frac{1}{2}}\right) \equiv u_J^{n+\frac{1}{2}} = \frac{u_J^{n+1}+u_J^n}{2},\ \left(\frac{\partial u}{\partial t}\right)^{n+\frac{1}{2}} = \frac{u_J^{n+1}-u_J^n}{\Delta t} \\ & \theta\left(y_J, t^{n+\frac{1}{2}}\right) \equiv \theta_J^{n+\frac{1}{2}} = \frac{\theta_J^{n+1}+\theta_J^n}{2},\qquad \left(\frac{\partial \theta}{\partial t}\right)^{n+\frac{1}{2}} = \frac{\theta_J^{n+1}-\theta_J^n}{\Delta t} \end{aligned}\right\} \tag{28}$$

Using equation (28) on equations (26) and (27) and applying the spectral method yields:

$$\left(D^2 - M^2 + d_{1,r}D\right)\left(\frac{u_{r+1}^{n+1}+u_{r+1}^n}{2}\right) + d_{0,r} = \left(\frac{u_{r+1}^{n+1}-u_{r+1}^n}{\Delta t}\right) \tag{29}$$

$$\left(e_{0,r}D^2 + e_{2,r}D\right)\left(\frac{\theta_{r+1}^{n+1}+\theta_{r+1}^n}{2}\right) + e_{1,r} = \left(\frac{\theta_{r+1}^{n+1}-\theta_{r+1}^n}{\Delta t}\right) \tag{30}$$

Subject to:

$$u_{r+1}(x_0,t) = 1,\ \ u_{r+1}(\infty,t) = 0 \tag{31}$$

$$\theta_{r+1}(x_0,t) = 1 + \varepsilon e^{nt},\ \ \theta_{r+1}(\infty,t) = 0 \tag{32}$$

Upon further simplifications yields:

$$D_1 U_{r+1}^{n+1} = E_1 U_{r+1}^n + K_1 \tag{33}$$

$$D_2\theta_{r+1}^{n+1} = E_2\theta_{r+1}^{n} + K_2 \tag{34}$$

Subject to:

$$u_{r+1}(x_0, t^n) = 1,\ u_{r+1}(xNx, t^n) = 0 \tag{35}$$

$$\theta_{r+1}(x_0, t^n) = 1 + \varepsilon e^{nt},\ \theta_{r+1}(xNx, t^n) = 0 \tag{36}$$

where

$$D_1 = \left(\frac{D^2 - M^2 + d_{1,r}D}{2}\right) - \frac{1}{\Delta t}, E_1 = -\left[\left(\frac{D^2 - M^2 + d_{1,r}D}{2}\right) + \frac{1}{\Delta t}\right], K_1 = -d_{0,r}$$

$$D_2 = \left(\frac{e_{0,r}D^2 + e_{2,r}D}{2} - \frac{Pr}{\Delta t}\right), \quad E_2 = -\left(\frac{e_{0,r}D^2 + e_{2,r}D}{2} + \frac{Pr}{\Delta t}\right), \qquad K_2 = -e_{1,r}$$

and

$$u_{r+1} = \begin{bmatrix} u_{r+1}(x_0, t) \\ u_{r+1}(x_1, t) \\ \vdots \\ u_{r+1}(xN_{x-1}, t) \\ u_{r+1}(xN_x, t) \end{bmatrix}, \qquad d_{0,r} = \begin{bmatrix} d_{0,r}(x_0, t) & & & & \\ & d_{0,r}(x_1, t) & & & \\ & & \ddots & & \\ & & & \ddots & \\ & & & & d_{0,r}(xN_x, t) \end{bmatrix}$$

$$\theta_{r+1} = \begin{bmatrix} \theta_{r+1}(x_0, t) \\ \theta_{r+1}(x_1, t) \\ \vdots \\ \theta_{r+1}(xN_{x-1}, t) \\ \theta_{r+1}(xN_x, t) \end{bmatrix}, \qquad e_{0,r} = \begin{bmatrix} e_{0,r}(x_0, t) & & & & \\ & e_{0,r}(x_1, t) & & & \\ & & \ddots & & \\ & & & \ddots & \\ & & & & e_{0,r}(xN_x, t) \end{bmatrix}$$

4. DISCUSSION OF RESULTS

The present study examined the effects of heat transfer on unsteady MHD boundary layer flow past a moving vertical plate in the presence of thermal radiation and viscous dissipation.

The equations which govern the flow model are systems of partial differential equation which was transformed to dimensionless equations by introducing non-dimensional variables and solved using Spectral Relaxation Method (SRM) which are known for efficiency, accurate and easy to compute method.

The spectral relaxation method (SRM) is an iterative method which employs the relaxation approach of Gauss-Seidel. This method was first described in Motsa (2012). In other to understand the physics of the problem under investigation, effects of all controlling parameters are presented in figures and tables. The numerical values for pertinent flow parameters in this paper are given by $Pr = 0.71, M = 1, R = A = 0.5, n = 0.1, Ec = 0.001$.

Therefore all Figures and Tables correspond to these values unless or otherwise stated. Table 1 displays the computed values of local skin-friction and local Nusselt number for different values of thermal Grashof number (Gr). The result shows that when thermal Grashof number is increased, the local skin-friction is also increased but negligible on the local Nusselt number.

Table 2 depicts values of local skin-friction and local Nusselt number for different values of thermal radiation parameter (T_R). It is observed that as the thermal radiation parameter increases the local skin-friction as well as the local Nusselt number increases. In Table 3, computed values for the local skin-friction and local Nusselt number for different values of the magnetic parameter (M) were shown.

The results in Table 3 reveal that as the magnetic parameter increases the local skin-friction reduces but negligible on the local Nusselt number. The applied transverse magnetic field gives rise to a drag-like force called Lorentz force which slows down the motion of an electrically conducting fluid. Table 4 displays the values of the local skin-friction and local Nusselt number for different values of thermal Grashof number and thermal radiation parameter.

The results show that in the absence of the Grashof number, that is $Gr = 0$, an increase in thermal radiation parameter is negligible on the local skin-friction but increases the local Nusselt number. It is observed in Table 4 that increase in thermal Grashof number and thermal radiation parameter further increases both the local skin-friction and local Nusselt number.

TABLE OF RESULTS

Table 1. Computed values for the local skin-friction and local Nusselt number for different values of thermal Grashof number (Gr) when $Pr = 0.71, R = A = 0.5, n = 0.1, M = 1.0$ and $Ec = 0.001$.

Gr	C_f	Nu
0.0	0.3314048088	0.6935811929
0.5	0.8897658912	0.6935811929
1.0	1.1689464324	0.6935811929
2.0	1.4481269736	0.6935811929

Table 2. Computed values for the local skin-friction and local Nusselt number for different values of thermal radiation parameter (T_R) when $Gr = 2.0, Pr = 0.71, A = 0.5, n = 0.1, M = 1.0$ and $Ec = 0.001$.

T_R	C_f	Nu
0.0	0.1284037962	0.5745550947
0.5	0.2460426881	0.6299620362
1.0	0.3314048088	0.6935811929
2.0	0.4519488455	0.8291131445

Table 3. Computed values for the local skin-friction and local Nusselt number for different values of magnetic parameter (M) when $Gr = 2.0, Pr = 0.71, T_R = A = 0.5, n = 0.1$, and $Ec = 0.001$.

M	C_f	Nu
0.0	0.4711170237	0.6935811929
0.5	0.3314048088	0.6935811929
1.0	0.2285629242	0.6935811929

Table 4. Computed values for the local skin-friction and local Nusselt number for different values of thermal Grashof number (Gr) and thermal radiation parameter (T_R) when $Pr = 0.71, A = 0.5, n = 0.1,\ and\ Ec = 0.001$.

Gr	T_R	C_f	Nu
0.0	0.0	1.4481269736	0.6299620362
	0.5	1.4481269736	0.6935811929
	1.0	1.4481269736	0.8291131445
0.5	0.0	1.1476059022	0.6299620362
	0.5	1.1689464324	0.6935811929

	1.0	1.1990824417	0.8291131445
1.0	0.0	0.8470848309	0.6299620362
	0.5	0.8897658912	0.6935811929
	1.0	0.9500379096	0.8291131445
2.0	0.0	0.2460426881	0.6299620362
	0.5	0.3314048088	0.6935811929
	1.0	0.4519488455	0.8291131445

Figures 2-6 show the effects of thermal Grashof number, Prandtl number, thermal radiation parameter, magnetic parameter and Eckert number on the velocity and temperature field. The thermal Grashof number effects on the velocity and temperature profiles are shown in Figure 2. The thermal Grashof number defines the ratio of buoyancy acting on the fluid and the viscous force in the boundary layer. It is observed from Figure 2 that as the thermal Grashof number is increased, the velocity in the boundary layer is also increased. It is noted that at higher thermal Grashof number the flow in the boundary layer is laminar. Figure 3 displays the effects of Prandtl number on the velocity and temperature profiles respectively. At higher Prandtl number both the velocity and temperature profiles reduces significantly. A fluid which possesses higher Prandtl number has greater viscosities which tend to lower the skin-friction.

The boundary layer is very conductive when the Prandtl number is of small values (say $Pr < 1$). Effects of thermal radiation parameter on the velocity and temperature profiles are displayed in Figure 4. Thermal radiation speeds up convective flow. Thermal radiation is of great importance when the temperature in the boundary layer is high. However, it is noticed from the Figure 4 that increases in thermal radiation leads to increase in both the velocity and temperature profiles. Figure 5 displays the effect of the magnetic parameter on both the velocity and temperature profiles. Magnetic parameter is deduced from the word "Magnetohydromagnetic" which explains the study of the motion of an electrically conducting fluid. It is obvious from the velocity equation that the magnetic field term is applied in opposite direction to the fluid flow in the boundary layer. Due to the applied magnetic field term in the opposite direction to the flow, a force called the Lorentz force is produced. Lorentz force is a drag-like/resistive force which slows down the motion of an electrically conducting fluid in the boundary layer. It is evident from the governing equations that magnetic parameter does not have any effect on the temperature profile. Figure 6 shows the effect of the viscous dissipative term on both velocity and temperature profiles respectively. The viscous dissipative term also known as the Eckert number connotes the relationship between the kinetic energy in the flow plus the enthalpy. It is observed from the Figure 6 that an increase in the Eckert number increases the velocity and temperature profiles respectively.

LIST OF FIGURES

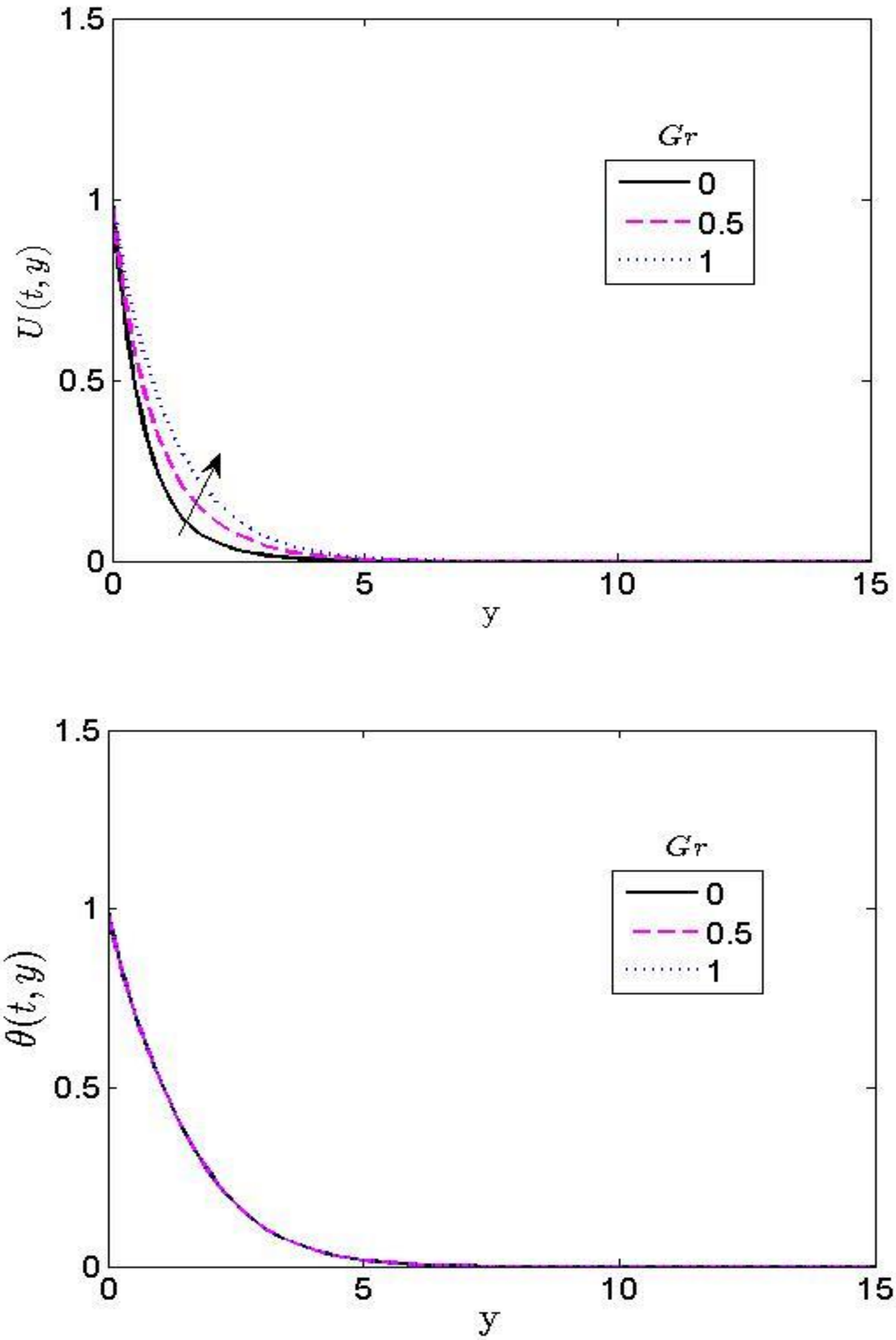

Figure 2. Effects of different values of thermal Grashof number on the velocity and temperature profiles

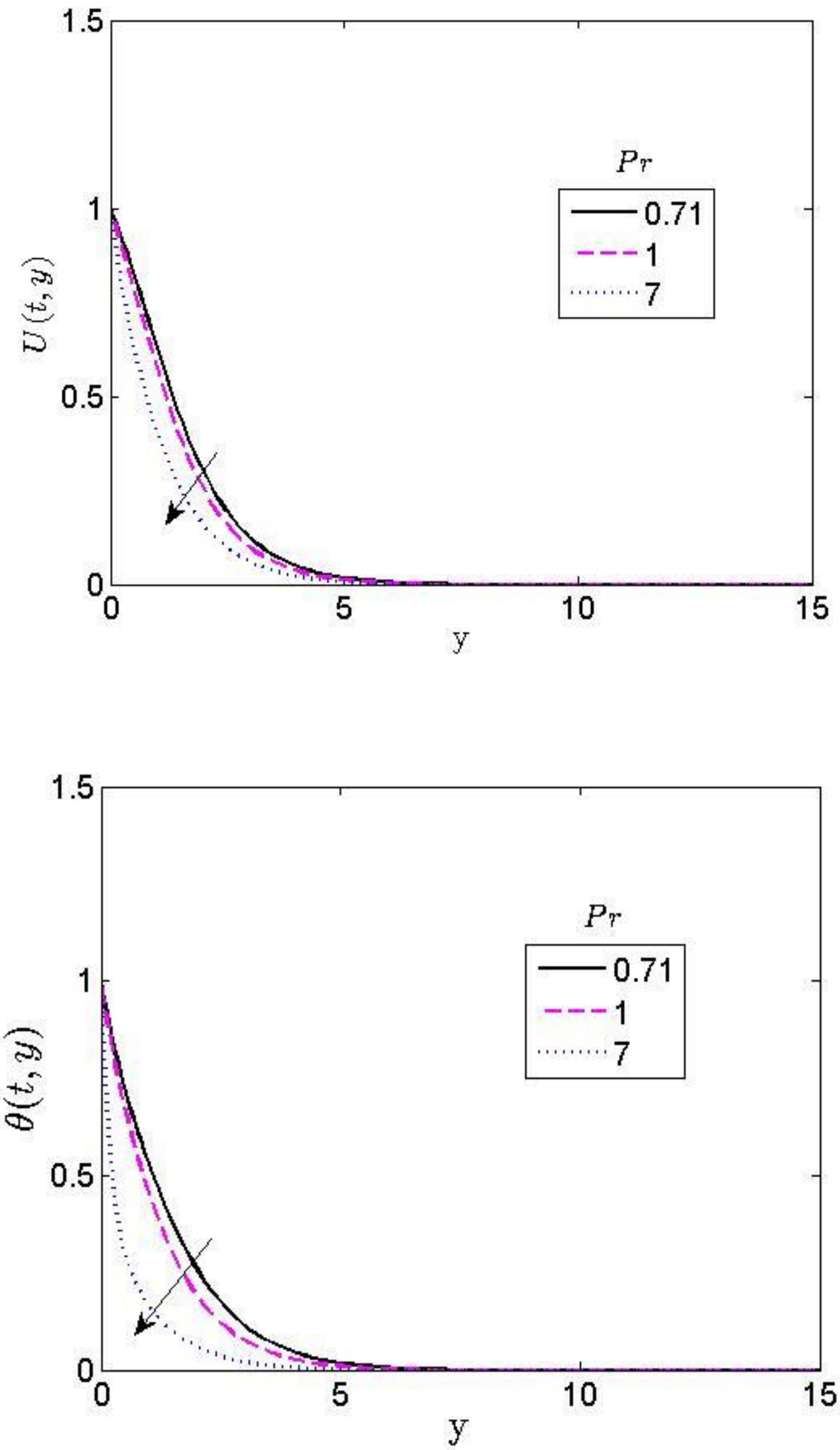

Figure 3. Effects of different values of Prandtl number on the velocity and temperature profiles

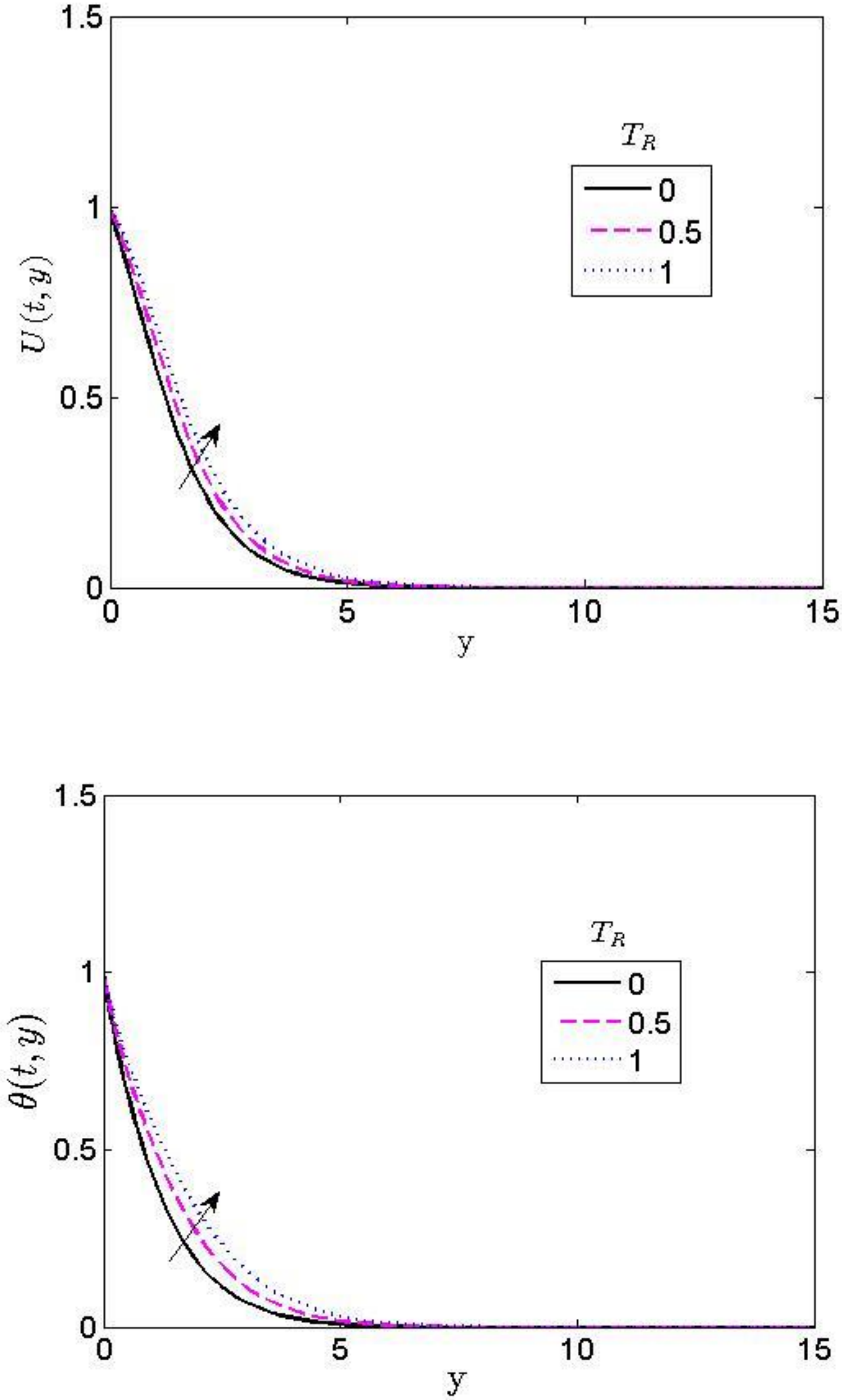

Figure 4. Effects of different values of thermal radiation parameter on the velocity and temperature profiles

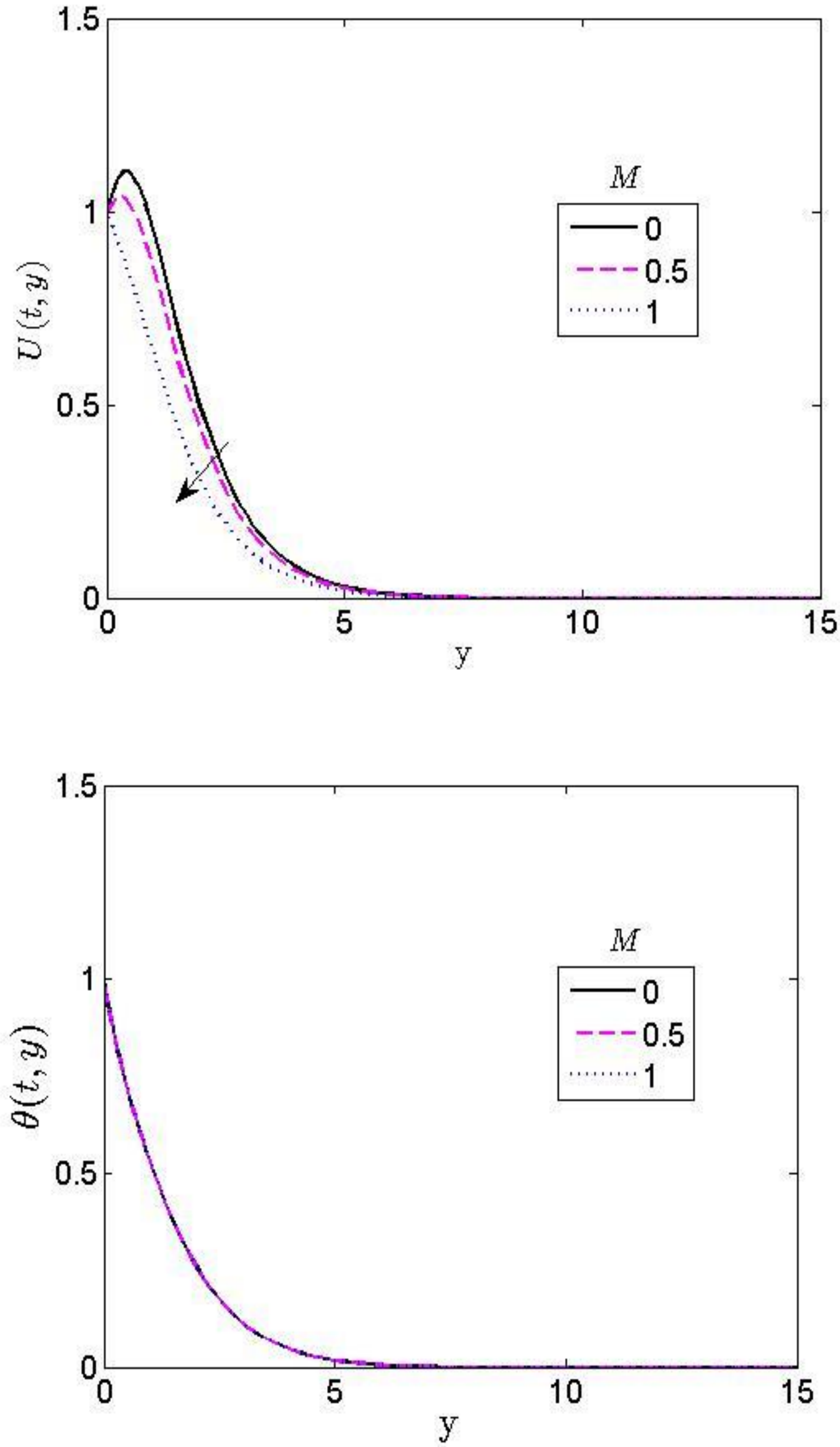

Figure 5. Effects of different values of Magnetic parameter on the velocity ‘and temperature profiles

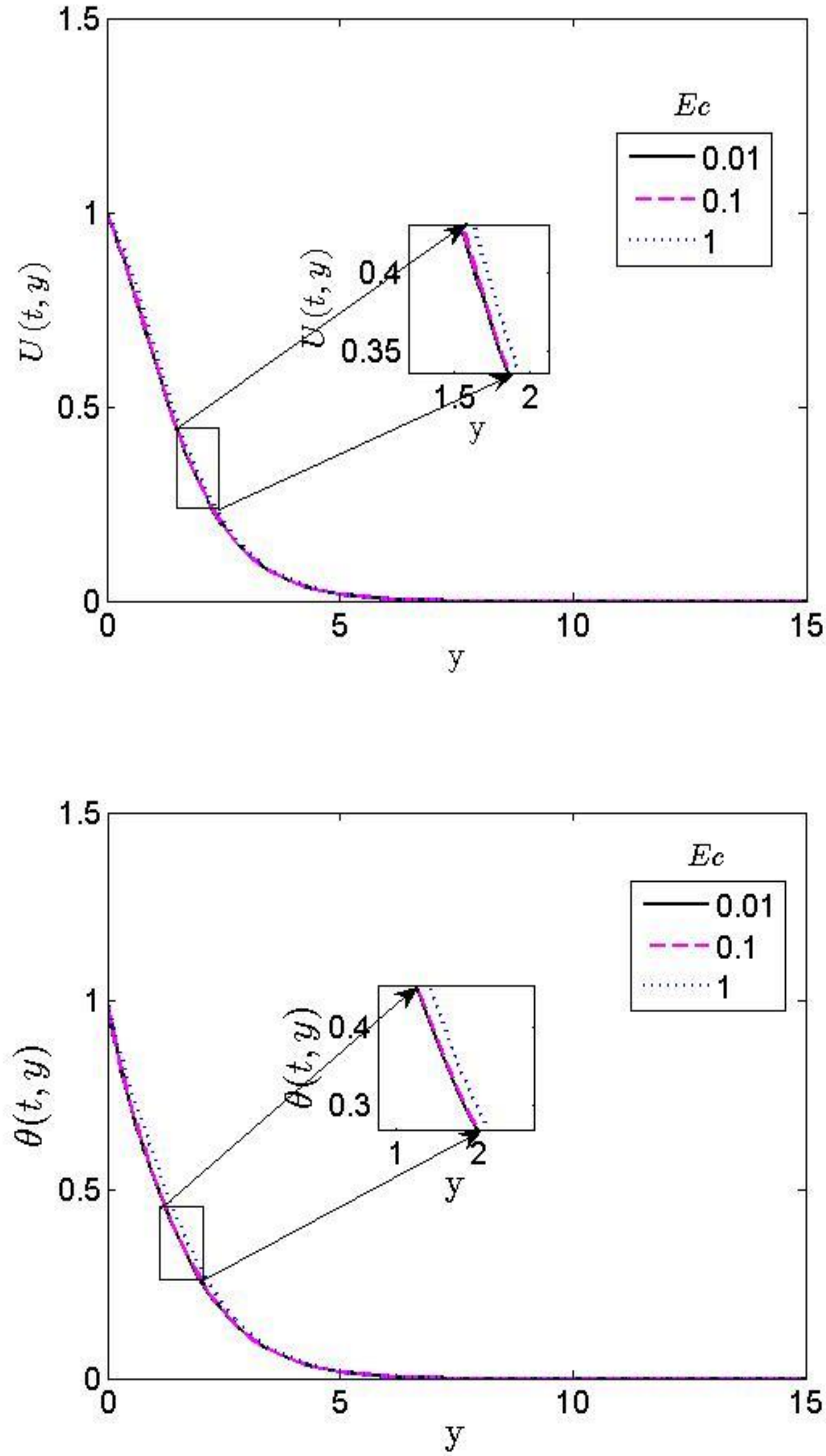

Figure 6. Effects of different values of Eckert number on the velocity and temperature profiles

5. CONCLUSIONS

This study examined the effects of heat transfer on unsteady MHD boundary layer flow past a moving vertical plate in the presence of thermal radiation and viscous dissipation. An efficient iterative method called spectral relaxation method (SRM) was used in solving the problem under investigation. This method was implemented in a symbolic mathematical programming language MATLAB R2012a (7.14.0.739) 32-bit (win 32). The choice of spectral relaxation method (SRM) in the present study was because the governing dimensionless equations are coupled nonlinear partial differential equations whose solution cannot be obtained analytically. With the help of the spectral relaxation method (SRM), these equations were decoupled and nonlinear. The domains of the governing equations with the boundary conditions were transformed into $[-1,1]$. Furthermore, implementing the spectral relaxation method (SRM), the following conclusions were drawn:

(i) Increase in the magnetic parameter reduces the velocity profile but negligible on the temperature profile.
(ii) Increased Prandtl number reduces both the velocity and temperature profiles respectively
(iii) The local skin-friction and the local Nusselt number increase with an increase in the thermal radiation parameter.
(iv) The magnetic parameter reduces the local skin-friction when it is increased but negligible on the local Nusselt number.

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