Anisotropic charged stars with quadratic equation state

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ABSTRACT

In this paper, we present a new model of static spherically symmetric relativistic compact stellar objects with anisotropic charged matter distribution and quadratic equation of state together with a prescribed form for the gravitational potential Z used by Bhar and Murad (2016). A graphical analysis of the physical properties indicate indicates that the new model well behaved and not admit singularities in the matter and the charge density.

*Keywords***:** Anisotropic charged matter distribution, compact stellar objects, energy density, quadratic equation of state

1. INTRODUCTION

From the development of Einstein´s theory of general relativity, the modelling of superdense mater configurations is an interesting research area [1,2]. In the last decades, such models allow explain the behavior of massive objects as neutron stars, quasars, pulsars, black holes and white dwarfs [3-5].

In theoretical works of realistic stellar models, is important include the pressure anisotropy [6-8]. The existence of anisotropy within a star can be explained by the presence of a solid core, phase transitions, a type III super fluid, a pion condensation [9] or another physical phenomenon as the presence of an electrical field [10]. The physics of ultrahigh densities is not well understood and many of the strange stars studies have been performed

within the framework of the MIT-Bag model [11]. In this model, the strange matter equation of state has a simple linear form given by $p = \frac{1}{3}(\rho - 4B)$ where ρ is the energy density, p is the isotropic pressure and B is the bag constant. Many researchers have used a great variety of mathematical techniques to try to obtain exact solutions for quark stars within the framework of MIT-Bag model; Komathiraj and Maharaj [11] found two new classes of exact solutions to the Einstein-Maxwell system of equations with a particular form of the gravitational potential and isotropic pressure. Malaver [12,13] also has obtained some models for quark stars considering a potential gravitational that depends on an adjustable parameter. Thirukkanesh and Maharaj [14] studied the behavior of compact relativistic objects with anisotropic pressure in the presence of the electromagnetic field. Maharaj et al. [15] generated new models for quark stars with charged anisotropic matter considering a linear equation of state. Thirukkanesh and Ragel [16] obtained new models for compact stars with quark matter. Sunzu et al. found new classes of solutions with specific forms for the measure of anisotropy [17].

With then use of Einstein´s field equations, important advances has been made to model the interior of a star. In particular, Feroze and Siddiqui [18,19] and Malaver [20-23] consider a quadratic equation of state for the matter distribution and specify particular forms for the gravitational potential and electric field intensity. Mafa Takisa and Maharaj [24] obtained new exact solutions to the Einstein-Maxwell system of equations with a polytropic equation of state. Thirukkanesh and Ragel [25] have obtained particular models of anisotropic fluids with polytropic equation of state which are consistent with the reported experimental observations. Malaver [26,27] generated new exact solutions to the Einstein-Maxwell system considering Van der Waals modified equation of state with and without polytropical exponent and [28] presented a anisotropic strange quark matter model by imposing a linear barotropic equation of state with Tolman IV form for the gravitational potential. Mak and Harko [29] found a relativistic model of strange quark star with the suppositions of spherical symmetry and conformal Killing vector. Bhar et al. [30,31] have studied extensively the behavior of static spherically symmetric relativistic objects with anisotropic matter distribution considering the Tolman VII space-time. Pant et al. [32] have found new exact solutions of the field equations for anisotropic neutral fluid in isotropic coordinates.

The aim of this paper is to obtain new exact solutions to the Maxwell-Einstein system for charged anisotropic matter with quadratic equation of state in static spherically symmetric spacetime using a prescribed form for the gravitational potential Z used by Bhar and Murad [33]. We have obtained a new classes of static spherically symmetrical models for relativistic stars in presence of an electromagnetic field. This article is organized as follows, in Section 2, we present Einstein´s field equations of anisotropic fluid distribution. In Section 3, we make a particular choice of gravitational potential Z(x) that allows solving the field equations and we have obtained new models for charged anisotropic matter. In Section 4, a physical analysis of the new solution is performed. Finally in Section 5, we conclude.

2. EINSTEIN FIELD EQUATIONS

We consider a spherically symmetric, static and homogeneous and anisotropic spacetime in Schwarzschild coordinates given by:

$$ds^2 = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{1}$$

where: $\nu(r)$ and $\lambda(r)$ are two arbitrary functions.

The Einstein field equations for the charged anisotropic matter are given by:

$$T_{00} = -\rho - \frac{1}{2}E^2 \tag{2}$$

$$T_{11} = p_r - \frac{1}{2}E^2 \tag{3}$$

$$T_{22} = T_{33} = p_t + \frac{1}{2}E^2 \tag{4}$$

where: ρ is the energy density, p_r is the radial pressure, E is electric field intensity and p_t is the tangential pressure, respectively. Using the transformations, $x = cr^2$, $Z(x) = e^{-2\lambda(r)}$ and $A^2y^2(x) = e^{2\nu(r)}$ with arbitrary constants A and c>0 the Einstein field equations can be written as:

$$\frac{1-Z}{x} - 2\dot{Z} = \frac{\rho}{c} + \frac{E^2}{2c} \tag{5}$$

$$4Z\frac{\dot{y}}{y} - \frac{1-Z}{x} = \frac{p_r}{c} - \frac{E^2}{2c} \tag{6}$$

$$4xZ\frac{\ddot{y}}{y} + (4Z + 2x\dot{Z})\frac{\dot{y}}{y} + \dot{Z} = \frac{p_t}{c} + \frac{E^2}{2c} \tag{7}$$

$$p_t = p_r + \Delta \tag{8}$$

$$\frac{\Delta}{c} = 4xZ\frac{\ddot{y}}{y} + \dot{Z}\left(1 + 2x\frac{\dot{y}}{y}\right) + \frac{1-Z}{x} - \frac{E^2}{c} \tag{9}$$

$$\sigma^2 = \frac{4cZ}{x}\left(x\dot{E} + E\right)^2 \tag{10}$$

σ is the charge density, $\Delta = p_t - p_r$ is the anisotropic factor and dots denote differentiation with respect to x. With the transformations of [32], the mass within a radius r of the sphere take the form

$$M(x) = \frac{1}{4c^{3/2}} \int_0^x \sqrt{x}\left(\rho_* + E^2\right) dx \tag{11}$$

where: $\quad \rho_* = \left(\frac{1-Z}{x} - 2\dot{Z}\right)c$

In this paper, we assume the following quadratic equation of state for charged anisotropic matter

$$p_r = \alpha\rho^2 \tag{12}$$

where: α is arbitrary constant and $\rho = \rho_* + E^2$

3. THE NEW PHYSICAL MODELS

To solve the systems of equations (5)-(10), following Bhar and Murad [33] we take the form of the gravitational potential given by:

$$Z(x) = \frac{1 + (a-b)x}{1 + ax} \tag{13}$$

where: a and b are real constants. This potential is well behaved and regular at the origin in the interior of the sphere.

We have considered the particular form of the electrical field proposed for Malaver [23]

$$E^2 = 2c(1 - Z) \tag{14}$$

Using $Z(x)$ and eq. (14) in eq. (5) we obtain

$$\rho = \frac{c\left[3b + (a-1)bx - abx^2\right]}{(1 + 2ax)^2} \tag{15}$$

Substituting (15) in eq.(12), the radial pressure can be written in the form:

$$p_r = \frac{\alpha c^2 \left[3b + (a-1)bx - abx^2\right]^2}{(1+2ax)^4} \tag{16}$$

Replacing (15) in eq. (11) we have for the mass function

$$M(x) = \frac{x^{1/2}}{\sqrt{c}}\left[\frac{2abx + 3a^2bx - a^2bx^2 + 3b}{6a^2(1+ax)}\right] - \frac{b\arctan(\sqrt{ax})}{2a^2\sqrt{ac}} \tag{17}$$

With (13) in eq.(14) we obtain for the electric field intensity

$$E^2 = \frac{2cbx}{1+ax} \tag{18}$$

and for charge density:

$$\sigma^2 = \frac{2bc^2\left[1 + (a-b)x\right]\left(9 + 12ax + 4a^2x^2\right)}{(1+ax)^4} \tag{19}$$

Substituting (13),(14) and (16) in (6), we have

$$\frac{\dot{y}}{y} = \frac{\left[\begin{array}{l}\left(a^2b^2c\alpha - a^3b\right)x^4 + \left(a^3b - 2a^2b^2c\alpha + 2ab^2c\alpha\right)x^3 + \left(a^2b^2c\alpha - 8ab^2c\alpha + b^2c\alpha + 5a^2b - 3ab\right)x^2 \\ + \left(6ab^2c\alpha - 6b^2c\alpha + 7ab - b\right)x + 9b^2c\alpha + 3b\end{array}\right]}{4(1+ax)^3\left[1 + (a-b)x\right]}$$
$$- \frac{b}{2(1-ax)\left[1 + (a-b)x\right]} \tag{20}$$

Integrating (20) , we obtain:

$$y(x) = d_1(ax - bx + 1)^A (1+ax)^B e^{C(x)} \tag{21}$$

where:

$$A = -\frac{\left(\begin{array}{l}8a^4b^2c\alpha - 40a^3b^3c\alpha + 74a^2b^4c\alpha - 60ab^5c\alpha + 18b^6c\alpha - 8a^2b^3c\alpha \\ 20ab^4c\alpha - 12b^5c\alpha + 2b^4c\alpha + 3a^2b^3 - 8ab^4 + 5b^5 - 6a^3b - 2a^3\alpha \\ + 6a^2b^2 + 6a^2b\alpha - 6a\alpha b^2 - 2b^4 + 2\alpha b^3\end{array}\right)}{2(a-b)^2 b^3}$$

$$B = -12ac\alpha + 9bc\alpha - \frac{a\alpha}{b^3} + 4\frac{a^2c\alpha}{b} + 2\frac{bc\alpha}{a} + \frac{bc\alpha}{a^2} - 4c\alpha - 3\frac{a}{b^2}$$

$$+ \frac{\alpha}{b^2} - \frac{3}{b} - \frac{3}{a} + \frac{3}{2}$$

$$C(x) = -\frac{\begin{pmatrix} 2a^2b^4c\alpha x^3 + 8a^4b^2c\alpha x - 24a^3b^3c\alpha x - 2a^3b^3x^3 + 16a^2b^4c\alpha x + 4ab^4c\alpha x^2 \\ -8a^2b^3c\alpha x + 8ab^4c\alpha x + 8a^3b^2c\alpha - 28a^2b^3c\alpha - 4a^2b^3x^2 + 20ab^4c\alpha + 2b^4c\alpha x \\ -8ab^3c\alpha + 8b^4c\alpha - 6a^3bx - 2a^3\alpha x - 6a^2b^2x + 2a^2b\alpha x + 10ab^3x - 6a^2b - 2a^2\alpha \\ -3ab^2 + 3ab\alpha + 9b^3 - b^2\alpha \end{pmatrix}}{2ab^2(a-b)(ax+1)^2}$$

(22)

and d_1 is the constant of integration.

The metric functions $e^{2\lambda}$ and $e^{2\nu}$ can be written as

$$e^{2\lambda} = \frac{1+ax}{1+(a-b)x}$$

$$e^{2\nu} = A^2 d_1^2 (ax - bx + 1)^{2A} (1+ax)^{2B} e^{2C(x)}$$

(23)

and for the tangential pressure we have

$$p_t = \frac{4xc[1+(a-b)x]}{1+ax}\frac{\ddot{y}}{y} + 2c\left[\frac{2+(4a-3b)x+2a(a-b)x^2}{(1+ax)^2}\right]\frac{\dot{y}}{y}$$

$$- \frac{b+bx(1+ax)}{(1+ax)^2}$$

(24)

The metric for this model is

$$ds^2 = -A^2 d_1^2 \left(1+(a-b)r^2\right)^{2A} \left(1+ar^2\right)^{2B} e^{2C(r^2)} dt^2 + \frac{\left(1+ar^2\right)}{\left(1+(a-b)r^2\right)} dr^2 + r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

(25)

The Figures 1, 2, 3, 4 and 5 represent the graphs of p_r, ρ, σ^2, $M(x)$ and radial speed sound v_{sr}^2 against the radial coordinate, respectively. The graphs has been plotted for a

particular choice of parameters a = 0.341 and b = 0.00454 with a stellar radius of r = 1.48 Km. Here the values of the constants are $c = \alpha = 1$.

4. PHYSICAL ANALYSIS OF THE NEW MODELS

For a solution of field equations to be physically acceptable, the following conditions should be satisfied [25,33]:

(i) The metric potentials should be free from singularities in the interior of the star.

(ii) The radial pressure and density must be finite and positive inside of the fluid sphere

(iii) $P_r > 0$ and $\rho > 0$ in the origin.

(iv) Monotonic decrease of the energy density and the radial pressure with increasing radius.

The new model satisfy the system of equations (5) - (12) and constitute another new family of solutions for anisotropic charged matter with quadratic equation of state. The metric functions $e^{2\lambda}$ and $e^{2\nu}$ can be written in terms of elementary functions and the variables energy density, pressure and charge density also are represented analytical. In this model

$$e^{2\lambda(0)} = 1 \ , \ e^{2\nu(0)} = A^2 d_1^{\ 2} e^{C(0)}$$

where:

$$C(0) = -\frac{\left(\begin{array}{c} 8a^3b^2c\alpha - 28a^2b^3c\alpha + 20ab^4c\alpha - 8ab^3c\alpha + 8b^4c\alpha - 6a^2b - 2a^2\alpha \\ -3ab^2 + 3ab\alpha + 9b^3 - b^2\alpha \end{array}\right)}{2ab^2(a-b)}$$

In the origin $\left(e^{2\lambda(r)}\right)'_{r=0} = \left(e^{2\nu(r)}\right)'_{r=0} = 0$ This shows that the gravitational potential is regular in $r = 0$. At the centre $\rho(0) = 3bc$, $p_r(0) = 9\alpha b^2 c^2$ both are positive if b >0. The mass function is continuous and behaves well inside of the star and the charge density σ not admits singularities at the center.

In Figure 1, the radial pressure is finite and decreasing with the radial coordinate. In Figure 2, that represent energy density we observe that is continuous, finite and monotonically decreasing function.

In Figure 3, the charge density σ is not singular at the origin, non-negative and decreases. In Fig. 4, the mass function is increasing function, continuous and finite. In Fig. 5, the condition $0 \le v_{sr}^2 \le 1$ is maintained inside the stellar interior, which is a physical requirement for the construction of a compact star [35].
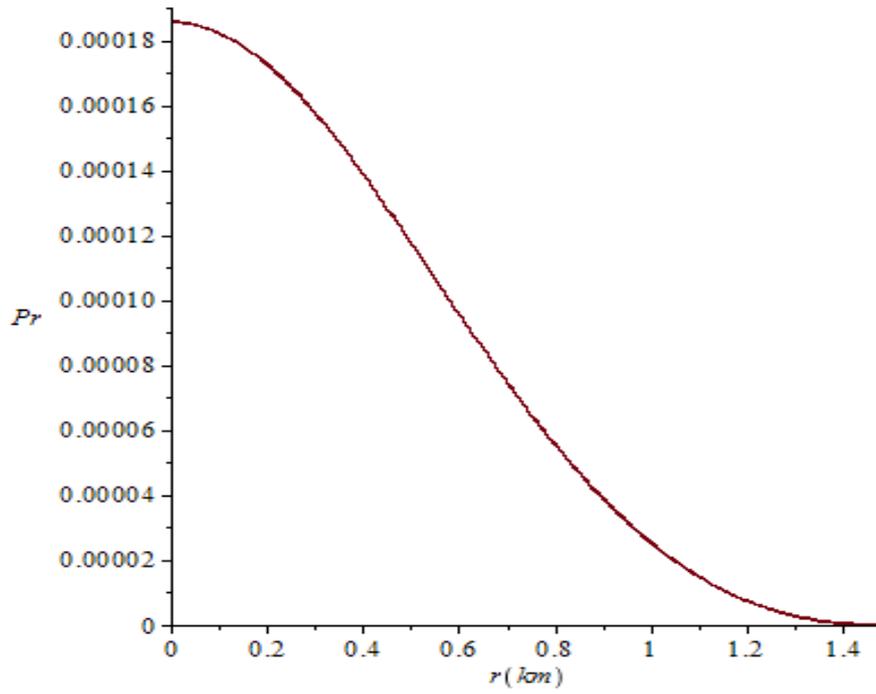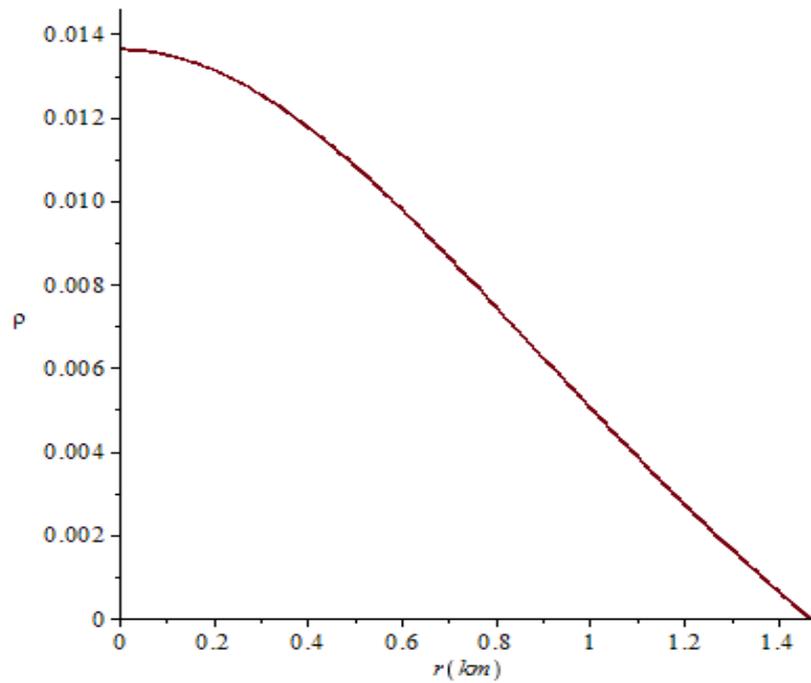
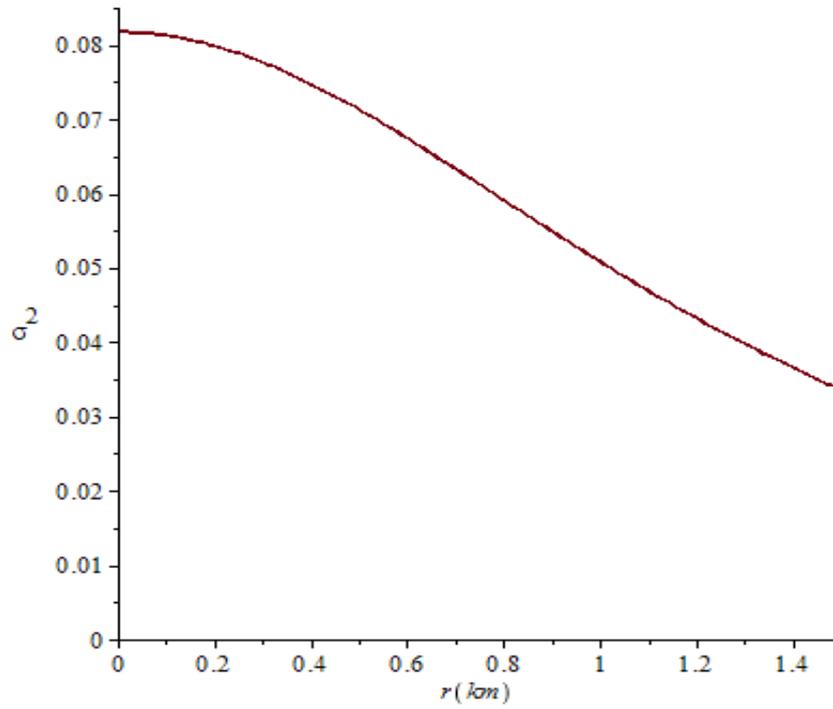**Figure 1.** Radial pressure.

Figure 2. Energy density
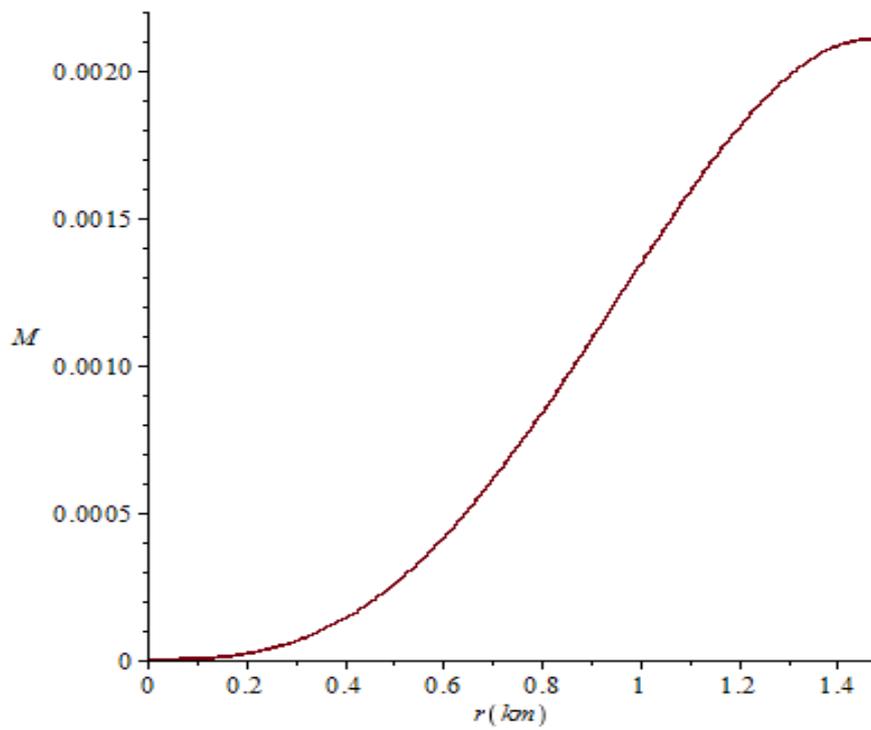
Figure 3. Charge density.
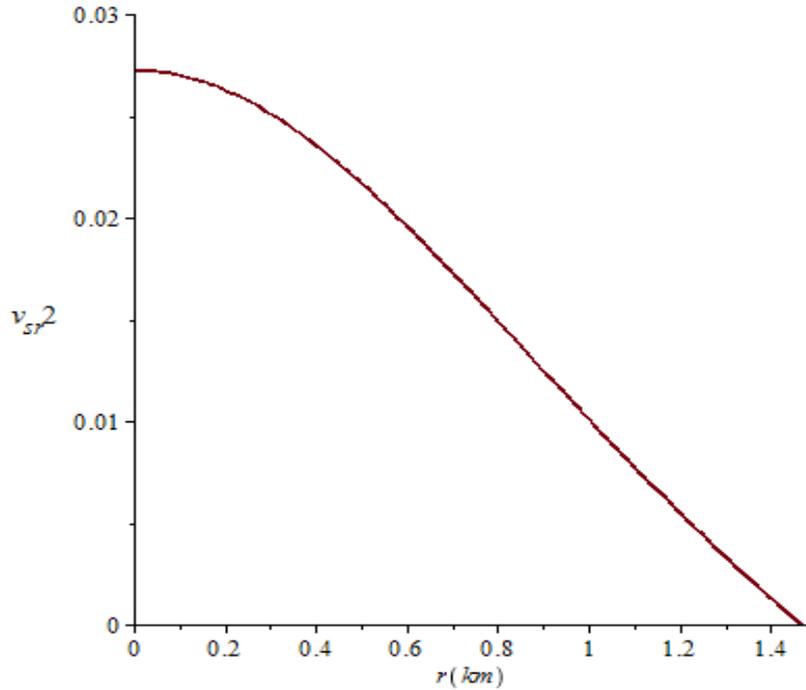

Figure 4. Mass.

Figure 5. Radial speed of sound.

5. CONCLUSION

In this paper, we have generated new exact solutions to the Einstein-Maxwell system considering a prescribed form for the gravitational potential Z and a quadratic equation of state which is very useful for realistic models of charged anisotropic matter. The radial pressure, energy density and the coefficients of the metric are well defined. The charge density σ not admits singularities at the center of the stellar object and the mass function is an increasing function, continuous and finite. The gravitational potentials are regular in the interior of the star and well behaved and the causality condition $0 \leq v_{sr}^2 \leq 1$ inside the radius of the fluid sphere. The new obtained model may be used to model relativistic charged compact objects in different astrophysical scenes as quark and neutron stars

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