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SHORT COMMUNICATION

Hamilton-Jacobi/ Canonical Phase Transitions of Ashtekar LQG-Polyakov Strings: World Sheet Dynamics as Regelation through the d = 3 + 1 Spacetime

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ABSTRACT

The following paper takes up an Ashtekar connectio-dynamics and algebraic quantum field theoretic treatment of a relativistic quantum string without SUSY and hence in four spacetime dimensions treats the Polyakov dynamics of a relativistic quantum string given explicitly by the Polyakov path integral in terms of a “canonical” phase transition where the string is imagined as a thread regelating through the “ice cube” of spacetime.

Keywords**:** Ashtekar connectio-dynamics, Loop Quantum Gravity, phase transitions, string theory, Nambu-Goto action, Polyakov path integral

1. INTRODUCTION

An interesting example of extended objects which permit a simple geometrical description is the relativistic quantum string. The minimal model incorporating relativistic dynamics is that of dimensions $d = 4 \equiv 3 + 1$ viz., the spacetime which is duely a pseudo-Euclidean manifold. It is the intent of the current paper to create such a minimal dynamical model for the Polyakov string in terms of a statistical description [1], which yields a different scenario in the formalism of Ashtekar connectio-dynamics and the consequent Loop Quantum Gravitational (LQG) quantization of string of the string theory. While dealing with the various mathematico-physical situations various hindrances and profound conceptual difficulties had been encountered; however, being pressed for time and demand, the following paper has though not glossed over the difficulties has managed to side-track these. However, these difficulties will not be ignored and will be the subject of a sequel paper. Hence, the problem of finite-temperature field theory and the consequent inverse-temperature- complex time equivalence is only mentioned in passing in this paper and the corresponding abelian $U(1)$ gauge will be the subject of a paper dedicated specifically to the problem of time [2]. All the while we do not drift away forgetting the main objective of the paper which is to provide a quantum mechanics-statistical thermophysics correspondence to the end of delivering the relativistic dynamics of quantum strings without SUSY. To this end, extra dimensions will also not be considered; let us hasten to add that we do not rule them out, just that we want to have a minimal simplistic model with extra constructions rendered redundant for the time being. This agrees quite well with the basic foundational principle of LQG: the principles of general relativity and quantum theory are combined consistently and driven to their logically limiting frontiers.

Now for a quantum string, more on the lines of Polyakov rather than the Nambu-Goto approach adopted by Thiemann (see [3] and references therein), which is shown to be marred with anomalies by a simple consideration of the harmonic oscillator by Helling and Policastro [4]. We can show by arguments similar to the above that the dynamics of the string is a first order phase transition given by

$$\frac{\partial T_c}{\partial p^{\mu}} = T_c \frac{disc\ X^{\mu}.}{disc\ S_{Polyakov}} \tag{1}$$

2. FULL TEXT: FORMULATION OF THE REGELATION DYNAMICS OF AN OPEN-ENDED QUANTUM STRING IN THE BACKGROUND FREE (BF) THEORY

The Thiemann [3] quantization of a string was a novel technique. However, as can never be enough stressed, he stated that the Nambu-Goto action for the string dynamics in d=3+1 dimensions is the only basis for a mathematically consistent background independent formulation. He also states that the version of string quantization that he gives for the flat target space – the Loop Quantization of a string is the only mathematically rigorous and consequently self-consistent quantum theory of string. The Pohlmeyer charges and the inherent group theoretic quantization of the string is then stated by Thiemann to be possessing a rich representation theoretic structure but he does not demonstrate the said richness. Even worse, he announces the sequel paper: "*The LQG String: Loop Quantum Gravitational Quantization of the String – Curved Target* Space", leaving all the scientific community interested in quantum gravity and string theory, mystified. Firstly, the assumption made by Thiemann are blatantly preposterous and outright wrong. The 2-D CFT (Conformal Field Theory) on which the Polyakov path integral is defined is equally mathematically rigorous. Basically, it is a construction of quantum field theory (QFT) on Riemann surfaces.which are conformal or one complex dimensional curves thereby making them two dimensional real surfaces. The theory is well constructed and robust. The problem is with the string theory defined on the world sheet which is apparently, a Riemann surface. The dynamics of a quantized string is expressed by a string perturbation series which is mathematically ill-defined. The perturbative format naturally assumes the curvature as arising due to the perturbations of the flat metric. Hence, the trouble; perturbations of these type are naturally background metric dependent. The format required here is the non-perturbative quantization of the string and its impressed dynamics through the d=3+1 spacetime. The phase transitional method outlined below for the dynamics of a quantum string, is one such attempt. The natural interpretation to be invoked in writing the partition function of a quantum string in the *Many Worlds Interpretation*.hence, mention is made of the copies of the quantum string. The Polyakov path integral integrates out all of these string amplitudes in the case of the open-ended quantum string. The string is envisaged to regelate through the spacetime continuum.

2. 1. THE ASHTEKAR-POLYAKOV CANONICAL PHASE TRANSITION EQUATION

Consider an ensemble or gas of copies of a relativistic quantum string— the tiny loop of energy of the string theory. Let E be the energy of the fluid/ensemble of the quantum copies of the string. The temperature of the ensemble is $T_{str} \geq T_{Pl}$ defined by

$$T_{str} = \left(\frac{\partial \sigma}{\partial E}\right)^{-1}, \tag{2}$$

where σ is the entropy.

Consider the variation of the temperature of the string-copy fluid. Let p^{μ} be the momentum of the string conjugate to its world sheet coordinate X^{μ}. If the temperature T_{str}, of the fluid of string-copies is decreased, the liquid condenses to form a rigid causal solid. The partition function of this gas is

$$z = Tr\, e^{-\beta(E_r - \mu N_r)}, \tag{3}$$

where μ is the chemical potential of the string ensemble. The partition function (3) changes in response to a change in world-sheet momentum co-ordinate, as

$$X^{\mu} = -\frac{\partial}{\partial p^{\mu}} \ln z. \tag{4}$$

A discontinuity in the action

$$S = \left(T\frac{\partial P}{\partial T} - P\right) \quad (5)$$

can be expressed by the gap in the entropy σ [3],

$$disc\ \sigma = T_c\ disc\ \frac{\partial P}{\partial T} \quad (6)$$

From (4) with $\ln z = (P - S_0)$ where S_0 is the ground state Polyakov action. To this corresponds $\langle 0|X^\mu|0\rangle$; it follows that

$$X^\mu = \frac{\partial S_0}{\partial p^\mu} - \frac{\partial P}{\partial p^\mu} = \langle 0|X^\mu|0\rangle - \frac{\partial P}{\partial p^\mu}. \quad (7)$$

If we expand the pressure in the vicinity of T_c according to

$$P = P_c + (T - T_c)\frac{\partial P}{\partial T}\bigg|_{T_c} + \ \dots. \quad (8)$$

P depends on E via P_c and T_c

$$\frac{\partial P}{\partial p^\mu} = \frac{\partial P_c}{\partial p^\mu} - \frac{\partial T_c}{\partial p^\mu}\frac{\partial P}{\partial T}\bigg|_{T=T_c} \quad (9)$$

Inserting (9) into (7) and applying *"disc"* to both the sides, the result is

$$disc\ X^\mu = \frac{\partial T_c}{\partial p^\mu} disc\ \frac{\partial P}{\partial T}\bigg|_{T=T_c}. \quad (10)$$

While P is discontinuous at T_c (as in the case of a first order phase transition), $\frac{\partial P}{\partial T}$ may jump. From (6) and (10), we finally obtain

$$\frac{\partial T_c}{\partial p^\mu} = T_c \frac{disc\ X^\mu.}{disc\ S_{Polyakov}}. \quad (1)$$

The world-sheet co-ordinate taken here as the order-parameter is amply justified as the order parameter is irreversible and the world-sheet co-ordinate is not retraced by the string and hence implies irreversibility making the world-sheet momentum a viable fugacity..

2. 2. INFERENCES FROM THE BF FORMULATION OF THE DYNAMICS OF THE QUANTUM STRING: REGELATION AS A MODE OF PROPAGATION OF A STRING IN D = 3 + 1 SPACETIME

For now, the current paper aims only at giving a simplistic model for string BF dynamics in terms of phase transitions in non-perturbative canonical gravity, here, loop quantum gravity. Again, the world sheet background independence claimed by Thiemann is not unique to his formulation since the Virasoro constraints delivered by the Nambu-Goto action which is divinely claimed to be BF by Thiemann is also equally well delivered by the Polyakov action and since the quantization of the resulting constraints doesn't introduce any further background, our quantization as well as Thiemann's quantization is as worldsheet background independent (BF) as the usual Fock space quantization.

Thus, the open-ended quantum string necessarily regelates through the body of the $d = 3 + 1$ spacetime just as a thread regelates through the bulk of the ice cube when pressed through it. The world-sheet momentum of the string is analogous to the pressure or rather fugacity in the usual Clausius-Clapeyron equation while X^{μ} is the order parameter.

It is however, mandate to couple this equation to the Background Free (BF) Loop Quantum Gravity (LQG) by considering the actual traditional Clausius-Clapeyron equation for a solid-liquid phase transition for a gas of "geometric particles"− the surface states, as dubbed by Major et. al., [5] as well as the grannulons − the nodes of the spin networks. It is in fact, the nodes that are important in the volume that is taken as the order parameter for the spacetime "solid⇌liquid" phase transition as the quantum string regelates through the geometry of spacetime. This equation reads

$$\frac{\partial T_c}{\partial P} = T_c \frac{disc\, V.}{disc\, \sigma} \tag{11}$$

The volume V appearing in the above equation is not an operator eigenvalue but a classical value that carries the properties of the BF quantum geometry in the statistical collective mode. The reason for the classicity of the volume operator is that the string charge defined as the ratio of the Planck length to the string length is in fact much less than one since it is the string charge that defines the electronic charge which is evidently much less than unity. The frame of reference of the string thus perceives a continuum of spacetime geometry and not a BF discrete quantum one.

3. CONCLUSIONS

Quantum gravity at the interface of the String Theory and Loop Quantum Gravity, has always been a fascinating esoteric and extremely sensitive topic to tackle. Loop Quantum Gravitational quantization of a string as proposed by Thiemann may be one good approach but does not shed light on any physical features of the string theory as a background independent theory. Too much physics and physical insight is lost in mathematical over-abstraction. The Thiemann approach is one such example. What has been gained? A proper way is to exploit the fact that the M-Theory, sprouting from the strong coupling limit Type IIA superstring theory and its T-dual Heterotic $E_8 \times E_8$ superstring theory, is indeed background independent. Consequently, one can enter from the higher dimensional realm and using the techniques of LQG, quantize the string. The theory formulated in this paper is done so as to provide a quantum model for the physical dynamics of the string. To answer the question – what does the physical picture of a quantum string travelling through the d=3+1 spacetime look like. And the answer the author thought best was *regelation*. A better understanding of this mode of propagation of the string through the spacetime may be possible. But the answer should most definitely be *regelation* as far as the constrained Hamiltonian dynamics of the LQG-String system is concerned.

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Biography

Koustubh Kabe is Dr. Phil (PhD) / Sc.D. in Theoretical Physics from the Advanced International School of Theoretical and Non-Linear Methodologies in Physics, Bari, Italy. His topic of Dissertation was "*Theoretical Investigations into the Fundamental Nature of Time and* Gravity". He has published several research papers investigating into the foundational issues of gravitational physics and the understanding of time and quantum gravity. He is also working on the problem of gravity and the cosmological implications in the framework of string theory. He is currently studying Quantum Measurement in addition to all of the above. His research interests are in the fields of General Theoretical Physics, Physical Mathematics, Theoretical Astrophysics, Theoretical High-energy Physics, Modern Theoretical Physics, Physical Cosmology, Geometric Analysis, Number Theory, Algebraic Geometry and lastly, Philosophy, Epistemology and Pedagogy behind Physical Theories. He is an author of a book titled "*Blackhole Dynamic Potentials and Condensed Geometry: New Perspectives on Blackhole Dynamics and Modern Canonical Quantum General Relativity*".

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