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Relativistic modeling of compact stars for charged anisotropic matter in a Tolman IV spacetime

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ABSTRACT

In this paper, we studied the behavior of relativistic objects considering Tolman IV form for the gravitational potential Z and a lineal equation of state within the framework of MIT-Bag model for the charged anisotropic matter. A physical analysis of electromagnetic field indicates that is regular in the origin and well behaved. The new obtained solution not admits singularities in the matter, charge density and metric functions. A graphical analysis indicates that the new stellar model satisfy all physical properties expected in a realistic star.

Keywords**:** Relativistic objects, Charged anisotropic matter, Tolman IV potential, lineal equation of state, electromagnetic field

1. INTRODUCTION

From the development of Einstein´s theory of general relativity, the modelling of superdense mater configurations is an interesting research area [1,2]. In the last decades, such models allow explain the behavior of massive objects as neutron stars, quasars, pulsars, black holes and white dwarfs [3-5]. Malaver [3] studied the behavior of the thermal capacity C_v for Schwarzschild´s black hole when $T>>T_C$ and $T<<T_C$ where T_C is the characteristic temperature of the Schwarzschild black hole and found that the value for C_v if $T>>T_C$ is the same that would be obtained in an ideal diatomic gas if only are considered the degrees of freedom rotational. Komathiraj and Maharaj [4] find new classes exact solutions to the Einstein-Maxwell system of equations for a charged sphere with a particular choice of the electric field intensity and one of the gravitational potentials. Sharma et al. [5] have obtained a class of solutions to the Einstein-Maxwell system assuming a particular form for the hypersurface (t = constant) containing a parameter λ .

In theoretical works of realistic stellar models, is important include the pressure anisotropy [6-8]. The existence of anisotropy within a star can be explained by the presence of a solid core, phase transitions, a type III super fluid, a pion condensation [9] or another physical phenomena as the presence of an electrical field [10]. The physics of ultrahigh densities is not well understood and many of the strange stars studies have been performed within the framework of the MIT-Bag model [11].

In this model, the strange matter equation of state has a simple linear form given by $p = \frac{1}{3}(\rho - 4B)$ where ρ is the energy density, p is the isotropic pressure and B is the bag constant. Many researchers have used a great variety of mathematical techniques to try to obtain exact solutions for quark stars within the framework of MIT-Bag model; Komathiraj and Maharaj [11] found two new classes of exact solutions to the Einstein-Maxwell system of equations with a particular form of the gravitational potential and isotropic pressure. Malaver [12,13] also has obtained some models for quark stars considering a potential gravitational that depends on an adjustable parameter. Thirukkanesh and Maharaj [14] studied the behavior of compact relativistic objects with anisotropic pressure in the presence of the electromagnetic field. Maharaj et al. [15] generated new models for quark stars with charged anisotropic matter considering a linear equation of state. Thirukkanesh and Ragel [16] obtained new models for compact stars with quark matter. Sunzu et al. found new classes of solutions with specific forms for the measure of anisotropy [17].

With then use of Einstein´s field equations, important advances has been made to model the interior of a star. In particular, Feroze and Siddiqui [18,19] and Malaver [20-23] consider a quadratic equation of state for the matter distribution and specify particular forms for the gravitational potential and electric field intensity. Mafa Takisa and Maharaj [24] obtained new exact solutions to the Einstein-Maxwell system of equations with a polytropic equation of state. Thirukkanesh and Ragel [25] have obtained particular models of anisotropic fluids with polytropic equation of state which are consistent with the reported experimental observations. Malaver [26,27] generated new exact solutions to the Einstein-Maxwell system considering Van der Waals modified equation of state with and without polytropical exponent and [28] presented a anisotropic strange quark matter model by imposing a linear barotropic equation of state with Tolman IV form for the gravitational potential. Mak and Harko [29] found a relativistic model of strange quark star with the suppositions of spherical symmetry and conformal Killing vector. Bhar et al. [30,31] have studied extensively the behavior of static spherically symmetric relativistic objects with anisotropic matter distribution considering the Tolman VII space-time.

The objective of this paper is to obtain new exact solutions to the Maxwell-Einstein system for charged anisotropic matter with lineal equation of state in static spherically symmetric spacetime using Tolman IV form for the gravitational potential Z. We have obtained a new classes of static spherically symmetrical models for relativistic stars in presence of an electromagnetic field. This article is organized as follows, in Section 2, we present Einstein´s field equations of anisotropic fluid distribution. In Section 3, we make a particular choice of gravitational potential Z(x) that allows solving the field equations and we have obtained a new model for charged anisotropic matter. In Section 4, a physical analysis of the new solution is performed. Finally in Section 5, we conclude.

2. EINSTEIN FIELD EQUATIONS

We consider a spherically symmetric, static and homogeneous and anisotropic spacetime in Schwarzschild coordinates given by

$$ds^2 = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{1}$$

where: $\nu(r)$ and $\lambda(r)$ are two arbitrary functions.

The Einstein field equations for the charged anisotropic matter are given by

$$T_{00} = -\rho - \frac{1}{2}E^2 \tag{2}$$

$$T_{11} = p_r - \frac{1}{2}E^2 \tag{3}$$

$$T_{22} = T_{33} = p_t + \frac{1}{2}E^2 \tag{4}$$

where: ρ is the energy density, p_r is the radial pressure, E is electric field intensity and p_t is the tangential pressure, respectively. Using the transformations, $x = cr^2$, $Z(x) = e^{-2\lambda(r)}$ and $A^2 y^2(x) = e^{2\nu(r)}$ with arbitrary constants A and c>0 the Einstein field equations can be written as

$$\frac{1-Z}{x} - 2\dot{Z} = \frac{\rho}{c} + \frac{E^2}{2c} \tag{5}$$

$$4Z\frac{\dot{y}}{y} - \frac{1-Z}{x} = \frac{p_r}{c} - \frac{E^2}{2c} \tag{6}$$

$$4xZ\frac{\ddot{y}}{y} + (4Z + 2x\dot{Z})\frac{\dot{y}}{y} + \dot{Z} = \frac{p_t}{c} + \frac{E^2}{2c} \tag{7}$$

$$\sigma^2 = \frac{4cZ}{x}\left(x\dot{E} + E\right)^2 \tag{8}$$

σ is the charge density and dots denote differentiation with respect to x. With the transformations of [32], the mass within a radius r of the sphere take the form

$$M(x) = \frac{1}{4c^{3/2}}\int_0^x \sqrt{x}\left(\rho_* + E^2\right)dx \tag{9}$$

where: $\rho_* = \left(\frac{1-Z}{x} - 2\dot{Z}\right)c$

In this paper, we assume the following lineal equation of state within the framework of MIT-Bag model:

$$p_r = \frac{1}{3}\rho \tag{10}$$

3. THE NEW MODEL

Following Tolman [33] and Thirukkanesh and Ragel [28], we take the form of the gravitational potential, $Z(x)$ as

$$Z(x) = \frac{(1+ax)(1-bx)}{(1+2ax)} \tag{11}$$

where: a and b are real constants. We have considered the form of the electrical field proposed for Malaver [23]

$$E^2 = 2c(1-Z) \tag{12}$$

Using $Z(x)$ and eq. (12) in eq. (5) we obtain

$$\rho = \frac{c\left[3(a+b) - \left(5ab - a - b + 2a^2\right)x + \left(4a^2 - 2a^2 - 3ab\right)x^2 - 2a^2bx^3\right]}{(1+2ax)^2} \tag{13}$$

Substituting (13) in eq.(10), the radial pressure can be written in the form:

$$P_r = \frac{c\left[3(a+b)-\left(5ab-a-b+2a^2\right)x+\left(4a^2-2a^2-3ab\right)x^2-2a^2bx^3\right]}{3(1+2ax)^2} \tag{14}$$

Replacing (13) in eq. (9) we have for the mass function

$$M(x) = \frac{x^{1/2}}{2\sqrt{c}}\left[-\frac{bx^2}{10}+\frac{(4ab-2a-b)}{12a}x+\frac{(b+2a)(1+2a)}{8a^2}-\frac{3b+4a}{8a(1+2ax)}\right] + \left[\frac{\sqrt{2}(ab-2a-b)}{32a^2\sqrt{ac}}\right]arctag\sqrt{2ax} \tag{15}$$

and for charge density:

$$\sigma^2 = \frac{2c^2(1+ax)(1-bx)\left[30a^3bx^3+\left(30a^2b+16a^3\right)x^2+\left(14a^2+16ab\right)x+3(a+b)\right]^2}{(1+2ax)^6\left[a+b+abx\right]} \tag{16}$$

The tangential pressure can be obtained from (7) with the help of (11) and (12)

$$p_t = \frac{4xc(1+ax)(1-bx)}{(1+2ax)}\frac{\ddot{y}}{y}+2c\left[\frac{2+(5a-3b)x+4a(a-2b)x^2-6a^2bx^3}{(1+2ax)^2}\right]\frac{\dot{y}}{y} - c\left[\frac{(a+b)+(2ab+a+b)x+\left(2a^2b+3ab+2a^2\right)x^2+2a^2bx^3}{(1+2ax)^2}\right] \tag{17}$$

Substituting (11),(12) and (13) in (6), we have

$$\frac{\dot{y}}{y} = \frac{(a+b+abx)}{4(1+ax)(1-bx)}+\frac{\left[3(a+b)+\left(5ab-a-b+2a^2\right)x+\left(4a^2b-2a^2-3ab\right)x^2-2a^2bx^3\right]}{12(1+ax)(1-bx)(1+2ax)} - \frac{\left[(a+b)x+abx^2\right]}{4(1+ax)(1-bx)} \tag{18}$$

Integrating (18), we obtain

$$y(x) = d_1(1+ax)^A(-1+bx)^B(1+2ax)^C e^{D(x)} \tag{19}$$

where: $A = \dfrac{3a - 2bc - ac + c + 3}{12(a+b)}$

$$B = \frac{4abc + 4a^2c + b^2c - 12ab^2 - 3b^3c - 8ab^2c - 6a^2bc + 3b^2 + 12ab + 12a^2 - 3b^3}{12b(a+b)(b+2a)}$$

$$C = \frac{(4a+3b)c}{12(2a+b)} \quad \text{and} \quad D(x) = \frac{x(c+3)}{12} \tag{20}$$

The metric functions $e^{2\lambda}$ and $e^{2\nu}$ can be written as

$$e^{2\lambda} = \frac{(1+2ax)}{(1+ax)(1-bx)} \tag{21}$$

$$e^{2\nu} = A^2 d_1^2 (1+ax)^{2A} (-1+bx)^{2B} (1+2ax)^{2C} e^{2D(x)} \tag{22}$$

The metric for this model is

$$ds^2 = -A^2 d_1^2 \left(1+ar^2\right)^{2A} \left(-1+br^2\right)^{2B} \left(1+2ar^2\right)^{2C} e^{D(r)} dt^2 + \frac{\left(1+2ar^2\right)dr^2}{\left(1+ar^2\right)\left(1-br^2\right)} + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \tag{23}$$

The Figures 1, 2, 3 and 4 represent the graphs of p_r , ρ , σ^2 and $M(x)$ against the radial coordinate, respectively. The graphs has been plotted for a particular choice of parameters a = 0.341 and b = 0.00454 with a stellar radius of r = 1.4 Km.

4. PHYSICAL FEATURES OF THE NEW MODEL

Any physically acceptable solutions must satisfy the following conditions [25]:

(i) Regularity of the gravitational potentials in the origin.
(ii) Radial pressure must be finite at the centre.
(iii) P_r > 0 and ρ >0 in the origin.
(iv) Monotonic decrease of the energy density and the radial pressure with increasing radius

The new model satisfy the system of equations (6) - (9) and constitute another new family of solutions for anisotropic charged matter with quadratic equation of state. The metric functions $e^{2\lambda}$ and $e^{2\nu}$ can be written in terms of elementary functions and the variables energy density, pressure and charge density also are represented analytical. In this model, $e^{2\lambda(0)}=1$, $e^{2\nu(0)}=A^2 d_1^{\,2}(-1)^{2B}$. In the origin $r=0$ and $\left(e^{2\lambda(r)}\right)'_{r=0}=\left(e^{2\nu(r)}\right)'_{r=0}=0$.

This shows that the gravitational potential is regular in $r=0$. In the centre $\rho(0)=3c(a+b)$, $p_r(0)=c(a+b)$. The mass function is continuous and behaves well inside of the star and the charge density σ not admits singularities at the center.

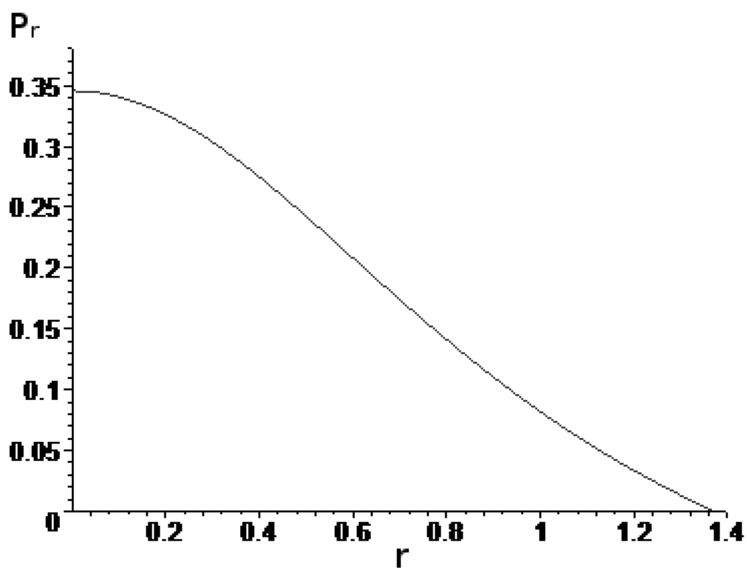

Fig. 1. Radial Pressure

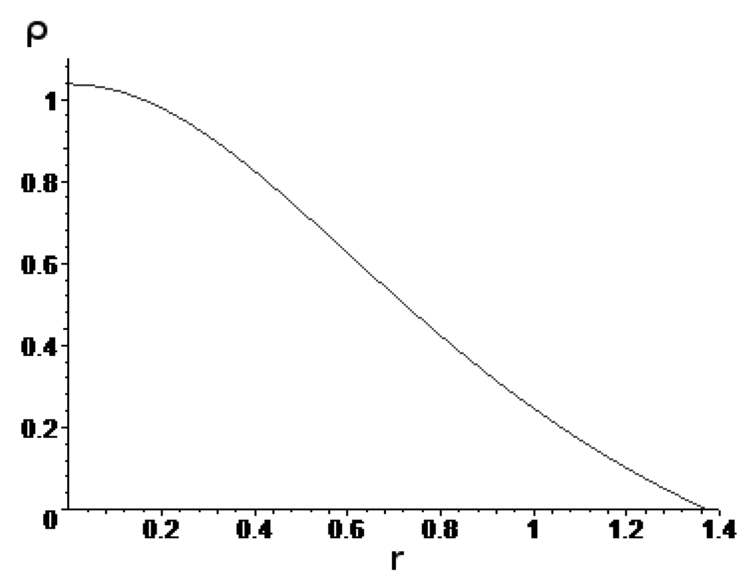

Fig. 2. Energy density

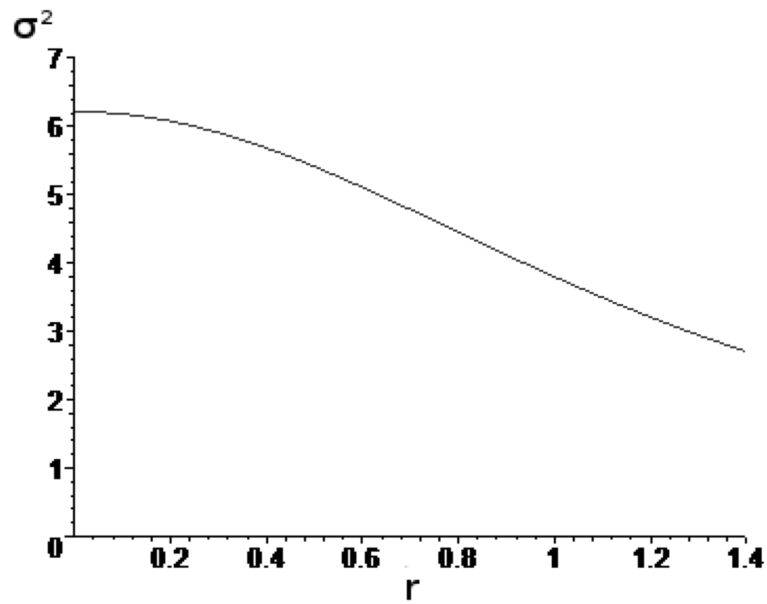

Fig. 3. Charge density

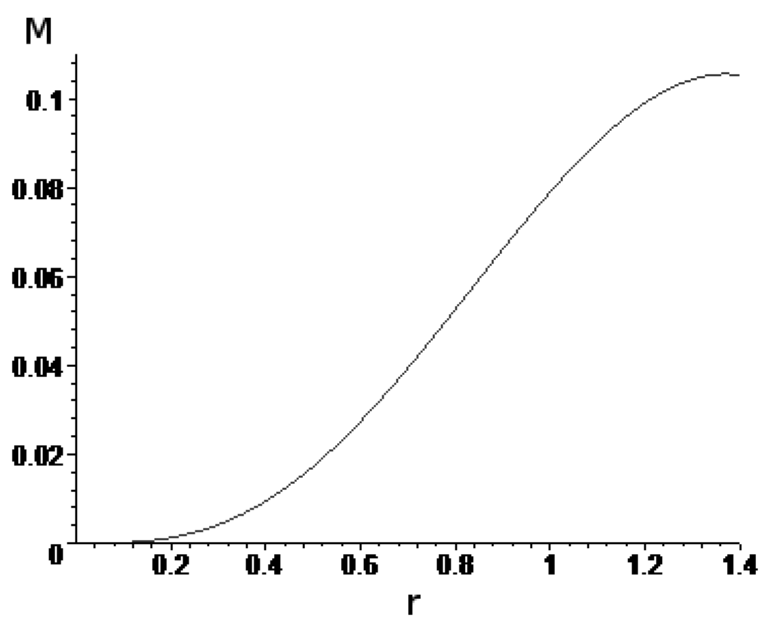

Fig. 4. Mass

In Figure 1, the radial pressure is finite and decreasing with the radial coordinate. In Figure 2, that represent energy density we observe that is continuous, finite and monotonically decreasing function. In Figure 3, the charge density σ is not singular at the origin, non-negative and decreases. In Figure.4, the mass function is increasing function, continuous and finite.

5. CONCLUSION

In this paper, we have generated new exact solutions to the Einstein-Maxwell system considering Tolman IV form for the gravitational potential Z and a lineal equation of state within the framework of MIT-Bag model which is very useful for realistic models of charged anisotropic matter. The radial pressure, energy density and the coefficients of the metric are well defined. The charge density σ not admits singularities at the centre of the stellar object and the mass function is an increasing function, continuous and finite. The gravitational potentials are regular at the centre and well behaved. The relativistic solution to the Einstein-Maxwell system presented are physically reasonable. The new obtained model may be used to model relativistic charged compact objects in different astrophysical scenes as quark and neutron stars.

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