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SHORT COMMUNICATION

Statistical Blackhole Dynamics

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ABSTRACT

In this paper, the non-generalized or restricted second law blackhole dynamics as given by Bekenstein in the beginning is restated, with a rigid proof, in a different form akin to the statement of the second law of thermodynamics given by Clausius. The various physical possibilities and implications of this statement are discussed therein. This paper is a not a mere venture into the restricted second law of blackhole dynamics pertaining to blackholes emitting Hawking radiation but rather a proper scheme in the development of a proper statistical blackhole dynamics. The paper thus considers an interesting reformulation of the second law of blackhole thermodynamics after some revisions and then several main results including the probabilistic formula for curvature at the horizon.

***Keywords*:** second law of blackhole dynamics; Hawking radiation, entropy, Clausius statement

1. INTRODUCTION

It is observed that dynamics of blackholes runs competitively on the same lines as that of classical thermodynamics, so that the former becomes a subject in itself. This subject is now referred to as *blackhole dynamics*. The theory of blackhole dynamics was arrived at due to the pioneering works of Bekenstein [1, 2], Hawking [3], Carter, Bardeen & others [4]. The so-called *No-Hair Theorem* (NHT) was enunciated by J. A. Wheeler and workable proofs, part wise were provided by Hawking, Carter, Bardeen & Israel [5] which only when integrated would account for the NHT. Just as the four laws of thermodynamics, there are the four laws of blackhole dynamics (for a proper well integrated, coherent & comprehensive reference to these laws see [6] & related references cited therein).

The following paper looks up at a statistical scheme for blackhole dynamics and an alternate statement of the second law of blackhole dynamics and its effect on the Hawking radiation is taken as a precursor.

2. THE CLAUSIUS' SECOND LAW OF BLACKHOLE DYNAMICS

The most general form of the second law is due to Hawking, and explicitly states that in any classical interaction of matter and radiation with blackholes, the total surface area of the boundaries of these holes (as formed by their horizons) can never decrease. In a more particular case of the Hawking radiation, the area of the event horizon of the blackoles emitting this radiation does decrease.

The energy is conserved for the blackholes as well as it is for the other phenomena that occur in the universe. This is embodied in the first law of blackhole dynamics;

$$d(M\,c^2) = \frac{\kappa\, c^2}{8\pi G}\,dA + \Omega\, dJ + \varphi\, dQ \tag{1}$$

where the symbols carry their usual meaning in accord with the conventions of blackhole dynamics. Here, we see that surface gravity corresponds to the temperature and area to the entropy. Within the classical framework, proportionality factors are undetermined. The Hawking radiation suggests that one should associate a temperature and entropy to a blackhole given by

$$T_H = \frac{\kappa\,\hbar}{2\,\pi\, k_B c} \tag{2}$$

and

$$S_{BH} = \frac{k_B}{4 l_{Pl}^2} A \tag{3}$$

where k_B is the Boltzmann, constant. S_{BH} is the so-called Bekenstein-Hawking entropy and T_H is the Hawking temperature. However, there is no hard and fast rule that the blackhole should radiate at the Hawking temperature.

We base this paper on the statement of the second law of thermodynamics as given by Clausius, stated explicitly as: *it is impossible for a self-acting machine, unaided by any external agency to convey heat from a body at a lower temperature to a body at a higher temperature.*

The self-acting machine as well as the unaided part therein can be omitted (the blackhole in itself is self-acting and unaided) since the most energetic phenomena in the universe are involved in blackhole physics and phenomenology, and hence no aid can be conceived and/ or be provided to any concerned blackhole: thus the aid is from the tidal forces near the event horizon of the blackhole in our case of emission by Hawking radiation. We state a similar form of the second law for blackhole dynamics corresponding to the rigid law of blackhole dynamics (of decrease in the area of event horizon) in the frame of the Hawking radiation and then give the proof of it and then restate the law in various different forms.

2. 1. Statement of the second law

No information equivalent to radiation (in particular, Hawking radiation) can be communicated by a blackhole to a blackhole with a relatively higher value of surface gravity.

Proof of the second law:

Let $A_1 \equiv A(\partial B_1)$ be the area of the blackhole whose boundary is ∂B_1 and to this we assign the surface gravity$\kappa_1 \equiv \kappa(\partial B_1)$, similarly $A_2 \equiv A(\partial B_2)$ is the area of the blackhole ∂B_2 with surface gravity$\kappa_2 \equiv \kappa(\partial B_2)$. Let ∂B_1 communicate a radiation by the Hawking mechanism to ∂B_2 subject to the assumed subsidiary condition: $\kappa_1 < \kappa_2$. The energy equivalent of the radiation is $E = mc^2 = \hbar\omega$.

The blackhole boundary ∂B_1 will then experience a decrease in its area A_1, as:

$$A_1' = A_1 - \delta A \tag{4}$$

and correspondingly, ∂B_2 will experience an areal increment, as:

$$A_2' = A_2 + \delta A \tag{5}$$

now, the temperature of a blackhole called its Hawking temperature is given by eq (2) and to a first approximation in eq (1), take $\Omega = \varphi = 0$ and/ or alternately take $J = Q = const.$ and we now have an inverse dependence of horizon area on the surface gravity which is true in general also i.e. without taking $\Omega = \varphi = 0$ or $J = Q = const.$ (the null and constancy assumption have been made to get a simpler mathematical picture and to avoid large expressions involving the J and Q terms in the proof given below), as:

$$dA = \frac{8\pi\, dM}{\kappa} \tag{6}$$

In the case of the blackholes ∂B_1 and ∂B_2,

$$S_2' > S_1' \Rightarrow A_2' > A_1' , \tag{7}$$

that is

$$\frac{dM_2}{\kappa_2} > \frac{dM_1}{\kappa_1} \tag{8}$$

or

$$\kappa_2 < \kappa_1 \tag{9}$$

which is clearly a contradiction to our earlier assumed subsidiary condition: $\kappa_1 < \kappa_2$.

Our newly enacted blackhole scenario is now theoretically true. Thus, if a blackhole emits Hawking radiation, the emitted particle cannot be trapped by the gravity of any other blackhole with a higher value of surface gravity. Else the particle would by some mechanism, without violating the law of conservation of energy, tunnel through the gravitational potential barrier and escape and simultaneously get red-shifted. If the blackhole with the lower value of surface gravity is rotating Kerr or Kerr-Newman blackhole then the particle can take a partner with it and escape by the *Penrose Mechanism* or by the *Blandford-Znacjek Mechanism*; however the area of event horizon of the emitting black hole increases in this case and our version of the second law is no longer applicable in these cases. Thus, every particle that had event horizon as its history is always connected with blackholes of higher value of surface gravity than that of the blackhole with a lower surface gravity that the particle approaches and it is possible that all the blackholes are connected in a higher dimensional physics (See for example [7] and references therein). Another way of stating our law would then be a combination of the original areal law and the surface gravity law as:

1. *In any blackhole involving Hawking radiation, the area of the boundary of a blackhole with a higher value of surface gravity cannot increase at the cost of decrease in the corresponding area of any other blackhole in the universe with a lower value of surface gravity in the sense of the restricted law stated by Bekenstein.*

or

2. *A blackhole cannot trap a particle emitted by another blackhole having a relatively lower value of surface gravity, by the Hawking mechanism. The law being true only for Hawking radiation.*

or

3. *A blackhole with a higher value of surface gravity has a zero capture cross-section for Hawking radiation (only) with respect to the Hawking radiation emitted by a blackhole with a relatively lower value of surface gravity.*

As earlier stated, this paper merely speculates the possibility of a different set of events associated with a particle, after its emission by a blackhole by the Hawking mechanism, as a purely statistical consequence. Now, we look to formulate a statistical description of blackhole dynamics.

2. 2. Statistical Geometrodynamics of blackholes and their horizons

The gravity of any energymomentum distribution is determined by the average Riemann i.e., the Einstein tensor describing the curvature of the spacetime due to the distribution. In the case of a blackhole this energymomentum distribution is the blackhole itself. And the only properties describing it are its mass, charge and angular momentum by the abovementioned No-Hair Theorem. In any case, the curvature is a degree of freedom of gravity best measured for a blackhole at its event horizon. The work of Ashtekar et. al. [8] suggests that this is indeed so. We allow the horizon to take information values of 1's and 0's at every point on its surface in Bit areas A_0. In the work of Ashtekar et. al. as well as the recent work on entropic gravity by E. Verlinde [9], it turns out that the bit area is the Planck area i.e., $A_0 = A_{Pl} = G\hbar$.

The area is a physical observable and is quantized in terms of the Planck area as:

$$\widehat{A}_J \coloneqq 8\pi G\hbar\gamma \sum_k \sqrt{j_k(j_k+1)}. \tag{10}$$

Thus, by Ashtekar's theory, a curvature degree of freedom can be assigned to each Bit.

The absence of a Bit i.e., a Bit 0 will mean a no-curvature at that quantum of area. Also, we can assign a geometrodynamic probability, $\Pi_{\mu\nu}$ with respect to the quantization condition prescribed by eq(10). The curvature at the horizon then becomes a probabilistic quantity and we have a relation like

$$\Pi_{\mu\nu} = exp\left({G_{\mu\nu}}/{\lambda_{Pl}} \right) \tag{11}$$

This then leads to the Boltzmann like relation

$$G_{\mu\nu} = \lambda_{Pl} \, ln\Pi_{\mu\nu} \tag{12}$$

where, the actual Boltzmann formula reads

$$S = k_B \ln W \tag{13}$$

where, k_B is the Boltzmann constant and W is the thermodynamic probability associated with the Bohr-Planck quantization condition, viz

$$E_s = n_s \hbar \omega_s \tag{14}$$

So, curvature is in (statistical) geometrodynamics what entropy is in (statistical) thermodynamics. Actually, the curvature is additive and with the joint geometrodynamic probability goes as

$$\boldsymbol{G} = \boldsymbol{G}(\boldsymbol{\Pi_r \Pi_s}) = \boldsymbol{G}(\boldsymbol{\Pi_r}) + \boldsymbol{G}(\boldsymbol{\Pi_s}) \tag{15}$$

Taking the respective partial derivatives w.r.t. $\boldsymbol{\Pi_{r,s}}$we have

$$\boldsymbol{\Pi_r G'}(\boldsymbol{\Pi_r \Pi_s}) = \boldsymbol{G'}(\boldsymbol{\Pi_s}) \tag{16-a}$$

$$\boldsymbol{\Pi_s G'}(\boldsymbol{\Pi_r \Pi_s}) = \boldsymbol{G'}(\boldsymbol{\Pi_r}) \tag{16-b}$$

Therefore,

$$\boldsymbol{\Pi_r \Pi_s G'}(\boldsymbol{\Pi_r \Pi_s}) = \boldsymbol{\Pi_s G'}(\boldsymbol{\Pi_s}) = \boldsymbol{\Pi_r G'}(\boldsymbol{\Pi_r}) \tag{17}$$

or therefore,

$$\boldsymbol{\Pi G'}(\boldsymbol{\Pi}) = \lambda_{Pl} \tag{18}$$

Separating to differential forms we have:

$\boldsymbol{dG} = \lambda_{Pl} \, {\boldsymbol{d\Pi}}/{\boldsymbol{\Pi}}$ which on integration yields eq (12) viz $\boldsymbol{G} = \lambda_{Pl} \, ln\boldsymbol{\Pi}$where, the boldface letters denote the tensor symbols in eq (12). A more physically and technically involved derivation is treated elsewhere [10]. Actually, $\Pi_{\mu\nu}$ is a matrix of different geometrodynamic probabilities for the various components of the Einstein curvature tensor appearing on the left hand side of eq(12). Thus, in the above enunciated second law of blackhole dynamics, the entropy reduction in the Hawking process, leads geometrodynamically to the curvature reduction in the very same process at the horizon. It is indeed the quanta of geometry that get converted to quanta of Hawking radiation emitted by the blackhole. Thus, as the horizon loses curvature, the blackhole evaporates and the radiation end state by this continuous Hawking process would smoothly erase the blackhole completely out of existence. There need not be any explosive ending to a Hawking evaporating blackhole. Also, there is in the total process a loss of curvature but a net gain in entropy w.r.t. the blackhole and its surroundings thereby rendering no contradiction to the generalized Bekenstein-Hawking second law of thermodynamics so far as the abovementioned second law is concerned. The probabilistic curvature formula given by eq (12) for curvature degree of freedom is valid for all kinds of energymomentum configurations and not just blackholes.

2. 3. Liouville Theorem and its consequences

The Liouville theorem states that the density of phase points contained in any volume in phase space remains constant in the phase space as the volume evolves with the dynamical evolution of the system tracing a trajectory in the configuration space. In the case of gravity, Ashtekar [11], showed that the phase space of the $SL(2,\mathbb{C})$, Yang-Mills gauge theory of gravity is the cotangent bundle over the configuration space of general relativity. Thus, the phase space of pure gravity is imbedded in the phase space of the Yang-Mills gauge theory of gravity. The Liouville theorem, thus, for the blackhole sector of gravitational phase space, asserts that as the blackhole evaporates by the Hawking mechanism, the density of the phase points should remain constant; in other words, the configuration of the blackhole changes but the Bekenstein-Hawking second law still holds and the density preservation implies also that the Hawking radiation lost will not go into another blackhole of relatively higher Hawking temperature viz., surface gravity.

3. CONCLUSION

As more and more forays are being made in the understanding of blackhole thermodynamics, some aspects of blackhole thermodynamics and their interpretations are glossed over. This paper took one such look. However, it is left to marvel what it truly is whose ensemble relates the blackhole Einstein curvature to the blackhole thermodynamic probability.

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Biography

Koustubh Kabe is Dr. Phil (PhD) / Sc.D. in Theoretical Physics. He has published several research papers investigating into the foundational issues of gravitational physics and the understanding of time and quantum gravity. He is also working on the problem of gravity and the cosmological implications in the framework of string theory. He is currently studying Quantum Measurement in addition to all of the above. His research interests are in the fields of General Theoretical Physics, Physical Mathematics, Theoretical Astrophysics, Theoretical High-energy Physics, Modern Theoretical Physics, Physical Cosmology, Geometric Analysis, Number Theory, Algebraic Geometry and lastly, Philosophy, Epistemology and Pedagogy behind Physical Theories. He is an author of a book titled "*Blackhole Dynamic Potentials and Condensed Geometry: New Perspectives on Blackhole Dynamics and Modern Canonical Quantum General Relativity*".

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