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Equalized mass can explain the dark energy or missing mass problem as higher density of matter in stars amplifies their attraction

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ABSTRACT

In order to compare properties of celestial bodies regardless of their motions, their masses, which represent also their energy spent on resistance to forced motions, should be expressed with respect to the same (for all of them) material substance, such as water, for instance. Then their density of matter, which has been defined as the ratio of their actual mass/bulk to an equivalent mass of water, should also be taken into account even in the radial Newtonian law of gravitation. Such an equalized mass adheres to the Newton's definition of "quantity of matter" as conjoined feature (i.e. product) of matter density and the bulk=mass of the body, even though he has never really operated on the notion so defined. The equalized mass conforms to the Einstein's concept of mass as equivalent of energy, both of which vary with speed. Hence the product of equalized mass and density of matter can increase the radial gravitational force of very dense stars, easing the problem of the allegedly missing mass that pushes galaxies too far away according to former estimates of their masses and their distances. Besides the equalized mass, the need for which has emerged from presence of nonradial potentials acting along paths on equipotential surfaces in radial/center-bound force fields, the repulsive effects of the nonradial angular potentials can also amplify the regular/innate spins of stars, black holes and galaxies, eliminating thus the need for the so-called dark matter/energy invented just to explain the allegedly missing mass/energy.

***Keywords*:** Equalized mass; complete total potential energy; dark matter/energy; missing mass

1. INTRODUCTION

In the very first definition of his *Principia*, Newton has defined the concept of "quantity of matter" as being determined jointly by density of the material substance that things are made of and by its bulk, which is nowadays rendered as mass [1]. Yet in primary equations of theories of gravitation developed prior to 2000 AD the effect of density of matter was never included until it has been demonstrated that density of matter of the source mass of locally dominant gravitational field matters in nonradial/equipotential (or tangential and binormal, in the parlance of differential geometry) gravitational interactions, and that various pieces of unbiased experimental evidence clearly suggest that average density of matter of the mass that generates the radial/center-bound gravitational force field affects the nonradial effects happening within the gravitational field [2].

In fact, the conclusion that density of matter was unnecessary when considering gravitational interactions, which is traditionally attributed to Galilei, was (mathematically) clearly wrong an inference when judged on its experimental merits [3]. Galilei would have to compare the impacts of planets and Sun on falling masses, not the incomparably small balls to the mass of Earth at the leaning tower of Pisa. Experiments to that effect have been conducted and their results contradict the Galilei's conclusion (drawn from his experiments) that density of matter does not matter [3].

To be honest, I must admit that without the oversimplified Galilei's conclusion (that density of matter did not affect gravitational interactions) neither Newton's nor Einstein's (nor any of the other alternative) theories of gravitation would have been invented, because the (new) synthetic mathematics necessary to account for all possible field's interactions was just developed in 21st century. But this delay does not justify propagation of almost four centuries old faulty thinking.

In various casual comparisons of electromagnetic and gravitational fields, the electric charge, which is the source of electrostatic fields, was routinely matched to gravitostatic mass, which is traditionally assumed as the source of gravitational fields. Since the electric charge is invariant in motion [10] p. 176f, whereas mass changes in motion [10] p. 177 just as electric field does [10] p. 180f, the charge should correspond to the density of matter of the stars and planets that are the actual sources of locally dominant stationary gravitational fields. Lorentz contraction refers both to distances between objects (such as charges) in moving reference frame and also to compound objects, hence to both mass and matter density [10] pp. 195, 454 at a deeper level of inquiry. But at the present, neoclassical level of inquiry, the density of matter is considered as being constant.

It has already been shown that nonradial gravitational interactions depend inversely on the density of matter [5], which fact was confirmed directly by results of some experiments [2,3]. Nevertheless, even the purely radial interactions must depend on the density of matter too, once an equalized mass is deployed [11]. To pretend that the (varying with speed) mass could reveal true measure of the substance out of which all stars and planets are made of, irrespective of their relative speeds and accelerations, is the oxymoron of the traditional astronomy and astrophysical sciences. It is an affront to God and insult to the intelligence of (alert) students to talk about the allegedly missing mass, while tacitly equating the notion of 'mass' with 'matter' (or a practically invariant amount of material substance with the mass-energy that varies in motion, to say it more precisely at the current level of inquiry) [12,13], and even devising clearly fake theorems [4].

Since the rate of work done within given radial field (and thus also the rate of change of the potential energy that is spent on the work done by the local radial/center-bound force field) was incompletely defined in the former physics [4], many former inferences derived from the faulty (and incomplete) rendition of the rates are incorrect. Yet operationally complete potential energy suggests that density of matter of the mass source of the field affects the exchange of potential energy for work done within the field [5], for the equipotential components of interactions taking place along equipotential surfaces introduced some angular nonradial potentials corresponding to the repulsive gravitational forces that act perpendicularly to the radial attractive force [6,7]. In addition to being experimentally confirmed, the new gravitational formulas can be theoretically derived from purely abstract mathematical considerations based only on operational rules of the differential calculus [8] and are quite consistent with the principles underlying the Frenet-Serret equations of differential geometry [9]. The theoretically derived and experimentally confirmed dependence of field interactions on density of matter prompts reevaluation of both physical and mathematical theories of gravitational fields conceived prior to 2000 AD, when presence of the nonradial effects of radial fields has been shown based on analyses of certain experiments [2].

2. POTENTIAL ENERGY IN RADIAL/CENTER-BOUND FORCE FIELDS

The operationally correct (and mathematically legitimate) product differentiation rule (PDR) requires that the rate dW of the work done W should be defined in terms relevant to radial force fields of a vector of the gravitational force **F** and the distance pointing vector **r**. It can be written:

$$dW(\mathbf{F} \bullet \mathbf{r}) \coloneqq d(\mathbf{F} \bullet \mathbf{r}) \coloneqq \mathbf{F} \bullet d\mathbf{r} \oplus \mathbf{r} \bullet d\mathbf{F} \qquad (1)$$

because the PDR is proven true and thus is mandatory to be always applied for all products. The symbol $\oplus$ means that formula (1) is not an equation but an operational template to be filled out with certain actual differentials once these are calculated [9] and substituted ($\coloneqq$) therein. And structurally, the formula (1) is just a prototype for building up abstract quasigeometric structures.

The traditional definition of work done rate, understood as the scalar product of force vector over the linear distance $r=|\mathbf{r}|$ to the given trajectory (or orbit) pointed to by the distance pointing vector **r**, was wrongly designated as the product of force over the distance change rate d**r** instead

$$dW \coloneqq \mathbf{F} \bullet d\mathbf{r} \qquad (2)$$

making it incomplete, and so was also the potential energy that is spent on the work done by the given radial/center-bound force field. The operationally wrong and conceptually faulty formula (2) has omitted thus the nonradial term $\mathbf{r} \bullet d\mathbf{F}$ which is unavoidable even for purely radial force fields, in general [9]. The traditional radial-only formula (2) is acceptable for just a single force vector, but it just cannot be used for central i.e. radial/center-bound fields of forces, which are hedgehog/porcupine-like agglomerations of forces. In terms of the field's interactions happening within a trihedron moving along trajectory pointed to by the radius vector **r** we should have four quite distinct (and different, in general) subcomponents of every interaction because each vector has two attributes: magnitude and angle. Hence, by combining them we expect 2×2 = 4 terms.

According to the proven PDR, operationally correct work done rate shall be evaluated [5] as:

$$dW = -F\cos 2\alpha \, dr + 2Fr\sin 2\alpha \, d\alpha - r\cos 2\alpha \, dF = -dW(r) + dW(\alpha) - dW(F) \quad (3)$$

where α is the angle of visibility of the given trajectory/orbit of a satellite (or that of an electron in an atom in the Bohr's classical picture) as seen from the local gravity center (or nucleus) when sweeping its perihelion radius. The formula (3) holds true for all kinds of radial/center-bound fields. It contains the usual radial work done rate dW(r) as well as two previously disregarded components of the total work done rate, namely the (doubled) angular nonradial rate dW(α) and a linear nonradial rate dW(F) of the total work done. Since the rates of work done pertain also to rates of the field's potential energy that is spent on the work being done by the radial force field, the notion of work done and that of the field's potential energy can be used interchangeably.

When it comes to counting terms/components, the faulty formula (2) actually amounts to an erroneous statement, namely that 2×2 = 1, because each vector comprises both a linear value (i.e. length) and an angle. The operationally correct formula (3) amounts to the correct statement that 2×2 = 4, for it contains 4 components, since the angle rate term $d\alpha$ is taken twice. André Mercier pointedly noted that it is not easy to decide what is potential [14,15], but he did not resolve the underlying issue that generated conceptual nonsenses or inconsistencies in various reasonings pertaining to mathematics, physics and even logic. Ignoring proven operational or structural laws of mathematics has thus adverse consequences, even though it was presumably quite unintended. Even otherwise intelligent mathematicians can make trivial mistakes and pretend not to see them.

3. THE RADIAL AND ANGULAR NONRADIAL IMPACT OF EQUALIZED MASS

Since density of matter is defined as the ratio of mass of the given body to the equivalent to it mass of water $Q = M/M_w$ I have designated for the time being the equalized mass $\mathcal{M} = M_w$ as equal to that of water under the same normal physical conditions. It is not about seeing stars or planets as if they were made of water, but to use common reference point. This way masses of all planets within our solar system could be quite meaningfully compared [3].

I am not doing it for a mere convenience. The traditional comparisons of masses of planets, each of which orbits our sun at different speeds, resulted in rather dubious inferences, some perpetuated for centuries. Only if all celestial bodies are considered as if their masses comprise the same material substance, we could avoid unnecessary bias. While material composition of planets is practically comparable, stars in different stages of their lives are far more difficult to compare fairly. Separating their mass and substance/matter is helpful for comparisons of stars. That is why the equalized mass is important.

Thus only an equalized mass could become the abstract mass common to all planets and other gravitating bodies in the universe, regardless of their (always relative) motions. The former physics was obsessed with proving that inertial and gravitational mass are exactly the same. That noble attempt was an exercise in futility. Only when the traditional notion of the inertial (i.e. motion-affected) mass is separated from the notion of material substance (that is usually determined in practice by average matter density, at least in macrophysics), we can split the concept of mass into two quite distinct magnitudes: an equalized mass and the density of matter. Both of these measures Newton combined into his so-called 'quantity of matter', which can influence gravitational interactions, but each impacts the gravitation differently. Now mass and matter have precise but distinctively different meanings, even though they jointly – as Newton had stipulated – should characterize celestial bodies. The equalized mass delivers the radial pull whereas the density of matter amplifies the pull (in denser than water stars) and also supplies the angular twist. Now we can handle both the notions mathematically, instead of merely talking about the imprecisely defined, essentially philosophical notions of inertial and gravitational mass. In order to see the distinction, just imagine two rivers: one of molten lava and the other with water flowing at the same speed. Yet due to different densities of their matter the two rivers will behave slightly differently, provided their matter densities remain practically almost unchanged for most purposes.

Using equalized mass $\mathcal{M}$ we combine the radial term W(r) (i.e. gravitational function) with the angular nonradial functional $W(\vartheta)$ by superposing their formally independent effects [11]:

$$W(r,\theta) := W(r) + W(\theta) = k\frac{G\mathcal{M}m}{r}(Q - K\theta) \tag{4}$$

which combines the two (radial and equipotential) effects into a single functional equation. Note that ϑ is a twisted 3D solid angle, not a planar one, even though in most practical applications it could be approximated by a ribbon or even by a planar angle (i.e. projection of the twisted solid angle onto a plane, such as the osculating plane tangent to the trajectory of the orbiting mass m). K is a proportionality coefficient, which shall handle also the chosen approximation level and proper units. Notice that the trajectory of the orbiting (or just passing by) small mass m will not only be twisted but also slightly deflected by the big source mass $\mathcal{M}$ that generates the locally dominant force field. The presence of the orbiting mass m and the coefficient k, which actually neutralizes its expression in terms of units and representation in order to maintain the whole eq. (4) as proper expression of potential energy. Notice that without m (and k) the $W(r,\vartheta)$ refers to potential energy of the field whereas with m and k it refers to potential energy of the interaction between the masses $\mathcal{M}$ and m. Other authors tried to do that in other ways – see [16] p. 225. My main reason for presenting it this way is to underscore the role of masses in forming the forces and the two interacting potentials $\mathcal{M}$/r and m/r. The mass m can be equalized too, of course. It can be viewed as a certain unit mass m in the eq. (4) in conjunction with the coefficient k, but for the sake of simplicity of this presentation I shall leave it as it was treated before.

For spherically symmetric spatial distributions of masses, the radius r and the twisted solid angle ϑ are practically independent at each point of the trajectory of the relatively small, subordinate mass m. I will henceforth assume that the density of matter Q is also uniformly distributed within each of these two bodies, for the sake of simplicity. At a deeper than the present, i.e. neoclassical level of inquiry, the structure of the matter composition shall also be taken into account, not only nonuniform distributions of mass and matter. And for fast rotating celestial bodies the twisted angle could be split into purely tangential/equatorial component and a (perpendicular to it) binormal component that acts in the polar plane.

4. RADIAL AND NONRADIAL EFFECTS OF EQUALIZED MASS

Traditional mathematical treatment of fields decomposed them into a scalar and a vector part (per Helmholtz's theorem [17]). Unfortunately, however, traditional mathematics has invented a lot of conceptually devastating ideas. The faulty formula (2), which may not work properly even for a sled, is just a tip of the proverbial iceberg of mathematical oversimplifications. When one goes radially in the direction of motion along a trajectory/orbit of an orbiting planet, for instance, the most influential factor is the (whether equalized or not) mass of the Sun as the mass source of the locally dominant gravitational field of the solar system. But once we turn around the Sun, the most influential factor is the density of matter of the Sun, whereas the Sun's mass becomes just a supporting factor for the interactions happening along the equipotential surface in the nonradial i.e. tangential and binormal directions. This (formerly unrecognized) mathematical fact has been confirmed in the experiments conducted by Sadeh and his teams [18,19], and first recognized in [2]. Hence without defying the Helmholtz's theorem I am just acknowledging superposition of scalar subcomponents rather than building up a vector potential in (spheroidal) three dimensions, which could also be done, but it would require an additional spatial structure. If so then why not? It is because the 4D spatial structure is not just a (3+1)D or a (1+3)D space (i.e. 4D spacetime or 4D timespace, respectively), but both of these spaces overlaid upon each other [20,21]. Hence staying within the conceptual confinement of the former single-space reality paradigm, which still "reigns" supreme in both mathematics and physics, I can superpose scalar components of the total scalar potential without doing injustice to the future (i.e. shifted) paradigm of a multispatial reality, which is both mathematically as well as physically far more complicated, and therefore shall be explained operationally elsewhere. We don't need that complexity for our purpose here.

Assuming symmetric shapes of all celestial bodies we can skip the W(F). Since the W(r) and W(ϑ) represent the radial and nonradial/equipotential components of combined scalar potential, respectively, we calculate the values of forces corresponding to these potentials:

$$\mathrm{F(r)} = \nabla \mathrm{W(r)} = \frac{\partial}{\partial \mathrm{r}} \frac{\mathrm{kG}\mathcal{M}\mathrm{Qm}}{\mathrm{r}} = -\frac{\mathrm{kG}\mathcal{M}\mathrm{Qm}}{\mathrm{r}^2} \tag{5}$$

$$\mathrm{F}(\vartheta) = \nabla \mathrm{W}(\vartheta) = -\frac{\partial}{\partial \vartheta} \frac{\mathrm{kG}\mathcal{M}\mathrm{mK}\vartheta}{\mathrm{r}} = -\frac{\partial}{\partial \vartheta} \frac{\mathrm{kG}\mathcal{M}\mathrm{mKr}\vartheta}{\mathrm{r}^2} = \frac{\partial}{\partial \lambda} \frac{\mathrm{kG}\mathcal{M}\mathrm{m}\lambda}{\mathrm{r}^2} = \frac{\mathrm{kG}\mathcal{M}\mathrm{m}}{\mathrm{r}^2} \quad \textit{with}\ \lambda = -\mathrm{Kr}\vartheta \tag{6}$$

with both forces acting along certain lines. The usual radial force F(r) acts along the radius r. The twisting angular force F(ϑ) acts against the twisting spheroidal angle ϑ along the equipotential surface, because of the opposite sign of the nonradial angular potential W(ϑ) to that of the radial potential W(r). Since the equipotential surface is placed at a constant radial distance r from its center, the distance r is not acting variable in the angular potential W(ϑ). Here the equipotential distance λ corresponds to the spheroidal twist caused by the angular potential in fairly practical approximation purposely simplified at the present level of inquiry.

Since all distant stars are for practical purposes fixed pointlike objects and their galaxies are agglomerations of such quite indistinguishable stars, the actual shape of the spheroidal path λ lying on few indistinguishable equipotential surfaces is immaterial. It was so approximated even in the vicinity of our Sun [2].

The formulas (4)-(6) have labels for the equalized mass $\mathcal{M}$ and the regular mass m, one of which can be a ratio/number. For the formulas actually pertain to interactions (which always involve two masses and thus – by extension – their fields and their two potentials) of the fields created by these masses. Even the Newton's inverse square law is a misnomer, for it refers to a force generated by two interacting distinct potentials: $F = |\boldsymbol{F}| = -\frac{GMm}{r^2} = G\frac{M}{r}\frac{m}{r}$. If instead of the mass m there would stand a massless "spiritual" body, no gravitational force would be acting on it and the spiritual body could go through material walls or even ascend to heaven without any spacecraft or fiery chariot just as Jesus Christ did after His resurrection. This does not mean that spiritual mass is null but that each generalized mass may have both: a material component that can be equal to zero and also a certain (noninteracting gravitationally) spiritual component.

Even Bible-believing Christians tend to misinterpret the operational and structural meanings of mathematical formulas to fit their unfaithful "scientific" minds' perceptions of what they tend to consider as reality. Potentials have mathematical existence whereas forces do emerge when at least two material potentials interact within the given field, for otherwise there would not be any interaction. Presumably, there is no separate material and a spiritual reality. Hence the abstract fundamental mathematical laws (underpinning the laws of physics) should be the same for both these realities. It is thus a remarkable fact that mathematicians and physicists made such infantile mistakes and for over two centuries (since 1773 AD, when it became feasible [10]) maintained the incomplete formula (2) as if having their – otherwise bright – minds enslaved, for as Apostle Paul explained, we wrestle not with humans but with rulers of the darkness who oppose God of heaven. Thus, every idea that might have vindicated God's claims became suppressed along with its prospective consequences, some of which may not even pertain to the spiritual realm at all. I am not trying to "preach" mathematics, but to point out that even the stupid, infantile and clearly illogical attainments (not to say "perverted achievements") can be explained by Satan's messing with conceited human minds. This does not mean that conceit makes one mercenary of Satan, but whoever stands against truth – whether in mathematics or in the other sciences – such ones relinquish their minds to become perverted by Satan, the adversary of truth and thus of God too.

Since forces – as vectors – act along lines/curves, not angles, the eq. (6) deploys the curved varying spheroidal distance λ instead of the spheroidal angle ϑ swept along the equipotential sphere (or maybe a part of the sphere) that is tangent to a varying yet still equipotential surface. Thus, the angular potential leads to clearly repulsive force twisting away the given trajectory [6]. With the traditional notion of mass, the nonradial angular potential depends explicitly on density of matter [5]; with the equalized mass $\mathcal{M}$ it still depends on the density of matter but now only implicitly so. Yet the radial Newtonian gravitational force as well as the radial potential energy depend explicitly on the average (assumed as uniform and constant) matter density of the mass source when combined with the equalized mass, which interchanges the dependence of the radial and nonradial terms. Note that the linear nonradial rate dW(F) of the total work done can become important in the cases when the given massive bodies possess significantly irregular shapes.

The eq. (5) suggests that in heavier stars with average density of their matter higher than that of water, their radial gravitational pull is increased by their density of matter, whereas the radial pull of lighter stars diminishes. On the other hand, the eq. (6) suggests that the angular nonradial twist disperses the masses, preventing thus gravitational collapse due to attraction in regular stars until their matter density reaches the point of no return, beyond which they became black holes. It also resolves the Newton's gravitational paradox, namely that in a sufficiently large cosmos, the average relative velocities can become infinite and thus lead to collapse of the universe [22], for which he was criticized. The critiques subsided due to successes of his theory, but the puzzle still needed an answer. The nonradial angular term also drives the creation of stars and planets.

5. SOME PROBLEMS WITH ESTIMATION OF MASSES OF DISTANT OBJECTS

Velocities of the galaxies in the Coma cluster (~300 million light-years away) were far too high when Zwicky studied them in 1930s. At the measured velocities, the galaxies in the cluster should fly apart and the cluster should evaporate [23]. Zwicky proposed the so-called dark matter as a solution to the Oort's discovery that there is apparently not enough mass to hold the matter in galaxies together. While admitting that some light from distant stars may be obscured, Oort's review [24] relied mainly on visual magnitudes, which were undeniably pretty unreliable. Since estimates of masses of nebulas based on luminosities were undependable, few new methods were proposed [25]. Zwicky realized that in very distant (over 1 million light years) nebulas/galaxies no individual Cepheids can be resolved and thus other methods must be used to estimate their distances [26]. One of then-new methods relied on the radial redshifts measured in their spectra [26]. The relations between velocities (assumed as circular) and the mass distributions in nebulas were studied extensively; one of such studies has found similarity between the velocity curve for M31 and that adopted for Milky Way [27]. These efforts were often hailed as sure indicators that presence of some dark matter could solve the problem of the allegedly missing mass, were it not for the fact that the usual radial attraction is not the only mathematically important (even though ignored by astronomers) factor to also be taken into account, even if their mathematics were not incomplete. Unfortunately, even now cosmic distances are known to no better that 10-15%, as there are many discrepancies of 0.2-0.3 mag between various accepted distance indicators [28].

Nevertheless, none of the astronomers (nor their aides) did take into account the possibility that – in addition to the radial gravitational redshift, which Einstein has proposed – there should be also a nonradial (i.e. tangential or binormal or both) frequency shift because light passing by a nearby star is also exposed to influence of the gravitational potential of the nearby star. Just as the rays' trajectory is deflected, their frequency is also decreased i.e. redshifted [2]. The actual presence of the nonradial redshifts (i.e. frequency decrease) has been discovered experimentally by Sadeh [18,19], whose results have been explained and reconciled in [2] and then derived also mathematically in [8]. Zwicky dismissed some alternative interpretations of the redshift, but he acted before the aforementioned Sadeh experiments have been conducted [29]. Some data do not support the claim that the redshift law is linear and isotropic in our neighborhood [30].

The two Sadeh's experiments have definitely confirmed presence of extra redshifts acquired along equipotential surfaces, which at present cannot be distinguished from the other frequency shifts (Doppler, cosmological, the [radial] gravitational [31], the local relativistic [32], and few electromagnetic [33]). Hence in cases where the line of sight of a star grazes the surface of yet another star, the observed redshift could place its star farther behind its actual position. This fact agrees with many previous *ad hoc* suspicions. For some quasars seem to be farther away than the galaxies in which they appear [34] and thus their apparent redshifts may be uncorrelated with their distances [35], for not all redshifts are due to expansion [36]. Note that although the two aforementioned Sadeh's experiments have already been published in 1968 AD, their actual and conceptual explanation (and reconciliation of the observed data) appeared in 2000 AD [2].

Presence of extra nonradial redshifts in spectra of distant stars can make the distances that are based only on their radial redshifts overestimated. Probably most cosmic distances estimated that way can suffer from that. For key predictions of the Hubble law are inconsistent with direct observations on equitable complete samples of extragalactic sources in the optical, infrared, and x-ray wave bands [37]. Both the radial and the nonradial redshifts would have to be taken into account in order to estimate distances reliably, but it is impossible – at present – to discern which spectra are distorted by stars placed nearby the line of sight of the spectra from distant stars. Yet presence of the nonradial angular twisting potential gives us the hope that sometime in the future it may become possible to distinguish the twist based on their acquired polarization, maybe with the exception of magnetic influences. I have no idea how to actually accomplish that, however.

Potter & Preston have introduced effective gravitational potential containing both the usual radial Newtonian attractive potential and a repulsive potential defined in a quantum-mechanical fashion for cosmology. Both potentials were presented as scalars in terms of magnitudes [38]. The authors have claimed that the presence of the repulsive potential in their quantum celestial mechanics (QCM) wave equation for the universe removes the need for invoking dark matter and dark energy and even accelerating universe among other features [39]. Their acknowledgement of possible presence of the repulsive potential is a departure from allegedly apodictic reliance on the radial Newtonian potential, but the repulsive potential has been derived from mathematically incomplete and approximate (Schwarzschild's metric) approach to radial-only general theory of relativity (GTR). For Einstein has quite deliberately omitted any possible tangential contributions in his GTR, which turned it into radial-only scheme of the GTR gravitational interactions [40].

Einstein in his GTR identified gravitation with the metric tensor of a Riemannian space, but this resulted in the gravitational field no longer being a physical field and thus the fundamental conservation laws were lost [41]. GTR is a field theory of gravitation, but it is not a field theory like the Maxell's theory of electromagnetic field; according to GTR gravitation is not localizable field [42]. Einstein abandoned the classical concept of a field for the sake of the local principle of equivalence of inertial and gravitational forces, raising this principle to the rank of a fundamental principle, despite lack of any physical grounds for doing that. It was this that led to the idea that gravitational energy cannot be localized in [length-based] space [43]. The equivalence of mass to energy suggests that the gravitational force between two objects ought to be proportional to the product of their energies and not to the product of their masses [44]. Moreover, the bending of spacetime into configurations of nontrivial topology requires the input of additional energy [45].

Although Elie Cartan did not insists on the notions of curvature and torsion (in reference to the GTR), he pointed out that the classical Riemannian space is without torsion and the space of absolute parallelism is without curvature [46]. Yet Einstein demanded teleparallel displacements [47]. General relativity is incomplete since it is necessary to supply additional assumptions about the relation between matter and spacetime, which are not contained in the theory [48]. One of the [most lamented] weaknesses of Einstein's theory of gravity is that although it furnishes the field equations, it does not provide boundary conditions for them and thus its model of the universe is not uniquely determined [49]. Despite its shortcomings, the GTR is an excellent, although radial-only, theory of gravitation. That is why it needs to be complemented by a nonradial theory [2]. It was Riemann who pointed out that if one takes radial variable on input and then displays it in a (X,Y,Z) frame, it is incomplete for it only decomposes the radial component into X,Y,Z [50].

Nevertheless, the QCM ignores the mathematical/operational incompleteness of the former definition of potential energy and the (mathematically derived [8], theoretically necessary [9], and experimentally confirmed [2,18,19]) fact that repulsive potential function in a realistic radial/center-bound force field is tied to matter density of the mass source [5]. Without nonradial impact that is due to exposure to the mass source of the locally dominant gravitational field, the idea of repulsive potential may be deficient conceptually [3,5], and operationally [8], because the exposure is codetermined by density of matter, not just by the mass alone. But existence of repulsive potentials seems undeniable and thus the QCM can be a helpful calculational device for cosmology. But split of the former mass into equalized mass and density of matter is advisable.

Yet the conclusion derived from the QCM that expansion of the universe is not accelerating would contradict the mathematical fact that presence of other external influences could add to the curvature and torsion of trajectories of neighboring celestial bodies. For presence of other mass sources whose nonradial (i.e. tangential and binormal) contributions to the global potential at any given point in space distorts the undisturbed/idealized trajectory of every single celestial body.

In other words, since any motion perturbed by the presence of all other bodies has screwlike character [9], it indicates the need for fluctuating yet persistent accelerations, and the impact of even constant density of matter makes the acceleration even more plausible. I presume that the QCM may not eliminate entirely the accelerating expansion of the universe. Nevertheless, it is true (and confirmed by the Sadeh's experiments [18,19], explained in [2]) that the expansion could be slower than previously estimated. This is because the extra nonradial (as opposed to the purely radial redshift predicted by Einstein's GTR) gravitational frequency shifts were not really recognized but have been routinely interpreted as due to increasing distance to stars, rather than as caused by proximity of other masses to the lines of sight of the stars whose spectra were found redshifted. From approximately valid premises no absolutely correct inferences could be drawn.

The postulated presence of nebulous or dark matter as a way to explain the allegedly missing mass that keeps the stars – orbiting far away from their galactic centers – on radial gravitational leash tethered to their mother-galaxies [24], has also a big theoretical problem. The problem was the (rather arrogant) presumption that astronomers and physicists have already discovered all the aspects of gravitation, despite the tacitly veiled mathematical fact that the mathematics applied to physics has huge conceptual and even operational inconsistencies, which could be traced back to Galilei [3], accepted by both Newton [111] and

Einstein [40]. Yet this fact was routinely ignored before presence of other than radial effects of gravity has been recognized [2]. Although there certainly is a lot of invisible matter, none of it could ever solve the aforesaid problems stemming from the faulty former radial-only formulas. Since the density of matter was once declared by Galilei as quite immaterial to gravitational interactions, adding more mass could only worsen the baffling problem. Only mass was entered into the primary equations of former theories of gravity even though the density of matter (or the space containing it) is also necessary to explain at least some clearly nonradial effects, whose actual existence was experimentally confirmed [2,3].

6. CAN DARK MATTER/ENERGY RECTIFY INCOMPLETE EQUATIONS?

Dark energy is regarded as the agent responsible for the observed acceleration of the Hubble expansion in the context of the GTR cosmological models [51]. The dark matter is something of a scandal in cosmology [52] since it can apparently exceed the presently visible matter even by a factor of 100, according to some estimates [53,54]. There are two levels of difficulties with the dark matter: missing matter vs. dark matter and the mass-to-light ratio. There is no correlation between the mass and light components [55]. The ratio of dark-to-luminous matter appears to be independent of galaxy mass and of luminosity [56]. Actually, there is a discrepancy between the dynamical and luminous mass of galaxies; the mass of a galaxy deduced via the virial theorem is larger than the mass deduced via mass-luminosity relation from all its stars; and there is also a discrepancy between the observed mean mass density of the whole universe (or, at least, of the presently visible part of it) and the critical one predicted by inflation – the latter part being larger [57]. However, a modified gravitational potential (i.e. Newtonian plus a Yukawa-type potential) $V = GM (1 + \sum\alpha_i \exp[-r/l_i])/r$ can explain the second, but not the first discrepancy, if the typical length scales are of the very same order of magnitude as the corresponding galaxies; fourth and higher order gravity (whose potential has many Yukawa-type terms) were also discussed [57].

Nevertheless, there is yet no firm evidence for the presence of any large (i.e. operationally significant) amounts of an invisible matter in the solar vicinity and in dwarf spheroidal galaxies [58] and no convincing or even unconvincing evidence for the possibility of presence of dark matter [59]. Maybe the dominant matter in the universe is nonbaryonic and thus could be either primordial black holes, massive neutrinos of some flavor, or perhaps even an exotic remnant of supersymmetry theories, such as gravitons [60]. Some scientists think it rather unlikely that baryons, neutrinos or black holes will be found to form the bulk of dark matter [61]. Perhaps most persuasive and fairly convincing evidence for the [presence of] dark matter in galaxies is provided by the rotation curves of spirals [16,62]. It is the purported need for the dark matter rather than it has allegedly been discovered; no dark matter/energy itself has ever been found.

Some observations suggest that the universe is dominated by dark energy component with negative pressure and so it is undergoing cosmic acceleration [63,64]. It is also proposed that maybe our universe could be closed by some exotic particles, which interact only gravitationally [65,66]. A nearly massless, slowly rolling scalar field may provide most of the energy density of the current universe [67]. An analysis of 27 clusters of galaxies indicates that about 90% of the mass is hidden in neutrinos, the member galaxies account for 5-7% and 3-5% comprises hot gas of very high entropy [68]. Primordial neutrinos could then make a major contribution to the mean density of matter in the universe, which is enough for the universe to be closed [69].

Dark matter may be real, for the shape and orientation of the hot gas cloud (around galaxy NGC720) require it to be confined by an egg-shaped dark matter halo, as the cloud is slightly flattened or ellipsoidal and has orientation slightly different from that of an optical image of the galaxy [70]. The point I am trying to make is that presence of the dark matter – no matter how abundant and conveniently placed it may be – could not rectify incomplete equations, for which the idea was invented. We need an intrinsic gravitational term, such as twisting nonradial angular potential, in order to explain this (and several other) phenomena. An extrinsic radial factor would only exacerbate the problem by demanding more and more of the dark matter-energy because in the radial-only theories (such as the GTR) energy (and thus its mass equivalent too) just cannot be localized. Proven mathematical laws cannot be just ignored while complaining about missing mass/energy. And the double meaning of mass as a dynamical stream of matter and the (static by definition) material substance of bodies only adds to the confusion. Even though the density of matter can change at a deeper than the present level of inquiry, it must be consistently treated at each of these levels. Otherwise, something will always be missing, hopefully not the mind itself.

Although the standard cosmological paradigm is a universe in which ordinary matter is just a minor constituent, with ~90% of the mass being in some nonbaryonic form, fairly recent data are consistent with a purely baryonic universe [71]. This might likely require presence of some kind of even higher clusters. Yet previous surveys of the universe claimed that there was no indication of existence of superclusters of galaxies [72] even though they seemingly do exist [30].

Gravitons and gravitinos and superweakly interacting massive particles (SWIMPs) in general have large masses but almost zero propensity to interact with regular matter [73]. This feature also demands the split of mass into equalized mass and density of matter. Since the dark matter is supposed only to fix the allegedly missing mass/energy problem in our former, operationally incomplete radial-only theories, it could otherwise stay quite inactive as it nicely does. Also, the heating of intracluster medium by dark matter collisions is not observed and absence of cool gas in centers of galaxy clusters revealed by Chandra and XXM observations goes contrary to some predictions suggested by some expected interactions of dark matter [74]. Massive neutrino could also solve the problem [75]. However, the COBE measurement strongly excludes the standard neutrino-dominated Hot Dark Matter model [76]. Both these assumptions, namely that the dark mass consists of neutrinos, and of some hypothetical noninteracting particles lead to problems; hence stars may contain more metals than their previous, accepted astrophysical estimates [77].

This conclusion can imply that the role that average density of matter plays in gravitational interactions needs thorough reevaluation. Dark matter produces numerous other problems [78]. Nevertheless, the postulated dark matter creates several difficulties because some conclusions about the components of dark matter are drawn not from experiments but from the assumption that without such inferences the hypothetical dark matter would not be plausible [79]. The notion of dark matter is just disguised existential postulate appearing to cover up faulty mathematics.

On the other hand, cosmic speed-up can be accommodated even within the GTR by a cosmic fluid with large, negative pressure, dubbed dark energy, for the cosmic acceleration can arise due to very tiny corrections to the usual gravitational, Einstein-Hilbert action of

GTR of the form R^n, with n<0, which eliminates the need for dark energy [80]. Since a modification to the action R^n with n<0 can lead to early-time inflation, the proposal provides a unified and purely gravitational origin for the early and late time accelerating phases of the universe, because the added term can strengthen as the universe flattens [80]. Thus, an intrinsic factor is preferable to an extrinsic one.

While redshift was interpreted as an expansion of the universe, in mass-boom nonexpanding model the redshift can be interpreted as shrinking quantum world [81]. However, regardless of its interpretation redshift cannot be used as a measure of distance because redshift could also be acquired when the rays pass by the belly of a star [2,3], which is experimentally confirmed fact [19]. Hubble himself did not claim any exactness but merely roughly linear correlation between distances and velocities, whether the latter are used directly or corrected for solar motion [82].

Since the proper-times of vibration of atoms situated at 0 and the equivalent point-source are the same (neglecting the Einstein effect due to mass) $\frac{\Delta\lambda}{\lambda} = \frac{dt'}{dt}$ this leads to a shift of the spectral line of wave length: $\frac{\Delta\lambda}{\lambda} = \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}} - 1$ [83]. This nonrelativistic redshift would be attributted in practice to the Doppler effect due to velocity of recession [83]. The cosmological redshift that is related to the scale of the universe at the time of emission, can be used as an indicator of both the distance and velocity of the source at any specified time. Parameter-free formulae can be derived for the distances and velocities at the time of the emission and the present epoch. Sources that recede faster than the speed of light can be observed in the former, but not in the latter [84]. Both assertions are valid, but the problem is that we always observe total cumulative redshift (minus a possible blueshift, if any) and there is no reliable way to distinguish all the contributing shifts.

Since the universe is not exactly uniform or isotropic, if Mach's principle holds, we might then expect that the slight asymmetries in the distribution of matter at large would result in slight deviations from at least some of the laws of mechanics which are commonly assumed to be quite exact [85] p.646. To good accuracy, the [radial] acceleration produced by a [radial] force points in the same direction as the force, and thus the inertial mass (as the ratio of force to acceleration) is independent of the direction of the force (i.e. inertia is isotropic) [85]. Nonetheless, if Mach's principle holds, the statement can be true only in the first approximation and asymmetries in the matter distribution at large, let's say concentration of matter near the center of our galaxy, could produce asymmetries in inertia [85] see p.647. Note that the notions of mass and matter are often used interchangeably, and thus some arguments are true only if the concept of 'mass' can also be understood as 'matter'. The ambiguity is indefensible. It only reinforces Newton's intuition and his insistence on defining 'quantity of matter' as mass/bulk conjoined with matter/substance [1].

If the current cosmic microwave background (CMB) anisotropy strongly constrains the mean spatial curvature of the universe to be near zero, or equivalently, the total energy density thereof to be near critical as predicted by inflation, this and few other lines of the argument indicate that the energy density of nonrelativistic matter is much less than critical; taken together, these results are considered as evidence, independent of supernovae data, for presence of dark energy in the universe [86]. The argument is admissible only if one may freely interchange mass and matter.

Protons and neutrons form either tiny clumps of matter (the various atomic nuclei) or very large clumps of matter (neutron stars), but nothing between these two. Yet the standard model seems to be consistent with the existence of new forms of nuclear matter that might populate the desert and thus could solve the mystery of missing mass [87]. Also in the so-called "brane world scenario" matter fields (such as quarks, leptons, gauge bosons, Higgs bosons) are supposed to be confined to live on a very abstract hypersurface (i.e. the brane) in a certain higher-dimensional space, where gravity field and possibly also some other fields can propagate also in directions transverse to the brane [88]. If the heterotic string theory is indeed correct, then the assumption that dark matter of the universe consists of axions agrees with experiments [89]. However, it has been definitely concluded that statistical analysis of magnitude and center-of-mass velocity data for binary galaxies will never provide adequate estimate of the mass within the orbit [90]. Again, a measure of material substance, such as density of matter, is needed to settle the whole issue.

It may be necessary to rethink the GTR with respect to the origin and possible variability of the concept of mass in both structural/geometric and operational/octonionic terms of [91-93], which permits to view the dark matter field as an electromagnetic field adjoint to gravity field considered in a complex octonionic space [94]; see also quaternionic representations [95].

Several alternative solutions to the dark matter/energy problem have already been proposed [96,97], but they did not recognize the mathematical incompleteness of previously accepted physical laws and thus assume that past theories of gravity are correct, which is not true. Also, a compression wave will produce a continual increase in the mass of the moving region during the time interval in which the region is traversed by the wave; the effect of this wave is therefore to augment the mass created in the region by the progressing universal expansion [98]. In classical Newtonian theory of gravitation, the universality of free fall (UFF) is an unexplained fact; yet if the UFF is violated then masses of particles would depend on the absolute value of the [radial] gravitational potential [99]. Hence most tidal dwarf galaxies may be less massive than generally advertised or substantial disklike dark matter components would have to be considered [100]. But no experimental evidence exists that the inverse square law of gravitation is valid at cosmic distances significantly larger than the diameter of the solar system; this fact raises the very real possibility that the "missing mass" (dark matter) may not exist [101]. In the scalar-vector-tensor theories [of gravitation] it is quite natural to attribute a significant fraction of the galactic halo "missing mass" to a breakdown of the inverse square law [101], to which the GTR reduces too.

There are other problems associated with the dark matter/energy, such as blackness problem: as luminosity of some supermassive objects (possibly black holes) should be ~1000 times bigger than that which is observed [102]. And there is the cosmological problem as the question about the structure of space and distribution of matter within the universe [103]. But introduction of a cosmological constant Λ requires new and completely unknown physics in the region of ultralow energies, for very small positive cosmological constant may be thought of as one corresponding to the energy density of a highly excited though still very symmetric "background" state, so it need not be very fundamental; but then it is natural to assume that it may not really be constant, in which case one may speak about cosmological term [104]. But the $\Lambda \neq 0$ models with baryon or stable neutrino missing mass disagree with current observations [105]. Accelerated expansion of the universe strongly suggests that the content of the universe is dominated by a nonclustered form of matter, the so-called dark energy, but the smallness of the cosmological constant is then problem here [106]. The hope that Dark Energy Survey map will help determine if the suggested by experimentally discovered facts cosmic acceleration is driven by a cosmological constant, by a new and dynamic form of energy, or by physics beyond the scope of the GTR [107] is surely exciting. But viable predictions require operationally correct mathematics. For valid predictions depend on operational procedures fitting the corresponding to them spatial/physical structures.

7. THE NOTION OF MASS DISAMBIGUATED

There is no question that much more mass could exist than is observed. But if mass equals to energy (as it certainly does, at least from quantum-mechanical and relativistic point of view) then perhaps more matter (not just mass, which is just a dynamical feature of matter) should also exist in order to keep all the mass moving. Yet traditional mass was equated with material substance, which could only be true if all celestial bodies would travel at the same speed and in practically the same orbit, from the standpoint of relativity. But this kind of uniform orbiting is impossible, especially when speaking about sparsely distributed stars moving within arms of spiral galaxies.

If we would like to have more matter (or perhaps more dense matter, to be exact) then where it would go since there was no place for density of matter in primary equations of virtually all former gravitational theories. Density of mass was sometimes understood as 'density of matter', but it was never about the material substance, for the mass density ρ=mass/volume was always about the distribution or the stream of matter, not about its material constitution. At first Newton has abandoned his idea of gravity due to erroneous measure of the Earth's radius [108] and then

postponed the publication of his *Principia* for some 20 years but he could not accommodate into his reasonings the density of matter, which he has postulated as conceptually necessary indeed for realistic definition of quantity of moving matter. If something is necessary and yet apparently absent, then perhaps it somehow works behind the scenes, so to say. If the former purely radial gravity worked fine without explicit presence of the density of matter, then perhaps the density of matter was already inadvertently included in the traditionally misdefined mass. That is why the equalized mass (used instead of the traditional mass) revealed the urgent need for the density of matter Q, which can amplify (if $Q>1$) the attractive effect of the equalized mass even for the usual radial interactions. Since mass relates gravitation to motion, then the mass (which varies with speed but only in the direction of motion) cannot pertain also to the material substance of gravitating bodies, whose measure per volume (density of mass) spreads in all three directions. It was Cavendish who measured the relative density of Earth as compared to that of water (i.e., the specific gravity) and then used it to determine the value of the gravitational constant G from it; in fact, he was using lead balls and then converted the results to water-based standard [109].

This simple mathematical reasoning, not to mention other, more sophisticated mathematical arguments, has stirred interest in treating gravitation in terms of potential/energy, which could not be localized in the GTR, however. This might sound like a joke, were it not for the fact that the GTR's radial-only predictions are experimentally confirmed [110]. Note that the nonradial angular term is inversely proportional to the density of matter [2] and thus it is enhanced in less dense (i.e. accreting matter) stars and/or planets.

The problem is the traditional mathematics, not the GTR *per se*. Even though at a deeper level of inquiry mass could vary, the equalized mass is not like the variable mass hypothesis [112] but still has many of its advantages for astrophysics.

As a matter of fact, it was Zwicky who required of every scientific statement that it be in accord with what might be called "the principle of flexibility in scientific truth" if we wish to be in an organic relation to experience [113]. While scientific 'truth' should not be "adjusted" to fit our theories, our theories could be made more flexible to fit prospective truths revealed in yet unanticipated experimental results. This belief was also emphasized by Dicke who wrote that a single observation can disprove a theory, but observational proofs seem to be impossible [114]. Given several operational disasters present in previously developed mathematics, it is no wonder that some philosophers made significant contributions to mathematics and its faulty applications even without guidance of direct experimentation. One of them is Mach, who cleared up some of the confusion in gravitational physics without mathematics but with great intuition and insight.

The Mach's principle (MP) asserts that the inertia of a body must increase when ponderable masses are piled up in its neighborhood and thus a material body in an otherwise empty space should have no inertial mass [115]. In this formulation, the MP avoids many of its controversial misinterpretations stemming from taking literally Mach's own words and claims, especially that distant stars somehow directly influence local massive bodies [116], [117]. But the claim that mass should vanish in empty space, which often enthused controversy [118,119], makes sense if the mass is interpreted not as a measure of material substance but as the resistance to change of motion propelled by all the external forces due to presence of other masses. For in absence of such external forces no need for so-understood mass would arise for no resistance is necessary if there is nothing to resist to. Mach exemplified it by appearance of fictitious forces [120] p.216. The MP produced the relation $GM=Rc^2$ where R is the radius and M the mass of the universe has been obtained by several authors, mostly by invoking the Mach's principle [115,121,122].

Physics frequently fights over fashionable interpretations of names/labels. Mach dismissed the notions of absolute time and space [120] p.213 and effectively reintroduced relativity before Einstein did it. For an acceleration would appear to be different, indeed, for any reference frame that is accelerated with respect to the former ones, which leads to the appearance of the fictitious forces due to an acceleration of the reference frame, while true forces are due to interactions of the body with other bodies [123]. The MP allows thus for interpretation of the inertial forces, like centrifugal and Coriolis forces, as induced by mutual interaction between the masses [124].

Any scalar theory of gravitation which is completely covariant must contradict the MP [125]. The Einstein's GTR appears to be in direct conflict with Mach's principle [126]. For the Mach's critique is only partly correct and the GTR only partly satisfies the Mach's principle [127]. The MP essentially means simultaneity of action and reaction of gravitational interactions between bodies in the sense of Newton's 3rd law [128]. As instantaneous it did not sit well with Einstein who interpreted the MP as negating the concept of absolute spacetime inherent in the STR and as justifying the treatment on an equal footing of inertial and noninertial frames of reference [129].

If Mach is right, then the acceleration given a particle by a force field ought to depend not only on the presence of the fixed stars but also, very slightly, on the distribution of matter in the immediate vicinity of the particle [130] and asymmetries in the matter distribution at large, say concentration of matter near the center of our galaxy, could produce asymmetries in inertia [85]. But this conclusion should not cause the rejection of the MP, which is just the messenger of the inadequacy of mass for representing both the distribution of matter as well as its substance, but rather to seek another measure for the latter. One cannot always "kill two birds with one stone".

8. GRAVITATIONAL ATTRACTION COEXISTS WITH TWISTING REPULSION

Although static gravitational force field generated by a single mass sources can produce only radial attractive force, gravitational interactions with other radial fields produce also nonradial components of the effective force acting between the source masses of the fields. The twisting nonradial effect of the extra angular nonradial component corresponds to an evidently repulsive gravitational force, which is much smaller than the overwhelmingly huge attractive radial/center-bound force of the mass source of the dominant force field. Nevertheless, the fact that the tiny angular nonradial component acts only along equipotential surfaces makes the angular nonradial force perpendicular to the radial component so that the effective gravitational force is twisted along the trajectory of the smaller, orbiting mass source m. The twisting creates thus permanent, even though relatively much smaller, effectively repulsive force whose perpendicularity makes it impossible to get eliminated by the primary radial force that always remains orthogonal to it. This specific geometric configuration equips gravitational interactions within radial force fields with three nonvanishing components: two attractive and one repulsive, which acts in a mutually perpendicular direction. Although the radial field is attractive, interactions within it are twisted in general, when an external force is applied. This is a consequence of the Frenet-Serret formulas of differential geometry [9], which are formulated in more abstract terms of curvature and torsion and claim that any externally influenced trajectories must always pursue screwlike motion. When more than one local mass source is present within the given force field (which is what determines interactions), then the other source(s) supply or generate external forces by their very presence. If a laser is applied (on atomic scale) its impact can emulate that of an extra source charge.

The gravitational field **g** is a polar vector directed towards the center of the Earth whereas the magnetic field **B** is an axial vector perpendicular to the equatorial plane [131]. Their confluence is fast when considered on cosmic time-scales, for in about 2000 years, the Earth's magnetic moment is due to vanish according to its present rate of decrease and presumably the reversal of magnetic poles will occur [132]. Octonionic approach offers merging the **g** and **B** field vectors [95] and so do also some gravito-electromagnetic theories [115] – see also references therein.

Since the universe is expanding with time the gravitational constant is changing with time but there are partial indications that the constant has not changed in that way [133]. Dirac proposed arguments in favor of a variation of the gravitational constant upon his study of the constants of nature [134]. With decrease in the constant G celestial bodies may have expanded [135] too for it is known that anomalies in Earth's gravity result from variations of density of rocks [136].

The dependence of the attractive pull of gravity on the density of mass (which is related to density of matter at relative rest) has also been detected by several spacecrafts orbiting

Moon. Fairly recent experimental data obtained by the NASA's dual Gravity Recovery and Interior Laboratory (GRAIL) spacecraft have confirmed existence of free-air gravity anomalies due to mass concentrations (referred to as mascons) in lunar impact basins [137]. Yet the influence of mascons was considered only in terms of the usual radial effects of gravity field thus far, because no "respectable" theory of former physics ever bothered to study any nonradial angular effects of the usual radial/center-bound gravitational fields. This particular experimental result definitely suggests that accounting for even local fluctuations of density of mater is indeed necessary for more precise estimations of orbits and trajectories traversing radial gravitational force fields.

The alleged universality of gravitational attraction is thus an illusion that probably emerged from (mathematically-averse) lines of thought, which was then thoughtlessly followed even by conceptually confused mathematicians. It was indirect but definitely illogical inference from the operationally incomplete traditional definition (2) of work done (and thus also that of potential energy) of purely radial force fields generated by single (or amalgamated nearby) mass sources.

Since energy exchange in the repulsive gravitational interactions grows stronger with lesser density of matter of the mass source of the locally dominant gravitational field [5,6], one could expect that the average density of matter in galaxies with more sparsely distributed stars shall be lower than in thick galaxies (i.e. with either denser stars or maybe with greater spatial abundance of equally dense stars as in sparsely populated galaxies). At a deeper than this present level of inquiry into interactions of the gravitational fields' one shall consider thus also the likely impact of changing density of matter and thus further enhance the static equations presented above. By the way, different methods used for accounting can produce discrepant outcomes. Some neutron stars, for example, seem to contain less nucleons counted by number than counted by mass [138].

Even in our small solar system any velocity $v = \sqrt{(GM(r)/r)}$ increases with increasing radius from our Sun, which orbits the galaxy at ~220 km/s (or maybe at ~320 km/s [139]), whereas the Earth also moves at ~30 km/s around the common gravity center of the Earth and Sun [140]. All the former estimates were only purely radial, however. It seems that for all the observed samples, typical models imply that even in the solar vicinity about half of the mass should be in the form of unobserved/dark matter [141]. The problem seems to be almost "universal", which fact might raise the question of whether or not our former mass/energy accounting laws were correct? If we compare the formulas (1) and (2), the answer to this question must be definitely resounding NO.

Cosmological models which are homogeneous in density are not fair representations for the observed universe in which most of the galaxies appear to be organized into clusters [142]. This hierarchical feature already stunned Laplace [143]. According to the hierarchy hypothesis there exists within an infinite cosmos a hierarchy of isolated systems, each specified by a charge, from which we can derive a radius, a time and a mass [144]. We just need correct mathematics.

9. RADIAL FORMULAS WITH EQUALIZED MASS AND DENSITY OF MATTER

If written in terms of potential energy U the values of radial and nonradial angular forces are

$$F(r) = |\mathbf{F}(\mathbf{r})| = \nabla U(r) = \frac{\partial}{\partial r}\frac{G\mathcal{M}Q}{r} = -\frac{G\mathcal{M}Q}{r^2} \tag{5a}$$

$$F(\vartheta) = |\mathbf{F}(\vartheta)| = \nabla U(\vartheta) = -\frac{\partial}{\partial \vartheta}\frac{G\mathcal{M}K\vartheta}{r} = -\frac{\partial}{\partial \vartheta}\frac{G\mathcal{M}Kr\vartheta}{r^2} = \frac{\partial}{\partial \lambda}\frac{G\mathcal{M}\lambda}{r^2} = \frac{\partial}{\partial \lambda}\frac{GM\lambda}{Qr^2} = \frac{GM}{Qr^2} \tag{6a}$$

where the nonradial angular term is rendered with both the equalized mass $\mathcal{M}$ and the traditional source mass M in order to show that it is inversely proportional to the density of matter Q of the source mass M. Here the spheroidal equipotential distance is $\lambda = -Kr\vartheta$ for the sake of simplicity. The radius r in the eq. (6a) is the parameter determining the equipotential surface, not a variable. Notice that F(r) is negative because the vectors of force and radius point in opposite directions.

The importance of the nonradial angular component $U(\vartheta)$ of the whole scalar potential U() is that it can be used to determine the density of matter of a “disturbing” star or planet just as it has happened in the experiment conducted by Sadeh in close vicinity of our Sun’s surface [18]. It is unclear to me how the coefficient K would be affected by the outward pointing distance from the surface of the disturbing star, for even the notion of ‘surface’ of a shining star can be perplexing.

Note that the nonradial angular potential term $U(\vartheta)$ diminishes very rapidly with increasing radial distance r from the center of a disturbing star [6], a fact confirmed in an experiment [145]. Simple explanation of the first two Sadeh experiments: [18] and [19], was presented in [2,3]. To explain them in one sentence, consider this: Since our Sun, which has 3.9 times less density of matter (Q = ~1.42) than the Earth (Q = ~5.52) had 3.9 stronger impact on the frequency decrease in the rays coming from Taurus A when the rays were passing in close vicinity of our Sun, then the practically tangential impact of our Sun was inversely proportional to its density of matter as in the eq. (6a) [2]. It is that simple. These experiments showed that Galilei, and both Newton and Einstein who accepted the experiments conducted at the leaning tower of Pisa at their face value, were wrong in regard to the nonradial effects of gravity and consequently also to the radial ones.

The formerly unanticipated results of these Sadeh experiments remained unreconciled and unexplained until 2000 AD [2], because they challenged some uncritically accepted assumptions of former theories of mathematics and physics, one of which was that claim of Galilei. We must not defy unbiased experimental evidence but rather use their feedback to complete theories that were somewhat incomplete and to correct theories that were insufficiently correct at the current – neoclassical – level of inquiry into the relevant phenomena. Development of our mathematical and physical theories should be guided by hints supplied by unanticipated experimental results.

If radial potential energy of a galaxy with radius R, equalized mass $\mathcal{M}$ and matter density Q is

$$U(r) = \frac{G\mathcal{M}Q}{r} \tag{7}$$

at a distance r, then using the methods espoused in [16] p.225 the rotational velocity is given by

$$v = r\sqrt{G\mathcal{M}Q/R^3} \tag{8}$$

for stars inside (r<R) and for stars located in spiral arms (where r>R by definition) it amounts to

$$v = \sqrt{G\mathcal{M}Q/r} \tag{9}$$

so that in both cases the estimated velocity is generally multiplied by √Q as compared with their previously calculated values in [16]. I shall leave it to observations to ascertain their feasibility.

Let us follow M. Ross in order to compare the difference that the equalized mass could make on the observed expansion of the universe as it was presented in [146]. The author considers a galaxy of gravitating mass m located at a radius r from the center of a sphere of mean density [of mass] ρ and mass $M = \frac{4}{3}\pi r^3 \rho$ with gravitational potential [of an interaction] of the galaxy

$$U = \frac{GMm}{r} = \frac{4}{3}\pi G m r^2 \rho \text{ and radial acceleration } \ddot{r} = \frac{GM}{r^2} = -\frac{4}{3}\pi\, G\rho r \tag{10}$$

[146] p. 20f. Here the gravitational potential U does not refer to the galaxy as in eq. (7) but rather to interaction between the masses M and m, which is fine with me for it can be regarded as a unit mass. I have used above the coefficient k in order to accommodate also the dimensional impact of the other mass m, whose presence underscores interacting potentials.

In a universe expanding according to Hubble's law, the kinetic energy T of the galaxy receding with velocity v is [146]:

$$T = \frac{1}{2}mv^2 = \frac{1}{2}mH^2r^2 \tag{11}$$

where m is the inert mass of the given galaxy and H is the Hubble parameter usually assumed as constant. Although there is no theoretical reason for the inertial mass m to equal the gravitational mass m above, careful tests have verified the equality to a precision better than one part in 10^{11} [146]. Thus, the total energy E as the sum of kinetic energy T and potential energy U is given by

$$E = T + U = \frac{1}{2}mH^2r^2 - \frac{4}{3}\pi G m r^2 \rho = mr^2\left(\frac{1}{2}H^2 - \frac{4\pi}{3}G\rho\right) \tag{12}$$

as it is presented in [146] p.21. Now, when substituting the equalized mass $\boldsymbol{\mathcal{M}}$ into the inert mass m we must complement it with the matter density Q(R,T) that depends on radius R of the given subset of the universe and on certain elapsing cosmic time T. But when substituting the equalized mass into the mass density ρ whose mass M refers to material substance, we should use a spatial distribution of matter Q(r,φ,θ,t) in spherical coordinates (r,φ,θ) at a certain elapsed time t. Even though for most practical purposes the distinction may be unimportant, our equalized energy is

$$E = \mathcal{M}Q(R,T)r^2\,[\tfrac{1}{2}H^2 - G\mathcal{M}Q(r,\varphi,\theta,t)] \tag{13}$$

with two quite distinct and possibly varying functions of densities of matter. The distribution of cosmic density of matter Q(R,T) is primarily the function of cosmic time and thus could also be written as Q(R(T)) where the radius R depends on the cosmic time T. The spatial distribution of matter Q(r,φ,θ,t) refers to the *status quo* at certain elapsed time t. In the sense, total radial energy depends on both: the evolution characterized by a certain expansion as well as the given outcome of the evolution. Note that the spatial distribution of matter density Q(r,φ,θ,t) is what we observe whereas the cosmic density of matter Q(R,T) is what we can tweak to arrive at present Q(r,φ,θ,t).

Though at the present, essentially neoclassical level of inquiry into gravitational phenomena the equalized mass is assumed as constant, at a deeper than the present level of inquiry it may be regarded as evolving i.e. as varying in both time and space. At the deeper level of inquiry all the present laws and currently attained formulas could undergo some modifications. The possibility of expansion (or an intrinsic drill-down capability) has been demonstrated in [8] on the basis of the differential calculus alone where physical considerations were used only for guidance and explanations. No law of physics or astrophysics is final; its form depends on how variables and constants/parameters are being treated at each level of inquiry. It is mathematics – no bargains.

Note that the energy E in the eq. (12) is only radial total, even though it is traditionally called total. In the eq. (13) the energy E contains also nonradial angular contributions, which were not considered in the GTR nor in any other previously developed (i.e. prior to 2000 AD) theories of radial-only gravitation. As being nonradial, the angular nonradial part of the total energy is not retrievable by forces acting radially or even tangentially – even though it is conserved – but it is conserved within the whole system. The traditional energy conservation law based on Noether's theorem should be renamed to energy retrieval law whereas the conservation law becomes more general in the context of the nonradial terms. When our Sun has "robbed" the rays coming from Taurus A by draining the rays' energy as in the experiment [19], sending the rays back to Taurus A would not let them regain their energy lost to our Sun, unless the ray would somehow become larger than our Sun and thus it would curve the space towards itself instead of seeing it curved by our Sun. The energy lost by the ray on its way to be intercepted on the Earth is still conserved, because our Sun keeps it, but because the robbery happened on tangential path crisscrossed the equipotential surfaces surrounding our Sun, which as the dominant mass source of the local gravitational field in its vicinity, curves the space onto itself, and the ray would not be able to regain its lost energy from our Sun without recurving the space back towards itself, which is impossible. The ray's energy lost to Sun has not been destroyed, but it could not be retrieved.

10. SOME APPARENT CONTROVERSIES

Some authors made bold claims. For example: it is true that from certain valid gravitational and/or electromagnetic equations no singularities (or just black holes, in the common parlance of physics) are deduced. But taming singularities is the "bread and butter" of complex analysis, so the fact that a particular theory does not admit them could mean that it still has an ample room to grow to make their theories more comprehensive. What they say is true but one cannot derive "generals from particulars", which is one of unquestionably true classical laws of medieval logic.

It is true that energy-momentum of the gravity field cannot be localized in the GTR but as the authors pointed out it is an issue of localizability not of presence or nonexistence of the energy [147] p.467. Besides they asserted that the imaginary time coordinate hides from view the [true] character being dealt with [147] p.51. Yet the 4D spacetime appears to be a double-sided spatial structure, one face of which looks like the spacetime whereas the other looks like a 4D timespace overlaid upon each other [20,21]. Other researchers have stumbled upon it too [148]. But the prospective solution to the conundrum is still elusive, because it would require a big paradigm shift. Instead of the currently espoused by scientists single-space reality paradigm, a multispatial reality paradigm would have to be adopted, which topic shall be further discussed elsewhere.

The other controversial issue is that of apparent inadequacies of theories whose predictions have gained at least partial experimental confirmation. In many cases the inadequacies were just suppressed as if they never emerged. In worst instances, some authors keep on claiming that the controversial issues have been resolved and the whole controversy was just misunderstanding or an early misconception. The latter was the fate of longitudinal mass versus transversal mass. It is an embarrassment to have different formulas for relativistic transformation of allegedly the same mass depending on direction in which it is transformed, especially if mass covers substance too.

With due respect to operational rules of mathematics the longitudinal mass is transformed as

$$m_{long} = \frac{m_0}{(1-\beta^2)^{\frac{3}{2}}} \tag{14}$$

by the Lorentz transformations, whereas the transversal mass is transformed as follows

$$m_{trans} = \frac{m_0}{(1-\beta^2)^{\frac{1}{2}}} \tag{15}$$

by the very same Lorentz transformations [149]. Here m_0 denotes the rest mass and $\beta = v/c$ is the rapidity, v is the constant velocity and c is the speed of light in vacuum. The derivation is not the problem – it is mathematically quite correct. But it is conceptually somewhat controversial, for how is it possible that the very same mass transforms differently depending on how it is treated if it is supposed to determine also the material substance? Similar issue arises with the moment of inertia that is the rotational analogue of mass [150]. Obviously, mass – as the alleged measure of material substance – would not change its physical nature just because it happens to be rotated.

The longitudinal mass refers to transformation of acceleration [151] whereas the transversal mass refers to transformation in the direction of [changing by the transformation] velocity. None of these transformations is questionable. The problem there is that the same notion of 'mass' tries to address two quite distinct and logically incompatible concepts. Therefore, the traditional mass must become equalized and then supplemented by density of matter of the given mass source.

11. CONCLUSIONS

The (formerly unaccounted for) repulsive interaction that is associated with density of matter could explain (at least partly) the apparently insufficient radial gravitational forces that might have demanded presence of some dark/matter/energy if the primary mathematical formulas of former gravitational theories were correct, which are not. One fairly simple way to accomplish that is to introduce the concept of equalized mass that is common to all material bodies within the universe. In such a case, the effective mass would be a product of the equalized mass and the average density of matter of the mass source in general and of massive bodies in particular. This substitution would make the effective scalar radial potentials (and thus also the corresponding to them radial attractive forces) amplified by their density of matter, provided the latter is higher than that of water, relative to which the density of matter is conventionally measured.

Since greater density of matter (in a product with equalized mass) of a star can increase its radial attractive force, this effect can help to lessen the missing mass problem without invoking any contribution from the conjectured dark matter and dark energy. Also stars and galaxies with lesser average density of matter shall have much more twisting repulsive forces than the more densely packed (with matter) ones. Hence the attraction of stars with higher density of matter could be considerably stronger than that of less dense stars even with the same equalized masses.

Here higher density pertains to matter denser than water and so does lesser. But the density of matter could be measured by reference to something other than water, as long as it pertains to the same reference point (to be established by convention) for all material bodies of the universe. For frequency shifts are not only indicative of distances but indirectly also of density of matter.

One should remember, however, that the actually observed in spectra (cumulative) redshift is not really the most reliable measure of true distances to distant stars, because in addition to the radial gravitational redshift, which Einstein has introduced, there could also be some nonradial redshifts due to the passage of the rays from the given distant star, whose spectra are observed, near other stars. The extra nonradial redshifts are tiny but could push distances to the affected by them stars or quasars far away from their actual positions in the universe (as in some quasars with much higher z than that of their apparent home galaxies). The nonradial redshift, which is induced by a "disruptive" (third party) stars located close to the line of sight of the star whose shifted spectra are observed, is only significant in very close vicinity of the line of sight to the disruptive star. By the same token, frequency decrease found near occultation with a disruptive star or planet could be used to fairly well estimate the disruptor's density of matter.

In none of the cited academic books and research articles the postulated dark matter/energy was ever discovered but merely hypothesized as a remedy for faulty operational mathematics. What was actually discovered is that certain astronomical observations were not supported by the formerly developed mathematics, whose obvious operational incompleteness was routinely, and maybe still is, being suppressed. I am not criticizing making mistakes and simplifications, both of which were often beneficial for development of new theories devised ahead of their time, but am against the tacit suppression of inconvenient truths in mathematics and physical sciences.

The currently unobserved or dark matter/energy that will certainly be discovered when our observational methods improve, would not solve the problem that is – or should have been – known to originate with operational incompleteness of all former theories of gravitation. The incompleteness has been inherited from faulty mathematical operations.

In the present paper, I have not sided with any particular interpretation of astrophysical or physical theories. I have shown that even the valid (at their face value) arguments lead to some inconclusive inferences when they are paired with few other (also apparently valid at their face value) arguments. The unspoken assumption common to all the reviewed here books and articles is that the mathematics used in physics, astrophysics, cosmology and even philosophical papers pertinent to the quest for dark matter/energy, is correct, in which case one might come up with the hypothesis that there should exist some unobserved yet matter/energy which could solve the observed deficiency of it. But the fact is that the mathematics is wrong and even some theorems are not only false but fake, because they have tacitly assumed what they were supposed to prove. If so, then the quest for yet to be discovered matter/energy may be legitimate, but to stipulate that it should correct incomplete equations is inadmissible both mathematically and conceptually.

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