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An analysis of the thermal capacity in Gravitons

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ABSTRACT

In this paper, we have considered a microscopic model based in the statistical mechanics for gravitons proposed for Viaggiu (2017). We obtain an analytical expression for the thermal capacity C_V in gravitons. It is predicted in this research the behavior of the thermal capacity in the limit of high and low temperature. We have also obtained the fluctuations of energy for gravitons with the equation for C_V . We found that the value of the fluctuation is the same that would be obtained in a gas with internal energy $U=2NK_BT$ when $T\rightarrow\infty$.

***Keywords*:** Statistical mechanics, Thermal capacity, Fluctuations, Gravitons, Microscopic model

1. INTRODUCTION

In the statistical and microscopic description of any system, the partition function plays determinant role and is defined as the total sum of states of the system [1]:

$$Z = \sum g(E_n) e^{-\frac{E_n}{K_B T}} \tag{1}$$

where n labels the total energy E_n , K_B is the Boltzman´s constant and $g(E_n)$ it is the number of degenerate states with the same energy E_n. The use of the equation (1) [1,2] allows to obtain a statistical and microscopic description of the energy and the thermal capacity.

The concepts of the statistical mechanics must be considered if we want to do a microscopic description of a physical system. Recently Mäkelä [3,4] constructed a microscopic model of "Stretched Horizon " of a Schwarzschild and Reissner-Nordström black holes and obtained an analytical expression for the partition function from the point of view of an observer on its stretched horizon. Malaver [5-7] studied the behavior of the thermal capacity C_V for Schwarzschild and Reissner-Nordström black holes when $T>>T_C$ and $T<<T_C$ where T_C is the characteristic temperature and found that the value for C_V if $T>>T_C$ is the same that would be obtained in an ideal diatomic gas if are considered the rotational and translational degrees of freedom, respectively. Viaggiu [8] present a statistical analysis in gravitons and derived equations for the partition function and the mean energy.

The graviton is an elementary hypothetical particle that mediates the gravitational interaction in the framework of quantum field theory [9]. If the graviton exist is expected to be massless particle and must be spin-2-boson [10]. The three other known forces of nature are mediated by elementary particles: electromagnetism by the photon, the strong interaction by the gluons and the weak interaction by the W and Z bosons. In the classical limit, the theory would reduce to general relativity and conform to Newton´s law of gravitation in the weak-field limit [11].

In this paper we have deduced an analytical expression for the thermal capacity for gravitons and we studied the behavior of C_V in the limit of high and low temperature. With this expression for C_V we have calculated the fluctuations of energy in gravitons. We found that the value of the fluctuation for high temperatures is the same that would be obtained in the classic limit where $U=2NK_BT$. This paper is outlined in the following manner: the section II we present the equations for statistical mechanics for gravitons. In section III it is shown the behavior of the thermal capacity with the temperature. In section IV we present an analysis of the fluctuations of the energy. Finally in section V, we conclude.

2. STATISTICAL MECHANICS OF GRAVITONS

According Viaggiu [8] the canonical partition function for gravitons in a box can be written as

$$Z_T = \left(\frac{e^{-\left(\frac{2c\pi\beta\hbar}{R}\right)}}{\left[1-e^{-\left(\frac{c\pi\beta\hbar}{2R}\right)}\right]} \frac{1}{\left[1-e^{-\left(\frac{c\pi\beta\hbar}{R}\right)}\right]} \right)^N \tag{2}$$

where $\beta = 1/K_BT$, $\hbar = h/2\pi$ and R is the radius of the spherical box where the gravitons are confined.

Defining the statistical energy with the following expression [1,2]

$$E_T = -\frac{\partial \ln Z}{\partial \beta} \tag{3}$$

With the eq. (2), for the internal energy $U=E$ we obtain

$$U = E = \frac{c\pi\hbar N}{2R\left[e^{\frac{\beta c\pi\hbar}{2R}} - 1\right]} + \frac{c\pi\hbar N}{R\left[e^{\frac{\beta c\pi\hbar}{R}} - 1\right]} \tag{4}$$

The thermal capacity to constant volume this defined as

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V \tag{5}$$

Then for C_V we can obtain for differentiation of (4)

$$C_V = \frac{c\pi\hbar N}{2R}\left[\frac{\partial}{\partial T}\left(\frac{1}{e^{\frac{c\pi\hbar\beta}{2R}} - 1}\right) + 2\frac{\partial}{\partial T}\left(\frac{1}{e^{\frac{c\pi\hbar\beta}{R}} - 1}\right)\right] \tag{6}$$

With the equation (6) we obtain the following expression for the thermal capacity in gravitons

$$C_V = NK_B\left[\left(\frac{c\pi\hbar\beta}{2R}\right)^2 e^{\frac{c\pi\hbar\beta}{2R}}\frac{1}{\left(e^{\frac{c\pi\hbar\beta}{2R}} - 1\right)^2} + \left(\frac{c\pi\hbar\beta}{R}\right)^2 e^{\frac{c\pi\hbar\beta}{R}}\frac{1}{\left(e^{\frac{c\pi\hbar\beta}{R}} - 1\right)^2}\right] \tag{7}$$

3. BEHAVIOR OF THE THERMAL CAPACITY IN GRAVITONS

In the equation (7) we define a specific parameter $\varepsilon = c\pi\hbar / RK_B$ where ε has the dimensions of temperature. In terms of ε we can write the expression for C_V of the following manner:

$$C_V = NK_B \left[\left(\frac{\varepsilon}{2T}\right)^2 e^{\frac{\varepsilon}{2T}} \frac{1}{\left(e^{\frac{\varepsilon}{2T}} - 1\right)^2} + \left(\frac{\varepsilon}{T}\right)^2 e^{\frac{\varepsilon}{T}} \frac{1}{\left(e^{\frac{\varepsilon}{T}} - 1\right)^2} \right] \qquad (8)$$

As the function e^x is equivalent to the sum of the infinite series $1 + x + x^2/2! + x^3/3! + ...$ we can determine the validity of (8) in two limiting cases.

In the first limiting case we consider the situation at very high temperature $T >> \varepsilon$. Developing the exponential terms in equation (8) we find

$$C_V = NK_B \left[\left(\frac{\varepsilon}{2T}\right)^2 \frac{\left(1 + \varepsilon/2T + (1/2!)(\varepsilon/2T)^2 + (1/3!)(\varepsilon/2T)^3 + \ldots\right)}{\left(1 + \varepsilon/2T + (1/2!)(\varepsilon/2T)^2 + (1/3!)(\varepsilon/2T)^3 + \ldots - 1\right)^2} + \left(\frac{\varepsilon}{T}\right)^2 \frac{\left(1 + \varepsilon/T + (1/2!)(\varepsilon/T)^2 + (1/3!)(\varepsilon/T)^3 + \ldots\right)}{\left(1 + \varepsilon/T + (1/2!)(\varepsilon/T)^2 + (1/3!)(\varepsilon/T)^3 + \ldots - 1\right)^2} \right] \qquad (9)$$

In the denominator it is needed to retain the term ε/T and $\varepsilon/2T$ since the higher powers of ε/T will be entirely negligible .In the numerator everything can cancel except the 1. With these approximations, we obtain for C_V

$$C_V = NK_B \left[\left(\frac{\varepsilon}{2T}\right)^2 \frac{1}{(\varepsilon/2T)^2} + \left(\frac{\varepsilon}{T}\right)^2 \frac{1}{(\varepsilon/T)^2} \right] = 2NK_B \qquad (10)$$

As in case of the black hole described by the Reissner-Nordström metric [6,7] the value of C_V obtained by this method is identical the one that would be obtained for an ideal gas with translational and rotational degrees of freedom for high temperatures.

In the second limiting case, let us consider the situation at very low temperature. When $T\to 0$ we have $T \ll \varepsilon$ and the equation (8) can be written as

$$C_V = NK_B\left[\left(\frac{\varepsilon}{2T}\right)^2 \frac{1}{e^{\frac{\varepsilon}{2T}}} + \left(\frac{\varepsilon}{T}\right)^2 \frac{1}{e^{\frac{\varepsilon}{T}}}\right] \tag{11}$$

Re-arranging the expression (11) the result is

$$C_V = NK_B\left[\begin{array}{l}\left(\frac{\varepsilon}{2T}\right)^2 \dfrac{1}{\left(1+\varepsilon/2T + \left(1/2!\right)\left(\varepsilon/2T\right)^2 + \left(1/3!\right)\left(\varepsilon/2T\right)^3 + \dots\right)} + \\ \left(\frac{\varepsilon}{T}\right)^2 \dfrac{1}{\left(1+\varepsilon/T + \left(1/2!\right)\left(\varepsilon/T\right)^2 + \left(1/3!\right)\left(\varepsilon/T\right)^3 + \dots\right)}\end{array}\right] \tag{12}$$

or equivalently

$$C_V = NK_B\left[\begin{array}{l}\dfrac{1}{\left(\left(\frac{2T}{\varepsilon}\right)^2 + 2T/\varepsilon + 1/2! + \left(1/3!\right)\left(\varepsilon/2T\right) + \dots\right)} + \\ \dfrac{1}{\left(\left(T/\varepsilon\right)^2 + T/\varepsilon + 1/2! + \left(1/3!\right)\left(\varepsilon/T\right) + \dots\right)}\end{array}\right] \tag{13}$$

when $T\to 0$ in the eq. (12) the first two terms in the denominators also approach zero but the fourth and all subsequent terms tend to infinity, hence if $T\to 0$ then $C_V\to 0$. This behavior is very similar to the observed in the thermal capacity of crystalline solids [1].

4. FLUCTUATIONS OF ENERGY

In agreement with (1), the dispersion of the energy can be written in the form [1] :

$$\overline{(\Delta E)^2} = \overline{(\Delta U)^2} = \frac{\partial^2 \ln Z}{\partial \beta^2} = -\frac{\partial E}{\partial \beta} \tag{14}$$

what is equivalent to

$$\overline{(\Delta U)^2} = \overline{U^2} - \overline{U}^2 = K_B T^2 \left(\frac{\partial E}{\partial T} \right)_V = K_B T^2 C_V \tag{15}$$

and the relative energy fluctuation is given by

$$\frac{\Delta U}{U} = \frac{T\sqrt{K_B C_V}}{U} \tag{16}$$

substituting (4) and (8) in eq. (16) we obtain

$$\frac{\Delta U}{U} = \frac{\sqrt{N}\left[e^{\frac{\varepsilon}{2T}} \left(e^{\frac{\varepsilon}{T}} - 1 \right)^2 + 4e^{\frac{\varepsilon}{T}} \left(e^{\frac{\varepsilon}{2T}} - 1 \right)^2 \right]^{1/2}}{N\left[\left(e^{\frac{\varepsilon}{T}} - 1 \right) + 2\left(e^{\frac{\varepsilon}{2T}} - 1 \right) \right]} \tag{17}$$

but the exponent $e^{\frac{\varepsilon}{T}} \approx 1 + \frac{\varepsilon}{T}$ and $e^{\frac{\varepsilon}{2T}} \approx 1 + \frac{\varepsilon}{2T}$ when $T >> \varepsilon$ and the fluctuation of energy can be written as

$$\frac{\Delta U}{U} = \frac{1}{\sqrt{2N}} \rightarrow 0 \quad \text{when} \quad \mathrm{N} \rightarrow \infty \tag{18}$$

The value of fluctuation is equal of that of an ideal gas with two translational and two rotational degrees of freedom, that is $U = 2NK_BT$

5. CONCLUSIONS

In this paper, we have deduced an expression for the thermal capacity in gravitons and have studied the behavior of C_V for $T \rightarrow 0$ and $T \rightarrow \infty$. Such a behavior is similar to the observed in the analysis of the thermal capacity of the solids. With this expression we have determined the fluctuations of energy in gravitons. We found that when $T >> \varepsilon$ the value of the fluctuations is the same that would be obtained for an ideal gas if only they are considered translational and rotational degrees of freedom.

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