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SHORT COMMUNICATION

# Bound State Solutions of the Schrödinger's Equation with Manning-Rosen Plus a Class of Yukawa Potential Using Pekeris-like Approximation of the Coulomb Term and Parametric Nikiforov-Uvarov

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**ABSTRACT**

The solutions of the Schrödinger equation with Manning-Rosen plus a class of Yukawa potential (MRCYP) have been presented using the Pekeris-like approximation and parametric Nikiforov-Uvarov (NU) method. The bound state energy eigenvalues and the corresponding un-normalized eigen functions are obtained in terms of Jacobi polynomials. Also, inversely quadratic Yukawa, Yukawa, Manning-Rosen and coulomb potentials have been recovered from the mixed potential and their Eigen values obtained. The Numerical results are computed for some values of n at l = 0 with α = 0.01, 0.1, 2 and 5 using python 3.6 programming, and these results could be applied to molecules moving under the influence of MRYP potential as negative energy eigenvalues obtained indicate a bound state system.

***Keywords*:** Schrödinger equation, Manning-Rosen potential, Yukawa potential, Pekeris-like approximation, Parametric Nikiforov-Uvarov method, Jacobi polynomials

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## 1. INTRODUCTION

Over the years theoretical physics and Chemistry have been successful in explaining the behaviour of different particles in different potentials. This has been made possible through obtaining exact or approximate solutions of the non-relativistic and relativistic wave equations for different physical systems of interest (Louis *et al*., 2016). In non-relativistic quantum mechanics, one of the interesting problems is to obtain exact solutions of the Schrodinger equation. In order to do this, a real potential is normally chosen to derive the energy eigenvalues and Eigen functions of the Schrodinger equation. The Schrödinger equation thereby reveals that the Eigen functions of the system can furnish us with information regarding the behaviour of such a system (Magu *et al*., 2017). Several authors have studied the bound states of the Schrödinger equation using different potentials and methods. Some of these potentials play very important roles in many fields of Physics such as Molecular Physics, Solid State and Chemical Physic (Ita *et al*., 2016). The Manning-Rosen potential has been deeply studied and applied in quantum systems. Also Yukawa potential and its classes have been studied in Schrodinger formalism (Ita and Ikeuba, 2015).

The purpose of the present paper is to solve the Schrödinger equation for the mixed potential MRCYP using the parametric NU method. The paper is organized as follows: After a brief introduction in section 1, the NU method is reviewed in section 2. In section 3, we solve the radial Schrödinger equation using the NU method. Finally, we discuss our results in section 4and a brief conclusion is then advanced in section 5.

## 2. NIKIFOROV-UVAROV METHOD

The Nikiforov-Uvarov (NU) method is based on the solutions of a generalized second-order linear differential equation with special orthogonal functions. The Schrödinger equation of the type as:

$$\psi''(r) + [E - V(r)]\psi(r) = 0, \qquad (1)$$

can be solved by this method. To do this equation (1) is transformed into equation of hypergeometric type with appropriate coordinate transformation $s = s(r)$ to get

$$\psi''(s) + \frac{\bar{\tau}(s)}{\sigma(s)}\psi'(s) + \frac{\bar{\sigma}(s)}{\sigma^2(s)}\psi(s) = 0, \qquad (2)$$

To solve equation (2) we can use the parametric NU method. The parametric generalization of the NU method is expressed by the generalized hypergeometrictype equation [19]

$$\psi''(s) + \frac{(c_1 - c_2 s)}{s(1 - c_3 s)}\psi'(s) + \frac{1}{s^2(1 - c_3 s)^2}[-\epsilon_1 s^2 + \epsilon_2 s - \epsilon_3]\psi(s) = 0, \qquad (3)$$

where$\sigma(s)$ and $\bar{\sigma}(s)$ are polynomials atmost second degree, and $\bar{\tau}(s)$ is a first degree polynomial. The eigenfunctions (equation 4) and corresponding eigenvalues (equation 5) to the equation become

$$\psi(s) = N_n s^{c_{12}} (1 - c_3 s)^{-c_{12} - \frac{c_{13}}{c_3}} P_n^{\left(c_{10}-1, \frac{c_{11}}{c_3} - c_{10} - 1\right)} (1 - 2c_3 s), \quad (4)$$

$$(c_2 - c_3)n + c_3 n^2 - (2n+1)c_5 + (2n+1)\left(\sqrt{c_9} + c_3\sqrt{c_8}\right) + c_7 + 2c_3 c_8 + 2\sqrt{c_8 c_9} = 0, \quad (5)$$

where

$$c_4 = \frac{1}{2}(1 - c_1), c_5 = \frac{1}{2}(c_2 - 2c_3), c_6 = c_5{}^2 + \epsilon_1, c_7 = 2c_4 c_5 - \epsilon_2, c_8 = c_4{}^2 + \epsilon_3, \quad c_9 = c_3 c_7 + c_2{}^2 c_8 + c_6, \quad c_{10} = c_1 + 2c_4 + 2\sqrt{c_8}, c_{11} = c_2 - 2c_5 + 2\left(\sqrt{c_9} + c_3\sqrt{c_8}\right), c_{12} = c_4 + \sqrt{c_8},$$

$$c_{13} = c_5 - \left(\sqrt{c_9} + c_3\sqrt{c_8}\right), \quad (6)$$

$N_n$ is the normalization constant and $P_n^{(\alpha,\beta)}$ are the Jacobi polynomials.

## 3. SOLUTIONS OF THE RADIAL PART OF SCHRÖDINGER EQUATION WITH MRCYP POTENTIAL:

The radial Schrödinger equation is given as

$$\frac{d^2 R_{nl}(r)}{dr^2} + \frac{2\mu}{\hbar^2}\left[E - V(r) - \frac{\lambda\hbar^2}{2\mu r^2}\right] R_{nl}(r), \quad (7)$$

where $\lambda = l(l+1)$ and $V(r)$ is the potential energy function. The Manning-Rosen potential (MRP) according to Louis *et al*., 2015, is given as

$$V(r) = -\left[\frac{Ce^{-\propto r} + De^{-2\propto r}}{(1 - e^{-\propto r})^2}\right] \quad (8)$$

The Class of Yukawa potential (CYP) is given as

$$V(r) = -\frac{V_o e^{-\propto r}}{r} - \frac{V_0{}' e^{-2\propto r}}{r^2}, \quad (9)$$

where $V_0$ and $V_0{}'$ are the potential depth of the CYP and $\alpha$ is an adjustable positive parameter. In equation (8)
$C$ and $D$ are constants. The sum of these potentials known as MRCYP is given as

$$V(r) = -\left[\frac{Ce^{-\propto r} + De^{-2\propto r}}{(1 - e^{-\propto r})^2}\right] - \frac{V_0 e^{-\propto r}}{r} - \frac{V_0{}' e^{-2\propto r}}{r^2} \quad (10)$$

Making the transformation $s = e^{-\alpha r}$ equation (10) becomes

$$V(s) = -\left[\frac{CS + DS^2}{(1-S)^2}\right] - \frac{V_0 \propto S}{1-S} - \frac{V_0{}' \alpha^2 S^2}{(1-S)^2} \quad (11)$$

Again, applying the transformation $s = e^{-\alpha r}$ to get the form that NU method is applicable, equation (7) gives a generalized hypergeometric-type equation as

$$\frac{d^2R(s)}{ds^2} + \frac{(1-s)}{(1-s)s}\frac{dR(s)}{ds} + \frac{1}{(1-s)^2s^2}[-(\beta^2 - F + B - G)s^2 + (2\beta^2 + A + B)s - (\beta^2)]R(s) = 0, \quad (12)$$

where

$$\lambda = 0, -\beta^2 = \frac{2\mu E}{\alpha^2\hbar^2}, A = \frac{2\mu C}{\alpha^2\hbar^2}, B = \frac{2\mu V_0}{\alpha\hbar^2}, F = \frac{2\mu D}{\alpha^2\hbar^2}, G = \frac{V_0{}'}{\hbar^2}, \frac{1}{r} \approx \frac{\propto}{(1-e^{-\propto r})} \approx \frac{\propto}{(1-S)}, \quad (13)$$

Comparing equation (12) with equation (3) yields the following parameters

$$c_1 = c_2 = c_3 = 1, c_4 = 0, c_5 = -\frac{1}{2}, c_6 = \frac{1}{4} + \beta^2 + B - F - G, c_7 = -2\beta^2 - A - B, c_8 = \beta^2, c_9 = \frac{1}{4} - (A + F + G), c_{10} = 1 + 2\sqrt{\beta^2}, c_{11} = 2 + 2\left(\sqrt{\frac{1}{4} - A - F - G} + \sqrt{\beta^2}\right), c_{12} = \sqrt{\beta^2}, c_{13} = -\frac{1}{2} - \left(\sqrt{\frac{1}{4} - A - F - G} + \sqrt{\beta^2}\right), \epsilon_1 = \beta^2 + B - F - G, \epsilon_2 = 2\beta^2 + A + B, \epsilon_3 = \beta^2, \quad (14)$$

Now using equations (5), (13) and (14) we obtain the energy eigen spectrum of the MRYP as

$$\beta^2 = \left[\frac{A + B - \left(n^2 + n + \frac{1}{2}\right) - (2n+1)\sqrt{\frac{1}{4} - A - F - G}}{(2n+1) + 2\sqrt{\frac{1}{4} - A - F - G}}\right]^2, \quad (15)$$

Equation (15) can be solved explicitly and the energy eigen spectrum of MRYP becomes

$$E = -\frac{\alpha^2\hbar^2}{2\mu}\left\{\left[\frac{\frac{2\mu C}{\propto^2\hbar^2} + \frac{2\mu V_0}{\alpha\hbar^2} - \left(n^2 + n + \frac{1}{2}\right) - (2n+1)\sqrt{\frac{1}{4} - \frac{2\mu C}{\propto^2\hbar^2} - \frac{2\mu D}{\alpha^2\hbar^2} - \frac{V_0{}'}{\hbar^2}}}{(2n+1) + 2\sqrt{\frac{1}{4} - \frac{2\mu C}{\propto^2\hbar^2} - \frac{2\mu D}{\alpha^2\hbar^2} - \frac{V_0{}'}{\hbar^2}}}\right]^2\right\} \quad (16)$$

We now calculate the radial wave function of the MRCYP as follows

The weight function $\rho(s)$ is given as [19]

$$\rho(s) = s^{c_{10}-1}(1 - c_3 s)^{\frac{c_{11}}{c_3} - c_{10} - 1}, \quad (17)$$

Using equation (14) we get the weight function as

$$\rho(s) = s^U(1 - s)^V, \quad (18)$$

where $U = 2\sqrt{\beta^2}$ and $V = 2\sqrt{\frac{1}{4} - A - F - G}$

Also we obtain the wave function $\chi(s)$ as [19]

$$\chi(s) = P_n^{c_{10}-1, \frac{c_{11}}{c_3} - c_{10} - 1}(1 - 2c_3 s), \tag{19}$$

Using equation (14) we get the function $\chi(s)$ as

$$\chi(s) = P_n^{(U,V)}(1 - 2s), \tag{20}$$

where $P_n^{(U,V)}$ are Jacobi polynomials

Lastly,

$$\varphi(s) = s^{c_{12}}(1 - c_3 s)^{-c_{12} - \frac{c_{13}}{c_3}}, \tag{21}$$

and using equation (14) we get

$$\varphi(s) = s^{U/2}(1 - s)^{V-1/2}, \tag{22}$$

We then obtain the radial wave function from the equation [19]

$$R_n(s) = N_n \varphi(s) \chi_n(s), \tag{23}$$

As

$$R_n(s) = N_n s^{U/2}(1 - s)^{(V-1)/2} P_n^{(U,V)}(1 - 2s), \tag{24}$$

where $n$ is a positive integer and $N_n$ is the normalization constant.

## 4. DISCUSSION

We have solved the radial Schrödinger equation and obtained the energy eigen values for the Manning-Rosen plus Yukawa potential (MRYP) in equation (16).

The following cases are considered:

Case 1: If $C = D = V_0{}' = 0$ in equation (10), the potential turns back into the Yukawa potential and equation (16) yields the energy eigen values of the Yukawa potential as

$$E = -\frac{\hbar^2}{2\mu}\left[\frac{\frac{2\mu V_0}{\hbar^2} - \alpha^2 (n+1)^2}{2(n+1)}\right]^2, \tag{25}$$

Equation (25) is similar to equation (30) of the Yukawa potential obtained by Ita *et al.*, 2015

Case 2: If $\propto \to 0, V_0 = Ze^2$ in equation (25), the energy eigen values for Coulomb potential becomes

$$E = -\frac{Z^2 e^4 \mu}{2\hbar^2 n'^2} \tag{26}$$

where $n' = n + 1$ in this case.

Case 3: If $V_0 = V_0{}' = 0$ the potential in equation (10) yields the Manning-Rosen potential with energy eigen values given as

$$E = -\frac{\alpha^2\hbar^2}{2\mu}\left\{\left[\frac{\frac{2\mu C}{\propto^2\hbar^2} - \left(n^2+n+\frac{1}{2}\right) - (2n+1)\sqrt{\frac{1}{4} - \frac{2\mu C}{\propto^2\hbar^2} - \frac{2\mu D}{\alpha^2\hbar^2}}}{(2n+1) + 2\sqrt{\frac{1}{4} - \frac{2\mu C}{\propto^2\hbar^2} - \frac{2\mu D}{\alpha^2\hbar^2}}}\right]^2\right\} \tag{27}$$

Eq. (27) is also similar to the bound state energy obtained by Louis *et al*., 2016 using the Manning-Rosen potential.

Case 4: If $C = D = V_0 = 0$; the potential in equation 10 yields the Inversely quadratic Yukawa potential with energy eigen value given as

$$E = -\frac{\alpha^2\hbar^2}{2\mu}\left\{\left[\frac{\left(n^2+n+\frac{1}{2}\right) + (2n+1)\sqrt{\frac{1}{4} - \frac{V_0{}'}{\hbar^2}}}{(2n+1) + 2\sqrt{\frac{1}{4} - \frac{V_0{}'}{\hbar^2}}}\right]^2\right\} \tag{28}$$

Case 5: If C = D = 0; the potential in equation 10 yields a class of the Yukawa potential with energy eigenvalue given as

$$E = -\frac{\alpha^2\hbar^2}{2\mu}\left\{\left[\frac{\frac{2\mu V_0}{\alpha\hbar^2} - \left(n^2+n+\frac{1}{2}\right) - (2n+1)\sqrt{\frac{1}{4} - \frac{V_0{}'}{\hbar^2}}}{(2n+1) + 2\sqrt{\frac{1}{4} - \frac{V_0{}'}{\hbar^2}}}\right]^2\right\} \tag{29}$$

Eq. (29) is similar to eq. (30) obtained by Antia *et al*., 2015 for the Yukawa potential bound state energies.

**Table 1.** Energy eigenvalues $E(eV)$ of the MRCYP potential for ђ = μ = 1, $V_0 = 0.2$, $V_0{}'$= 0.5, C = -0.5, D = 0.1 with different α values.

| $n$ | $\alpha = 0.01$ | $\alpha = 0.03$ | $\alpha = 0.1$ |
|---|---|---|---|
| 1 | −0.1582008 | −0.1624298 | −0.1803667 |
| 2 | −0.1603611 | −0.1692804 | −0.2071690 |
| 3 | −0.1625505 | −0.1763832 | −0.2365925 |
| 4 | −0.1647688 | −0.1837352 | −0.2686024 |
| 5 | −0.1670157 | −0.1913334 | −0.3031757 |
| 6 | −0.1692912 | −0.1991758 | −0.3402965 |
| 7 | −0.1715951 | −0.2072602 | −0.3799535 |

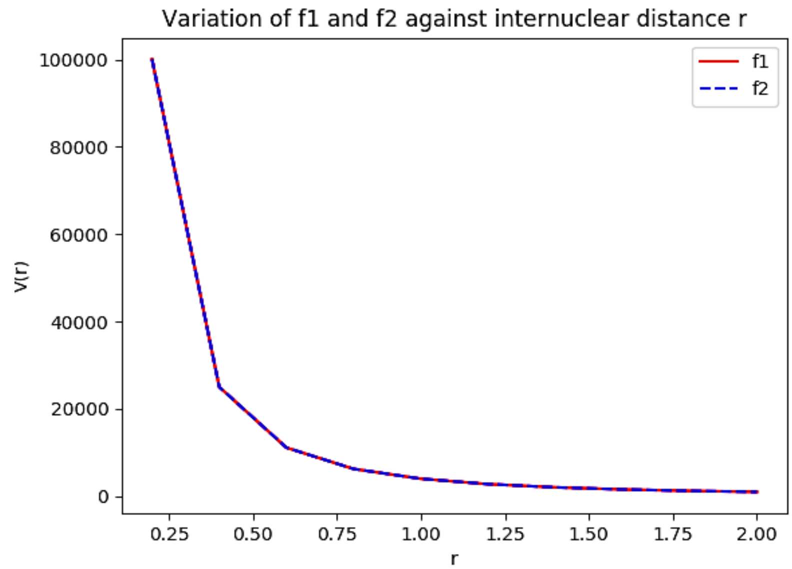


**Fig. 1.** Comparison of the centrifugal term f1 = $\frac{1}{r^2}$ with approximation f2 for α = 0.01

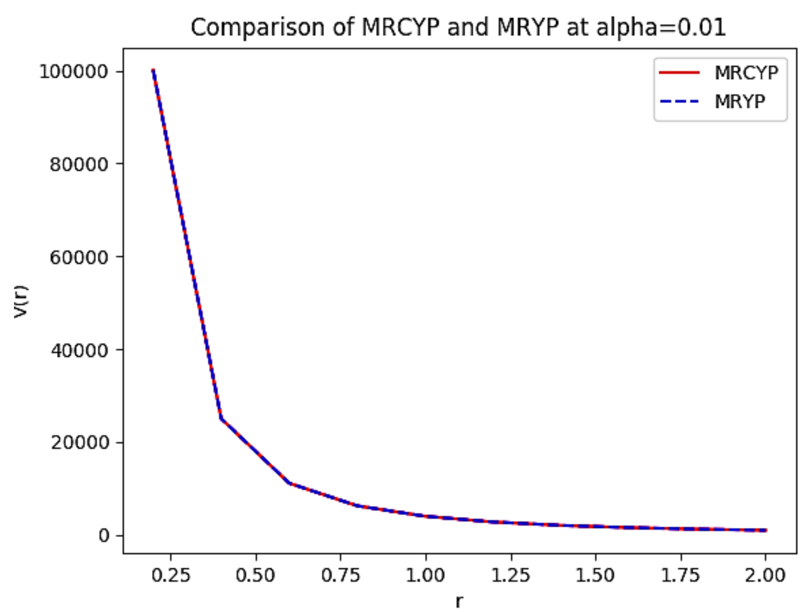


**Fig. 2.** The behaviour of the MRCYP potential compared with MRYP at alpha = 0.01

## 5. CONCLUSION

We have obtained the energy eigen values and the corresponding un-normalized wave function using the parametric NU method for the Schrödinger equation with MRYP. Special cases of the potential have also been considered.

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