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$^{\#}$gp-locally closed sets and $^{\#}$gp-locally closed functions

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ABSTRACT

The aim of this paper is to introduce and study $^{\#}$gp-locally closed sets. Basic characterizations and several properties concerning them are obtained. Further, $^{\#}$gp - locally closed function is also defined. Some of the properties are investigated.

Keywords**:** locally closed set; $^{\#}$g-locally closed set; $^{\#}$gp-closed set and $^{\#}$gp-locally closed set

1. INTRODUCTION

Norman Levine [7] introduced generalized closed (briefly g-closed) sets in 1970.A subset A of a topological space is said to be locally closed [4] if it is the intersection of an open set and a closed set. Ganster and Reilly [5] introduced and studied three different notations of generalized continuity, namely, LC-continuity, LC-irresoluteness and sub-LC-continuity.Balachandran et al [3] introduced the concept of generalized locally closed sets and obtained seven different notions of generalized continuities.Palaniappan and Rao [9] introduced regular generalized closed sets. Recently Arockiarani et al [2] introduced regular generalized locally closed sets and obtained six new classes of generalized continuous using the concept of regular generalized closed sets. M.K.R.S. Veerakumar [13,15] and [16] introduced $g^{\#}$-locally closed sets and $G^{\#}$LC-functions, $g^{\wedge}$-locally closed sets $G^{\wedge}$LC-functions and g^{*}-locally closed sets and G^{*}LC-functions respectively. The authours [1] introduced $^{\#}$gp-closed sets in topological spaces.

In this chapter we introduce #gp-locally closed sets and #gp-locally closed functions and study some of their properties.

2. PRELIMINARIES

Throughout this paper, (X,τ)(or X) represents topological spaces on which no separation axioms are assumed unless otherwise mentioned. A subset A of a space (X,τ), cl(A), int(A) and A^c denote the closure of A, the interior of A and the complement of A respectively. We recall the following definitions which are useful in the sequel.

Definition 2.1. A subset A of a space (X,τ) is called a

(i) **generalized closed** (brifly **g-closed**) set [7] if cl(A) ⊆U whenever A⊆U and U is open in (X,τ); the complement of a g-closed set is called a **g-open** [7] set.

(ii) **regular open** [6] set if A=int(cl(A)) and **regular closed** [6] set if cl(int(A))=A.

(iii) **regular generalized closed** (briefly **rg-closed**) set [9] if cl(A) ⊆U whenever A⊆U and U is regular open in (X,τ); the complement of a rg-closed set is called a **rg-open** [9] set.

iv) **α-generalized closed** (brifly **αg-closed**) set [8] if αcl(A) ⊆U whenever A⊆U and U is open in (X,τ); the complement of an αg-closed set is called an **αg-open** [8]set.

v) **$g^{\#}$-closed** set [12] if cl(A) ⊆U whenever A⊆U and U is αg-open in (X,τ); the complement of a $g^{\#}$-closed set is called a **$g^{\#}$-open** [12] set.

vi) **#gp-closed** set [1] if pcl(A)⊆U whenever A⊆U and U is αg--open in (X,τ); the complement of a #gp-closed set is called a **#gp-open** [1] set.

(vii) **g^-closed** set [14] if cl(A) ⊆U whenever A⊆U and U is semi open in (X,τ); the complement of a g^-closed set is called a **g^-open** [14] set.

(viii) **g*-closed** set [11] if cl(A) ⊆U whenever A⊆U and U is g-open in (X,τ); the complement of a g*-closed set is called a **g*-open** [11] set.

Definition 2.2. A subset S of a space (X,τ) is called a

(i) **regular generalized locally closed** (briefly **rglc**) set [2] if S=G∩F, where G is rg-open and F is rg-closed in (X,τ).

(ii) **rglc*** set [2] if there exist a rg-open set G and a closed set F of (X,τ) such that S=G∩F.

(iii) **rglc**** set [2] if there exist an open set G and a rg-closed set F of (X,τ) such that S=G∩F.

(iv) **generalized locally closed** (briefly **glc**) set [4] if S=G∩F, where G is g-open and F is g-closed in (X,τ). The class of all generalized locally closed sets in (X,τ) is denoted by GLC(X,τ).

(v) **GLC*** [4] set if there exist a g-open set G and a closed set F of (X,τ) such that S=G∩F

(vi) **GLC**** [4] set if there exist an open set G and a g-closed set F of (X,τ) such that S=G∩F.

(vii) **$g^{\#}$-locally closed** [13] (briefly **$g^{\#}$lc**) set if S=G∩F, where G is $g^{\#}$-open in (X,τ) and F is $g^{\#}$-closed in (X,τ). The class of all $g^{\#}$-locally closed sets in (X,τ) is denoted by $G^{\#}$LC(X,τ).

(viii) **$G^{\#}$LC*** [13] set if there exists a $g^{\#}$-open set G and a closed set F of (X,τ) such that S=G∩F.

(ix) **$G^{\#}$LC**** [13] set if there exists an open set G and a $g^{\#}$-closed set F of (X,τ) such that S=G∩F.

(x) **g*-locally closed** [16] (briefly **g*lc**) set if S=G∩F, where G is g*-open in (X,τ) and F is g*-closed in (X,τ). The class of all g*-locally closed sets in (X,τ) is denoted by G*LC(X,τ).

(xi) **G*LC*** [16] set if there exists a g*-open set G and a closed set F of (X,τ). such that S=G∩F.

(xii) **G*LC**** [16] set if there exists an open set G and a g*- closed set F of (X,τ) such that S=G∩F.

(xiii) **g^-locally closed** [15] (briefly **g^lc**) set if S=G∩F, where G is g^-open in (X,τ) and F is g^-closed in (X,τ). The class of all g^-locally closed sets in (X,τ) is denoted by G^LC(X,τ).

(xiv) **G^LC*** [15] set if there exists a g^-open set G and a closed set F of (X,τ). such that S=G∩F.

(xv) **G^LC**** [15] set if there exists an open set G and a g^-closed set F of (X,τ) such that S=G∩F.

Definition 2.3. A topological space (X,τ) is called

(i) **submaximal** if every dense subset is open and

(ii) **rg-submaximal** [2] if every dense subset is rg-open.

Definition 2.4. A function f:(X,τ) → (Y,σ) is called

(i) **LC-continuous** [5] if $f^{-1}(V) \in LC(X,\tau)$ for each open set V of (Y,σ).

(ii) **GLC-continuous** [4] if $f^{-1}(V) \in GLC(X,\tau)$ for each open set V of (Y,σ).

(iii) **GLC*-continuous** [4] if $f^{-1}(V) \in GLC^*(X,\tau)$ for each open set $V \in (Y,\sigma)$.

(iv) **GLC**-continuous** [4] if $f^{-1}(V) \in GLC^{**}(X,\tau)$ for each open set $V \in (Y,\sigma)$.

(v) **G^LC-continuous** [15] if $f^{-1}(V) \in G\hat{}LC(X,\tau)$ for each open set V of (Y,σ).

(vi) **G^LC*-continuous** [15] if $f^{-1}(V) \in G\hat{}LC^*(X,\tau)$ for each open set $V \in (Y,\sigma)$.

(vii) **G^LC**-continuous** [15] if $f^{-1}(V) \in G\hat{}LC^{**}(X,\tau)$ for each open set $V \in (Y,\sigma)$.

(viii) **$G^{\#}$LC-continuous** [13] if $f^{-1}(V) \in G^{\#}LC(X,\tau)$ for each open set V of (Y,σ).

(ix) **$G^{\#}$LC*-continuous** [13] if $f^{-1}(V) \in G^{\#}LC^*(X,\tau)$ for each open set $V \in (Y,\sigma)$.

(x) **$G^{\#}$LC**-continuous** [13] if $f^{-1}(V) \in G^{\#}LC^{**}(X,\tau)$ for each open set $V \in (Y,\sigma)$.

(xi) **G*LC-continuous** [16] if $f^{-1}(V) \in G^*LC(X,\tau)$ for each open set V of (Y,σ).

(xii) **G*LC*-continuous** [16] if $f^{-1}(V) \in G^*LC^*(X,\tau)$ for each open set $V \in (Y,\sigma)$.

(xiii) **G*LC**-continuous** [16] if $f^{-1}(V) \in G^*LC^{**}(X,\tau)$ for each open set $V \in (Y,\sigma)$.

3. $^{\#}$GP-LOCALLY CLOSED SETS AND SOME OF THEIR PROPERTIES

In this section we study $^{\#}$gp-locally closed sets and some of their properties. We introduce the following definition.

Definition 3.1. A subset S of a space (X,τ) is called **$^{\#}$g-pre locally closed** if S=G∩F, where G is $^{\#}$gp -open and F is $^{\#}$gp -closed in (X,τ).
The class of all $^{\#}$g-pre locally closed sets in (X,τ) is denoted by $^{\#}$GPLC(X,τ).

Definition 3.2. For a subset S of (X,τ), **$S \in {}^{\#}GPLC^{*}(X,\tau)$** if there exists a $^{\#}$gp -open set G and a closed set F of (X,τ) such that S=G∩F.

Definition 3.3. For a subset S of (X,τ), **$S \in {}^{\#}GPLC^{**}(X,\tau)$** if there exist an open set G and a $^{\#}$gp-closed set F of (X,τ) such that S=G∩F.

Proposition 3.4.

i. If $S \in GLC^*(X,\tau)$, then $S \in {}^\#GPLC(X,\tau)$, $S \in {}^\#GPLC^*(X,\tau)$ and $S \in {}^\#GPLC^{**}(X,\tau)$.

ii. If $S \in LC(X,\tau)$, then $S \in {}^\#GPLC(X,\tau)$, $S \in {}^\#GPLC^*(X,\tau)$ and $S \in {}^\#GPLC^{**}(X,\tau)$.

iii. If $S \in {}^\#GLC(X,\tau)$ [resp. ${}^\#GLC^*(X,\tau)$ and ${}^\#GLC^{**}(X,\tau)$] , then $S \in {}^\#GPLC(X,\tau)$, $S \in {}^\#GPLC^*(X,\tau)$ and $S \in {}^\#GPLC^{**}(X,\tau)$.

iv. If $S \in G^\wedge LC(X,\tau)$ [resp.$G^\wedge LC^*(X,\tau)$ and $G^\wedge LC^{**}(X,\tau)$],then $S \in {}^\#GPLC(X,\tau)$, $S \in {}^\#GPLC^*(X,\tau)$ and $S \in {}^\#GPLC^{**}(X,\tau)$.

v. If $S \in rglc^*(X,\tau)$ [resp. $rglc^{**}(X,\tau)$], then $S \in {}^\#GPLC(X,\tau)$, $S \in {}^\#GPLC^*(X,\tau)$ and $S \in {}^\#GPLC^{**}(X,\tau)$.

vi. If $S \in G^*LC(X,\tau)$ [resp. $G^*LC^*(X,\tau)$ and $G^*LC^{**}(X,\tau)$], then $S \in {}^\#GPLC(X,\tau)$, $S \in {}^\#GPLC^*(X,\tau)$ and $S \in {}^\#GPLC^{**}(X,\tau)$.

The proof is obvious from the definitions 2.2, 3.1, 3.2 and 3.3.

The converses of the proposition 3.4 need not be true as can be seen from the following examples.

Example 3.5.

Let $X=\{a,b,c\}$ and
$\tau=\{X,\Phi,\{a\},\{a,b\}\}$
${}^\#GPLC = P(X)$
${}^\#GPLC^* = P(X)$
${}^\#GPLC^{**} = P(X)$
$GLC^* =\{X,\Phi,\{a\},\{b\},\{c\}\{a,b\},\{b,c\}\}$
Let $A=\{a,c\} \in {}^\#GPLC$, ${}^\#GPLC^*$, and ${}^\#GPLC^{**}$, but $\{a,c\} \notin GLC$*****

Example 3.6.

Let $X=\{a,b,c\}$ and
$\tau=\{X,\Phi,\{a\},\{b,c\}\}$,
$G^\#PLC = P(X)$
$G^\#PLC^* = P(X)$
$G^\#PLC^{**}=P(X)$
$LC =\{X,\Phi,\{a\},\{b,c\}\}$
Let $A=\{a,b\} \in {}^\#GPLC$, ${}^\#GPLC^*$ and ${}^\#GPLC^{**}$, but $\{a,b\} \notin LC$.

Example 3.7.

X and τ be as in the example 3.6
${}^\#GPLC = P(X)$

$^{\#}GPLC^{*}$ = P(X)
$^{\#}GPLC^{**}$ = P(X)
$^{\#}GLC$ = {X,Φ,{a},{b,c}}
$^{\#}GLC^{*}$ = {X,Φ,{a},{b,c}}
$^{\#}GLC^{**}$ = {X,Φ,{a},{b,c}}
Let A = {a,c} ∈ $^{\#}GPLC$, $^{\#}GPLC^{*}$ and $^{\#}GPLC^{**}$, but{a,c} ∉ $^{\#}GLC$, $^{\#}GLC^{*}$ and $^{\#}GLC^{**}$.

Example 3.8.

Let X={a,b,c} and
τ={X,Φ,{a}}
$^{\#}GPLC$ = P(X)
$^{\#}GPLC^{*}$ = P(X)
$^{\#}GPLC^{**}$ = P(X)
$G^{\wedge}LC$ = {X,Φ,{a},{b,c}}
$G^{\wedge}LC^{*}$ = {X,Φ,{a},{b,c}}
$G^{\wedge}LC^{**}$ = {X,Φ,{a},{b,c}}
Let A={b} ∈ $^{\#}GPLC$, $^{\#}GPLC^{*}$ and $^{\#}GPLC^{**}$, but {b} ∉ $G^{\wedge}LC$, $G^{\wedge}LC^{*}$ and $G^{\wedge}LC^{**}$.

Example 3.9.

Let X={a,b,c} and
τ={X,Φ,{a},{b},{a,b }}
#GPLC = P(X)
$^{\#}GPLC^{*}$ = P(X)
$^{\#}GPLC^{**}$ = P(X)
$rglc^{*}$ = {X,Φ,{c},{a,c},{b,c}}
Let A={b} ∈ $^{\#}GPLC$, $^{\#}GPLC^{*}$ and $^{\#}GPLC^{**}$, but {b} ∉ $rglc^{*}$.

Example 3.10.

Let X and τ be as in the example 3.9
$^{\#}GPLC$ = P(X)
$^{\#}GPLC^{*}$ = P(X)
$^{\#}GPLC^{**}$ = P(X)
$rglc^{**}$ = {X,Φ,{a},{b}, {a,b}}
Let A={c} ∈ $^{\#}GPLC$, $^{\#}GPLC^{*}$ and $^{\#}GPLC^{**}$, but {c} ∉ $rglc^{**}$.

Example 3.11.

Let X and τ be as in the example 3.8
$^{\#}GPLC$ = P(X)
$^{\#}GPLC^{*}$ = P(X)
$^{\#}GPLC^{**}$ = P(X)
$G^{*}LC$ = {X,Φ,{a},{b,c}}
$G^{*}LC^{*}$ = {X,Φ,{a},{b,c}}

$G^{*}LC^{**}$ = {X,Φ,{a},{b,c}}
Let A={b} ∈ $^{\#}GPLC$, $^{\#}GPLC^{*}$ and $^{\#}GPLC^{**}$, but {b} ∉ $G^{*}LC$, $G^{*}LC^{*}$ and $G^{*}LC^{**}$.

Theorem 3.12.

For a subset S of (X,τ) the following are equivalent
(i) S ∈ $^{\#}$**G**PLC*(X,τ).
(ii) S=P∩cl(S) for some $^{\#}$gp -open set P.
(iii) cl(S)-S is $^{\#}$gp-closed.
(iv) SU(X-cl(S)) is $^{\#}$gp-open.

Proof:

(i)=> (ii) Let S ∈ $^{\#}GPLC^{*}$(X,τ).
Then there exist a $^{\#}$gp -open set P and a closed set F in (X,τ) such that S=P∩F.
Since S ⊆ P and S ⊆ cl(S), we have S ⊆ P∩cl(S).
Conversely, since cl(S) ⊆ F, P∩cl(S) ⊆ P∩F
=S, we have that S=P∩cl(S).
(ii) => (i) Since P is $^{\#}$gp -open and cl(S) is closed, we have P∩cl(S) ∈ $^{\#}GPLC^{*}$(X,τ)
(iii) => (iv) Let F=cl(S)-S.
By assumption F is $^{\#}$gp -closed.
Now X-F=X∩F^{c}
=X∩$(cl(S)-S)^{c}$
= SU(X-cl(S)).
Since X-F is $^{\#}$gp -open, we have that SU(X-cl(S)) is $^{\#}$gp-open.
(iv) =>(iii) Let U=SU(X-cl(S)).
By assumption U is $^{\#}$gp -open.
Then X-U is $^{\#}$gp -closed.
Now X-U=X-(SU(X-cl(S))
=cl(S)∩(X-S)
=cl(S)-S.
Therefore cl(S)-S is $^{\#}$gp-closed.
(iv) => (ii) Let U=SU(X-cl(S)).
By assumption, U is $^{\#}$gp-open.
Now U∩cl(S)= SU(X-cl(S))∩cl(S)
=(cl(S)∩S)U(cl(S)∩(X-cl(S)))
=SUΦ
=S.
Therefore S=U∩cl(S) for the $^{\#}$gp-open set U.
(ii) => (iv) Let S=P∩cl(S) for some $^{\#}$gp-open set P.
Now SU(X-cl(S))=P∩cl(S)U(X-cl(S))
=P∩(cl(S)U(X-cl(S)))
=P∩X
=P is $^{\#}$gp-open.
We introduce the following definition.

Definition 3.13. A topological space (X,τ) is called **#gp-submaximal** if every dence set is #gp-open.

Theorem 3.14.

Let (X,τ) be a topological space. Then
(i) If (X,τ) is submaximal, then it is #gp -submaximal.
(ii) If (X,τ) is rg-submaximal, then it is #gp -submaximal.

Proof:

(i) Given (X,τ) is submaximal
To prove (X,τ) is #gp -submaximal
Let A be a dense set of (X,τ) .
Since (X,τ) is submaximal ,then A is an open set of (X,τ)
By theorem 3.2 [1] every open set is #gp -open, then A is a #gp - open set of (X,τ)
Thus we get every dense set of (X,τ) is a #gp - open
Hence (X,τ) is #gp -submaximal.

(ii) Given (X,τ) is rg- submaximal
To prove (X,τ) is #gp-submaximal
Let A be a dense set of (X,τ) .
Since (X,τ) is rg- submaximal ,then A is a rg-open set of (X,τ)
Since every rg-open set is #gp -open, then A is a #gp - open set of (X,τ)
Thus we get every dense set of (X,τ) is a #gp - open
Hence (X,τ) is #gp-submaximal.

Remark 3.15.

The Converses of the theorem 3.14 need not be true as can be seen from the following examples.

Example 3.16.

Let X={a,b,c} and
τ={Φ,X,{a},{a,c}}.
Closed sets of (X,τ) = {Φ,X,{b},{b,c}}
Dense sets of (X,τ) = {Φ,X,{a},{c},{a,b},{a,c}}
#gp -open sets of (X,τ) = {Φ,X,{a},{c},{a,b},{a,c}}
Here every dense set of (X,τ) is a #gp -open .
Hence (X,τ) is #gp -submaximal. But it is not submaximal, since {c} is a dense set of (X,τ) but it is not open

Example 3.17.

Let X and τ be as in the example 3.9,
rg-open sets of (X,τ) = {Φ,X,{c},{b,c},{a,c}}
Dense sets of (X,τ) = {Φ,X,{a},{b},{a,b}}
$^{\#}$gp -open sets of (X,τ) = {Φ,X,{a},{b},{a,b}}
Here every dense set of (X,τ) is a $^{\#}$gp - open .
Hence (X,τ) is $^{\#}$gp -submaximal. But it is not rg- submaximal,since {a} is a dense set of (X,τ) but it is not rg-open

Corollary 3.18.

A topological space (X,τ) is $^{\#}$gp -submaximal⇔$^{\#}$GPLC*(X,τ)=P(X).

Proof:

Necessity- Let S∈P(X).
Let U=SU(X-cl(S)).
Then cl(U)=X.
Therefore U is dense in (X,τ)
Since (X,τ) is $^{\#}$gp -submaximal, U is $^{\#}$gp -open.
By the theorem 3.12, S∈$^{\#}$GPLC*(X,τ) .

Sufficiency- Let S be dense subset of (X,τ).
Then SU(X-cl(S))=SUΦ
=S.
Since S∈$^{\#}$GPLC*(X,τ), by the theorem 3.12 again, S is $^{\#}$gp -open in (X,τ).

Theorem 3.19.

For a subset S of (X,τ),if S∈$^{\#}$GPLC**(X,τ), then there exists an open set Q such that S=Q∩cl$^{\#}$p(S), where cl$^{\#}$p(S) is the $^{\#}$gp-closure of S (i.e) the intersection of all $^{\#}$gp-closed subsets of (X,τ) that contain S.

Proof:

Let S∈$^{\#}$GPLC**(X,τ).
Then there exists an open set Q and a $^{\#}$gp -closed set F of (X,τ) such that S=Q∩F.
Since S⊆Q and S⊆cl$^{\#}$p(S), we have S⊆Q∩cl$^{\#}$p(S).
Since cl$^{\#}$p(S) ⊆ F, We have Q∩cl$^{\#}$p(S) ⊆ Q∩F=S.
Thus S=Q∩cl$^{\#}$p(S).

Proposition 3.20.

Let A and B be any two subsets of (X,τ). If A∈$^{\#}$GPLC(X,τ) and B is $^{\#}$gp-open or $^{\#}$gp -closed, then A∩B∈$^{\#}$GPLC(X,τ).

Proof:

$A \in {}^{\#}GPLC(X,\tau)$ implies that $A \cap B=(G \cap F) \cap B$ for some ${}^{\#}gp$ -open set G and for some ${}^{\#}gp$ -closed set F.
If B is ${}^{\#}gp$ -open, then $G \cap B$ is ${}^{\#}gp$ -open.
Then $A \cap B=(G \cap B) \cap F \in {}^{\#}GPLC(X,\tau)$.
If B is ${}^{\#}gp$ -closed, then $A \cap B=G \cap (F \cap B) \in {}^{\#}GPLC(X,\tau)$, since $F \cap B$ is ${}^{\#}gp$ -closed.

Theorem 3.21.

Let A and B be any two subsets of (X,τ). If $A,B \in {}^{\#}GPLC^{*}(X,\tau)$, then $A \cap B \in {}^{\#}GPLC^{*}(X,\tau)$.

Proof:

Let $A,B \in {}^{\#}GPLC^{*}(X,\tau)$
Then there exists ${}^{\#}gp$ -open sets P and Q such that $A=P \cap cl(A)$ and $B=Q \cap cl(B)$ by the theorem 3.12.
$P \cap Q$ is also ${}^{\#}gp$ -open.
Then $A \cap B=(P \cap Q) \cap (cl(A) \cap cl(B)) \in {}^{\#}GPLC^{*}(X,\tau)$.

Theorem 3.22.

Let A and B be any two subsets of (X,τ). If $A \in {}^{\#}GPLC^{**}(X,\tau)$ and B is closed or open, then $A \cap B \in {}^{\#}GPLC^{**}(X,\tau)$.

Proof:

Let $A \in {}^{\#}GPLC^{**}(X,\tau)$
Then there exists an open set G and a ${}^{\#}gp$-closed set F of (X,τ) such that $A \cap B=G \cap (F \cap B)$. If B is open, then $A \cap B=(G \cap B) \cap F \in {}^{\#}GPLC^{**}(X,\tau)$.
If B is closed, then $A \cap B=G \cap (B \cap F) \in {}^{\#}GPLC^{**}(X,\tau)$.

Theorem 3.23.

Let A and Z be any two subsets of (X,τ) and let $A \subseteq Z$. If Z is ${}^{\#}gp$-open in (X,τ) and $A \in {}^{\#}GPLC^{*}(Z,\tau/Z)$, then $A \in {}^{\#}GPLC^{*}(X,\tau)$.

Proof:

Let $A \in {}^{\#}GPLC^{*}(Z,\tau/Z)$
Then there exists a ${}^{\#}gp$ -open set G in $(Z,\tau/Z)$ such that $A=G \cap cl_z(A)$, where $cl_z(A)=Z \cap cl(A)$. Since G and Z are ${}^{\#}gp$ -open, then $G \cap Z$ is ${}^{\#}gp$-open.
This implies that $A=(Z \cap G) \cap cl(A) \in {}^{\#}GPLC^{*}(X,\tau)$.

Theorem 3.24.

If Z is $^{\#}$gp -closed, open in (X,τ) and A∈$^{\#}$GPLC*(Z,τ/Z), then A∈$^{\#}$GPLC(X,τ).

Proof:

Let A∈$^{\#}$GPLC*(Z,τ/Z).
Then A=G∩F for some $^{\#}$gp-open set G in (Z,τ/Z) and for some closed set F in (Z,τ/Z). Since F is closed in (Z,τ/Z), F=B∩Z for some closed set B of (X,τ).
Now F is the intersection of $^{\#}$gp -closed sets B and Z in (X,τ).
So F is also $^{\#}$gp -closed in (X,τ).
Therefore, A=G∩(B∩Z) ∈$^{\#}$GPLC(X,τ).

Theorem 3.25.

If Z is closed and open in (X,τ) and A∈$^{\#}$GPLC(Z,τ/Z), then A∈$^{\#}$GPLC(X,τ).

Proof:

Let A∈$^{\#}$GPLC(Z,τ/Z).
Then there exist a $^{\#}$gp -open set G and a $^{\#}$gp -closed set F of (Z,τ/Z) such that A=G∩F. Since F is $^{\#}$gp -closed set in (Z,τ/Z), there exists a closed set B of (X,τ) such that F=B∩Z. Clearly F is $^{\#}$gp -closed in (X,τ).
Now A=G∩B∩Z∈$^{\#}$GPLC(X,τ).

Theorem 3.26.

If Z is g$^{\#}$-closed, open subset of (X,τ) and A∈$^{\#}$GPLC**(Z,τ/Z), then A∈$^{\#}$GPLC**(X,τ).

Proof:

Let A∈$^{\#}$GPLC**(Z,τ/Z).
Then A=G∩F, where G is open and F is $^{\#}$gp -closed in (Z,τ/Z).
Since Z is $^{\#}$gp -closed, we have F is $^{\#}$gp -closed in (X,τ).
Therefore A∈$^{\#}$GPLC**(X,τ).

Proposition 3.27.

Let A and B be any two subsets of (X,τ). Let A , B∈$^{\#}$GPLC* (X,τ). If A and B are separated in (X,τ), then AUB∈$^{\#}$GPLC* (X,τ).

Proof:

Let A , B∈$^{\#}$GPLC* (X,τ).
By the theorem 3.12, there exist $^{\#}$gp -open sets G and H of (X,τ) such that A=G∩cl(A) and B=H∩cl(B).

Now $U=G\cap(X\text{-}cl(B))$ and $V=H\cap(X\text{-}cl(A))$ are $^{\#}$gp-open subsets of (X,τ).
Then $U\cap V$ is also $^{\#}$gp -open set in (X,τ).
It is clear that $A=U\cap cl(A)$,
$B=V\cap cl(B)$,
$U\cap cl(B)=\Phi$,
$V\cap cl(A)=\Phi$.
Consequently, $AUB=(UUV)\cap(cl(AUB))$,
Showing that $AUB\in {}^{\#}GPLC^{*}(X,\tau)$.

Theorem 3.28.

Let $\{Z_i: i\in\Lambda\}$ be a finite $^{\#}$gp -closed set of (X,τ) and let A be a subset of (X,τ). If $A\cap Z_i\in {}^{\#}GPLC^{**}(Z_i ,\tau/Z_i)$ for every $i\in\Lambda$,then $A\in {}^{\#}GPLC^{**}(X,\tau)$.

Proof:

For each $i\in\Lambda$, there exist an open set $U_i\in\tau$ and a $^{\#}$gp -closed set F_i of $(Z_i ,\tau/Z_i)$ such that $A\cap Z_i=U_i\cap(Z_i\cap F_i)$.
Then $A=U\{A\cap Z_i : i\in\Lambda\}$
$=[U\{U_i: i\in\Lambda\}]\cap[U\{Z_i\cap F_i: i\in\Lambda\}]$.
This shows that $A\in {}^{\#}GPLC^{**}(X,\tau)$.

Theorem 3.29.

Let (X,τ) and (Y,σ) be any two topological spaces. Then,
(i) If $A\in {}^{\#}GPLC(X,\tau)$ and $B\in {}^{\#}GPLC(X,\tau)$, then $AXB\in {}^{\#}GPLC(XXY,\tau X\sigma)$.
(ii) If $A\in {}^{\#}GPLC^{*}(X,\tau)$ and $B\in {}^{\#}GPLC^{*}(X,\tau)$, then $AXB\in {}^{\#}GPLC^{*}(XXY,\tau X\sigma)$.
(iii) If $A\in {}^{\#}GPLC^{**}(X,\tau)$ and $B\in {}^{\#}GPLC^{**}(X,\tau)$, then $AXB \in {}^{\#}GPLC^{**}(XXY,\tau X\sigma)$.

Proof:

(i) Let $A\in {}^{\#}GPLC(X,\tau)$ and $B\in {}^{\#}GPLC(X,\tau)$.
Then there exist $^{\#}$gp-open sets V and V' of (X,τ) and (Y,σ) respectively and $^{\#}$gp - closed sets W and W' of (X,τ) and (Y,σ) respectively such that $A=V\cap W$ and $B=V'\cap W'$. Then $AXB=(V\cap W)X(V'\cap W')=(VXV')\cap(WXW')$ holds and hence $AXB\in {}^{\#}GPLC(XXY,\tau X\sigma)$.
(ii) Let $A\in {}^{\#}GPLC^{*}(X,\tau)$ and $B\in {}^{\#}GPLC^{*}(X,\tau)$.
Then there exist $^{\#}$gp -open sets V and V' of (X,τ) and (Y,σ) respectively and closed sets W and W' of (X,τ) and (Y,σ) respectively such that $A=V\cap W$ and $B=V'\cap W'$. Then $AXB=(V\cap W)X(V'\cap W')=(VXV')\cap(WXW')$ holds and hence $AXB\in {}^{\#}GPLC^{*}(XXY,\tau X\sigma)$.
(iii) Let $A\in {}^{\#}GPLC^{**}(X,\tau)$ and $B\in {}^{\#}GPLC^{**}(X,\tau)$.
Then there exist an open sets V and V' of (X,τ) and (Y,σ) respectively and $^{\#}$gp-closed sets W and W' of (X,τ) and (Y,σ) respectively such that $A=V\cap W$ and $B=V'\cap W'$. Then $AXB=(V\cap W)X(V'\cap W')=(VXV')\cap(WXW')$ holds and hence $AXB\in {}^{\#}GPLC^{**}(XXY,\tau X\sigma)$.

Remark 3.30.

The following diagram shows the relationships between $^{\#}$gp -locally closed sets and some other sets.

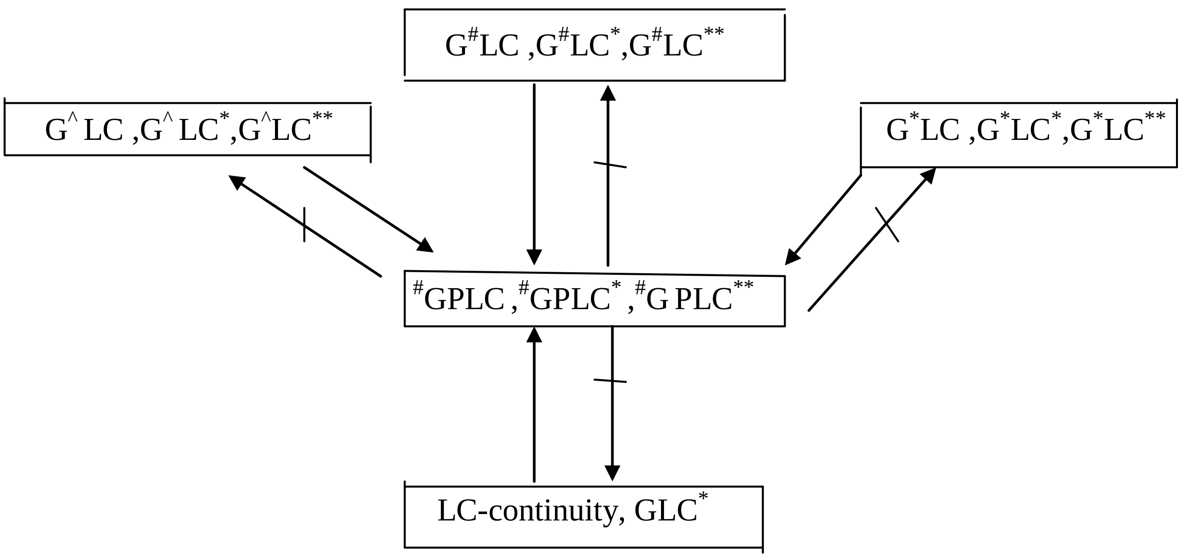

Figure 1

Where: A $\rightarrow$ B (A $\nrightarrow$ B) represents A implies B (A does not imply B).

4. $^{\#}$GP -LOCALLY CLOSED FUNCTIONS AND SOME OF THEIR PROPERTIES

In this section, the concept of $^{\#}$gp -locally closed function have been introduced and investigated the relation between $^{\#}$gp-locally closed functions and some other locally closed functions.

Definition 4.1. A function f: $(X,\tau)\rightarrow(Y,\sigma)$ is called $^{\#}$**GPLC-irresolute** (resp.$^{\#}$**GPLC*-irresolute,** $^{\#}$**GPLC**-irresolute**) if $f^{-1}(V) \in {}^{\#}GPLC(X,\tau)$ (resp. $f^{-1}(V) \in {}^{\#}GPLC^{*}(X,\tau)$, $f^{-1}(V) \in {}^{\#}GPLC^{**}(X,\tau)$) for each $V \in {}^{\#}GPLC(Y,\sigma)$ (resp.$V \in {}^{\#}GPLC^{*}(Y,\sigma)$, $V \in {}^{\#}GPLC^{**}(Y,\sigma)$.

Definition 4.2. A function f: $(X,\tau)\rightarrow(Y,\sigma)$ is called $^{\#}$**GPLC-continuous** (resp.$^{\#}$**GPLC*-continuous,** $^{\#}$**GPLC**-continuous**) if $f^{-1}(V) \in {}^{\#}GPLC(X,\tau)$ (resp. $f^{-1}(V) \in {}^{\#}GPLC^{*}(X,\tau)$, $f^{-1}(V) \in {}^{\#}GPLC^{**}(X,\tau)$) for each open set V of (Y,σ).

Proposition 4.3.

(i) If f is GLC*-continuous, then it is $^{\#}$GPLC, $^{\#}$GPLC* and $^{\#}$GPLC**-continuous.

(ii) If f is LC-continuous, then it $^{\#}$GPLC, $^{\#}$GPLC* and $^{\#}$GPLC**-continuous.

(iii) If f is $G^{\#}LC$ [resp. $G^{\#}LC^{*}$ and $G^{\#}LC^{**}$] -continuous, then it is $^{\#}GPLC$, $^{\#}GPLC^{*}$ and $^{\#}GPLC^{**}$-continuous.

(iv) If f is $G^{\wedge}LC$ [resp. $G^{\wedge}LC^{*}$ and $G^{\wedge}LC^{**}$]-continuous, then it is $^{\#}GPLC$, $^{\#}GPLC^{*}$ and $^{\#}GPLC^{**}$-continuous.

(v) If f is $G^{*}LC$ [resp. $G^{*}LC^{*}$ and $G^{*}LC^{**}$]-continuous, then it is $^{\#}GPLC$, $^{\#}GPLC^{*}$ and $^{\#}GPLC^{**}$-continuous.

Proof:

(i) Let f: $(X,\tau)\rightarrow(Y,\sigma)$ be GLC^{*}-continuous
To prove f: $(X,\tau)\rightarrow(Y,\sigma)$ is $^{\#}GPLC$ [resp. $^{\#}GPLC^{*}$ and $^{\#}GPLC^{**}$] - continuous
Let V be an open set of (Y,σ)
Since f: $(X,\tau)\rightarrow(Y,\sigma)$ is GLC^{*}-continuous, then $f^{-1}(V) \in GLC^{*}(X,\tau)$
By proposition 3.4(i), $f^{-1}(V) \in {}^{\#}GPLC(X,\tau)$ [resp. $^{\#}GPLC^{*}$ and $^{\#}GPLC^{**}$]
Therefore f: $(X,\tau)\rightarrow(Y,\sigma)$ is $^{\#}GPLC$ [resp. $^{\#}GPLC^{*}$ and $^{\#}GPLC^{**}$]-continuous

(ii) Let f: $(X,\tau)\rightarrow(Y,\sigma)$ be LC-continuous
To prove f: $(X,\tau)\rightarrow(Y,\sigma)$ is $^{\#}GPLC$ [resp. $^{\#}GPLC^{*}$ and $^{\#}GPLC^{**}$] - continuous]
Let V be an open set of (Y,σ)
Since f: $(X,\tau)\rightarrow(Y,\sigma)$ is LC-continuous,then $f^{-1}(V) \in LC(X,\tau)$
By proposition 3.4(ii), $f^{-1}(V) \in {}^{\#}GPLC(X,\tau)$ [resp. $^{\#}GPLC^{*}$ and $^{\#}GPLC^{**}$]
Therefore f: $(X,\tau)\rightarrow(Y,\sigma)$ is $^{\#}GPLC$ [resp. $^{\#}GPLC^{*}$ and $^{\#}GPLC^{**}$]-continuous

(iii) Let f: $(X,\tau)\rightarrow(Y,\sigma)$ be $G^{\#}LC$-continuous [resp.$G^{\#}LC^{*}$ and $G^{\#}LC^{**}$]- continuous
To prove f: $(X,\tau)\rightarrow(Y,\sigma)$ is $^{\#}GPLC$ [resp. $^{\#}GPLC^{*}$ and $^{\#}GPLC^{**}$] - continuous]
Let V be an open set of (Y,σ)
Since f: $(X,\tau)\rightarrow(Y,\sigma)$ is $G^{\#}LC$ [resp.$G^{\#}LC^{*}$ and $G^{\#}LC^{**}$]-continuous,then
$f^{-1}(V) \in G^{\#}LC(X,\tau)$ [resp.$G^{\#}LC^{*}$ and $G^{\#}LC^{**}$]
By proposition 3.4(iii), $f^{-1}(V) \in {}^{\#}GP\,LC(X,\tau)$ [resp. $^{\#}GPLC^{*}$ and $^{\#}GPLC^{**}$]
Therefore f: $(X,\tau)\rightarrow(Y,\sigma)$ is $^{\#}GPLC$ [resp. $^{\#}GPLC^{*}$ and $^{\#}GPLC^{**}$]-continuous

(iv) Let f: $(X,\tau)\rightarrow(Y,\sigma)$ be $G^{\wedge}LC$-continuous [resp.$G^{\wedge}LC^{*}$ and $G^{\wedge}LC^{**}$]- continuous
To prove f: $(X,\tau)\rightarrow(Y,\sigma)$ is $^{\#}GPLC$ [resp. $^{\#}GPLC^{*}$ and $^{\#}GPLC^{**}$] - continuous]
Let V be an open set of (Y,σ)
Since f: $(X,\tau)\rightarrow(Y,\sigma)$ is $G^{\wedge}LC$ [resp.$G^{\wedge}LC^{*}$ and $G^{\wedge}LC^{**}$]-continuous,then
$f^{-1}(V) \in G^{\wedge}LC(X,\tau)$ [resp.$G^{\wedge}LC^{*}$ and $G^{\wedge}LC^{**}$]
By proposition 3.4(iv), $f^{-1}(V) \in {}^{\#}GPLC(X,\tau)$ [resp. $^{\#}GPLC^{*}$ and $^{\#}GPLC^{**}$]
Therefore f: $(X,\tau)\rightarrow(Y,\sigma)$ is $^{\#}GPLC$ [resp. $^{\#}GPLC^{*}$ and $^{\#}GPLC^{**}$]-continuous

(v) Let f: $(X,\tau)\rightarrow(Y,\sigma)$ be $G^{*}LC$-continuous [resp.$G^{*}LC^{*}$ and $G^{*}LC^{**}$]- continuous
To prove f: $(X,\tau)\rightarrow(Y,\sigma)$ is $^{\#}GPLC$ [resp. $^{\#}GPLC^{*}$ and $^{\#}GPLC^{**}$] - continuous
Let V be an open set of (Y,σ)
Since f: $(X,\tau)\rightarrow(Y,\sigma)$ is $G^{*}LC$ [resp.$G^{*}LC^{*}$ and $G^{*}LC^{**}$]-continuous,then
$f^{-1}(V) \in G^{*}LC(X,\tau)$ [resp.$G^{*}LC^{*}$ and $G^{*}LC^{**}$]

By proposition 3.4(vi), $f^{-1}(V) \in {}^{\#}GPLC\ (X,\tau)$ [resp. $^{\#}GPLC^{*}$ and $^{\#}GPLC^{**}$]
Therefore f: $(X,\tau)\to(Y,\sigma)$ is $^{\#}GPLC$ [resp. $^{\#}GPLC^{*}$ and $^{\#}GPLC^{**}$]-continuous

The converses of the proposition 4.3 need not be true as seen from the following examples.

Example 4.4.

Let X={a,b,c}=Y
$\tau=\{\Phi,X,\{a,b\}\}$ and
$\sigma=\{\Phi,Y,\{b,c\}\}$.
Define f: $(X,\tau)\to(Y,\sigma)$ by f(a)=a, f(b)=b and f(c)=c.
GLC^{*} sets of (X,τ) $=\{X,\Phi,\{a\},\{b\},\{c\},\{a,b\}\}$
LC sets of (X,τ) $=\{X,\Phi,\{c\},\{a,b\}\}$
$^{\#}GPLC$ sets of (X,τ) =P(X)
$^{\#}GPLC^{*}$ sets of (X,τ)=P(X)
$^{\#}GPLC^{**}$ sets of (X,τ)=P(X)
$f^{-1}(\Phi) = \Phi \in {}^{\#}GPLC\ (X,\tau)$ [resp. $^{\#}GPLC^{*}(X,\tau)$ and $^{\#}GPLC^{**}(X,\tau)$]
$f^{-1}(Y) = X \in {}^{\#}GPLC\ (X,\tau)$ [resp. $^{\#}GPLC^{*}(X,\tau)$ and $^{\#}GPLC^{**}(X,\tau)$]
$f^{-1}(\{b,c\}) = \{b,c\} \in {}^{\#}GPLC\ (X,\tau)$ [resp. $^{\#}GPLC^{*}(X,\tau)$ and $^{\#}GPLC^{**}(X,\tau)$]
Thus we get $f^{-1}(V) \in {}^{\#}GPLC\ (X,\tau)$ [resp. $^{\#}GPLC^{*}(X,\tau)$ and $^{\#}GPLC^{**}(X,\tau)$] for every open set V of (Y,σ) .
Hence f is $^{\#}GPLC$, $^{\#}GPLC^{*}$ and $^{\#}GPLC^{**}$-continuous. But it is not GLC^{*}-continuous and LC-continuous, since {b,c} is an open set of (Y,σ) but $f^{-1}\{b,c\}=\{b,c\} \notin GLC^{*} \in (X,\tau)$ and $LC(X,\tau)$.

Example 4.5.

Let X={a,b,c}=Y,
$\tau=\{\Phi,X,\{b\},\{b,c\}\}$ and
$\sigma=\{\Phi,Y,\{a,b\}\}$.
Define f: $(X,\tau)\to(Y,\sigma)$ by f(a)=a, f(b)=b and f(c)=c
$G^{\#}LC$ sets of $(X,\tau)=\{X,\Phi,\{a\},\{b\},\{c\},\{a,c\},\{b,c\}\}$
$G^{\#}LC^{*}$ sets of (X,τ) $=\{X,\Phi,\{a\},\{b\},\{c\},\{a,c\},\{b,c\}\}$
$G^{\#}LC^{**}$ sets of (X,τ) $=\{X,\Phi,\{a\},\{b\},\{c\},\{a,c\},\{b,c\}\}$
$^{\#}GPLC$ sets of (X,τ) =P(X)
$^{\#}GPLC^{*}$ sets of (X,τ)=P(X)
$^{\#}GPLC^{**}$ sets of (X,τ) =P(X)
$f^{-1}(\Phi) = \Phi \in {}^{\#}GPLC\ (X,\tau)$ [resp. $^{\#}GPLC^{*}(X,\tau)$ and $^{\#}GPLC^{**}(X,\tau)$]
$f^{-1}(Y) = X \in {}^{\#}GPLC\ (X,\tau)$ [resp. $^{\#}GPLC^{*}(X,\tau)$ and $^{\#}GPLC^{**}(X,\tau)$]
$f^{-1}(\{a,b\}) = \{a,b\} \in {}^{\#}GPLC\ (X,\tau)$ [resp. $^{\#}GPLC^{*}(X,\tau)$ and $^{\#}GPLC^{**}(X,\tau)$]
Thus we get $f^{-1}(V) \in {}^{\#}GPLC\ (X,\tau)$ [resp. $^{\#}GPLC^{*}(X,\tau)$ and $^{\#}GPLC^{**}(X,\tau)$] for every open set V of (Y,σ) .
Hence f is $^{\#}GPLC$, $^{\#}GPLC^{*}$ and $^{\#}GPLC^{**}$-continuous. But it is not $G^{\#}LC$, $G^{\#}LC^{*}$ and $G^{\#}LC^{*}$-continuous, since {a,b} is an open set of (Y,σ) but $f^{-1}\{a,b\}=\{a,b\} \notin G^{\#}LC\ (X,\tau)$, $G^{\#}LC^{*}(X,\tau)$ and $G^{\#}LC^{**}(X,\tau)$.

Example 4.6.

Let $X=\{a,b,c\}=Y$,
$\tau=\{\Phi,X,\{a\}\}$ and
$\sigma=\{\Phi,Y,\{a,b\}\}$.
Define $f:(X,\tau)\rightarrow(Y,\sigma)$ by $f(a)=a$, $f(b)=b$ and $f(c)=c$
$G^{\wedge}LC$ sets of (X,τ) $=\{X,\Phi,\{a\},\{b,c\}\}$
$G^{\wedge}LC^{*}$ sets of (X,τ) $=\{X,\Phi,\{a\},\{b,c\}\}$
$G^{\wedge}LC^{**}$ sets of (X,τ) $=\{X,\Phi,\{a\},\{b,c\}\}$
$^{\#}GPLC$ sets of (X,τ) $=P(X)$
$^{\#}GPLC^{*}$ sets of $(X,\tau)=P(X)$
$^{\#}GPLC^{**}$ sets of $(X,\tau)=P(X)$
$f^{-1}(\Phi) = \Phi\in {^{\#}GPLC}(X,\tau)$ [resp. $^{\#}GPLC^{*}(X,\tau)$ and $^{\#}GPLC^{**}(X,\tau)$]
$f^{-1}(Y) = X\in {^{\#}GPLC}(X,\tau)$ [resp. $^{\#}GPLC^{*}(X,\tau)$ and $^{\#}GPLC^{**}(X,\tau)$]
$f^{-1}(\{a,b\}) = \{a,b\}\in {^{\#}GPLC}(X,\tau)$ [resp. $^{\#}GPLC^{*}(X,\tau)$ and $^{\#}GPLC^{**}(X,\tau)$]
Thus we get $f^{-1}(V)\in {^{\#}GPLC}(X,\tau)$ [resp. $^{\#}GPLC^{*}(X,\tau)$ and $^{\#}GPLC^{**}(X,\tau)$] for every open set V of (Y,σ) .
Hence f is $^{\#}GPLC$, $^{\#}GPLC^{*}$ and $^{\#}GPLC^{**}$-continuous. But it is not $G^{\wedge}LC$, $G^{\wedge}LC^{*}$ and $G^{\wedge}LC^{**}$-continuous, since $\{a,b\}$ is an open set of (Y,σ) but $f^{-1}\{a,b\}=\{a,b\}\notin G^{\wedge}LC(X,\tau)$, $G^{\wedge}LC^{*}(X,\tau)$ and $G^{\wedge}LC^{**}(X,\tau)$.

Example 4.7.

Let X,Y,τ,σ and f be as in the example 4.6
$G^{*}LC$ sets of $(X,\tau)=\{X,\Phi,\{a\},\{b,c\}\}$
$G^{*}LC^{*}$ sets of (X,τ) $=\{X,\Phi,\{a\},\{b,c\}\}$
$G^{*}LC^{**}$ sets of (X,τ) $=\{X,\Phi,\{a\},\{b,c\}\}$
$^{\#}GPLC$ sets of (X,τ) $=P(X)$
$^{\#}GPLC^{*}$ sets of $(X,\tau)=P(X)$
$^{\#}GPLC^{**}$ sets of $(X,\tau)=P(X)$
$f^{-1}(\Phi) = \Phi\in {^{\#}GPLC}(X,\tau)$ [resp. $^{\#}GPLC^{*}(X,\tau)$ and $^{\#}GPLC^{**}(X,\tau)$]
$f^{-1}(Y) = X\in {^{\#}GPLC}(X,\tau)$ [resp. $^{\#}GPLC^{*}(X,\tau)$ and $^{\#}GPLC^{**}(X,\tau)$]
$f^{-1}(\{a,b\}) = \{a,b\}\in {^{\#}GPLC}(X,\tau)$ [resp. $^{\#}GPLC^{*}(X,\tau)$ and $^{\#}GPLC^{**}(X,\tau)$]
Thus we get $f^{-1}(V)\in {^{\#}GPLC}(X,\tau)$ [resp. $^{\#}GPLC^{*}(X,\tau)$ and $^{\#}GPLC^{**}(X,\tau)$] for every open set V of (Y,σ) .
Hence f is $^{\#}GPLC$, $^{\#}GPLC^{*}$ and $^{\#}GPLC^{**}$-continuous. But it is not $G^{*}LC$, $G^{*}LC^{*}$ and $G^{*}LC^{**}$-continuous, since $\{a,b\}$ is an open set of (Y,σ) but $f^{-1}\{a,b\}=\{a,b\}\notin G^{*}LC(X,\tau)$, $G^{*}LC^{*}(X,\tau)$ and $G^{*}LC^{**}(X,\tau)$.

Theorem 4.8.

Let $f:(X,\tau)\rightarrow(Y,\sigma)$ and $g:(Y,\sigma)\rightarrow(Z,\eta)$ be any two functions. Then

(i) g0f is $^{\#}GPLC$ -irresolute if f and g are $^{\#}GPLC$-irresolute.

(ii) g0f is $^{\#}GPLC^{*}$-irresolute if f and g are $^{\#}GPLC^{*}$-irresolute.

(iii) g0f is $^{\#}$GPLC **-irresolute if f and g are $^{\#}$GPLC**-irresolute.

(iv) g0f is $^{\#}$GPLC -continuous if f is $^{\#}$GPLC -continuous and g is continuous.

(v) g0f is $^{\#}$GPLC *-continuous if f is $^{\#}$GPLC *-continuous and g is continuous

(vi) g0f is $^{\#}$GPLC **-continuous if f is $^{\#}$GPLC **-continuous and g is continuous.

(vii) g0f is $^{\#}$GPLC -continuous if f is $^{\#}$GPLC -irresolute and g is $^{\#}$GPLC -continuous .

(viii) g0f is $^{\#}$GPLC *-continuous if f is $^{\#}$GPLC *-irresolute and g is $^{\#}$GPLC *-continuous.

(ix) g0f is $^{\#}$GPLC **-continuous if f is $^{\#}$GPLC **-irresolute and g is $^{\#}$GPLC **-continuous.

Proof:

(i) Given f: $(X,\tau)\rightarrow(Y,\sigma)$ and g: $(Y,\sigma)\rightarrow(Z,\eta)$ are $^{\#}$GPLC -irresolute.
To prove g0f: $(X,\tau)\rightarrow(Z,\eta)$ is $^{\#}$GPLC -irresolute
Let $V\in{}^{\#}$GPLC (Z,η)
Since g: $(Y,\sigma)\rightarrow(Z,\eta)$ is $^{\#}$GPLC -irresolute, then $g^{-1}(V)\in{}^{\#}$GPLC (Y,σ)
Since f: $(X,\tau)\rightarrow(Y,\sigma)$ is $^{\#}$GPLC -irresolute, then $f^{-1}(g^{-1}(V))\in{}^{\#}$GPLC (X,τ)
i.e $(g0f)^{-1}(V)\in{}^{\#}$GPLC (X,τ)
Thus we get $(g0f)^{-1}(V)\in{}^{\#}$GPLC (X,τ) for every $V\in{}^{\#}$GPLC (Z,η)
Hence g0f :$(X,\tau)\rightarrow(Z,\eta)$ is $^{\#}$GPLC -irresolute

(ii) Given f: $(X,\tau)\rightarrow(Y,\sigma)$ and g: $(Y,\sigma)\rightarrow(Z,\eta)$ are $^{\#}$GPLC *-irresolute.
To prove g0f: $(X,\tau)\rightarrow(Z,\eta)$ is $^{\#}$GPLC *-irresolute
Let $V\in{}^{\#}$GPLC $^{*}(Z,\eta)$
Since g: $(Y,\sigma)\rightarrow(Z,\eta)$ is $^{\#}$GPLC *-irresolute, then $g^{-1}(V)\in{}^{\#}$GPLC $^{*}(Y,\sigma)$
Since f: $(X,\tau)\rightarrow(Y,\sigma)$ is $^{\#}$GPLC *-irresolute, then $f^{-1}(g^{-1}(V))\in{}^{\#}$GPLC $^{*}(X,\tau)$
i.e $(g0f)^{-1}(V)\in{}^{\#}$GPLC $^{*}(X,\tau)$
Thus we get $(g0f)^{-1}(V)\in{}^{\#}$GPLC * (X,τ) for every $V\in{}^{\#}$GPLC $^{*}(Z,\eta)$
Hence g0f :$(X,\tau)\rightarrow(Z,\eta)$ is $^{\#}$GPLC*-irresolute

(iii) Given f: $(X,\tau)\rightarrow(Y,\sigma)$ and g: $(Y,\sigma)\rightarrow(Z,\eta)$ are $^{\#}$GPLC **-irresolute.
To prove g0f : $(X,\tau)\rightarrow(Z,\eta)$ is $^{\#}$GPLC **-irresolute
Let $V\in{}^{\#}$GPLC $^{**}(Z,\eta)$
Since g: $(Y,\sigma)\rightarrow(Z,\eta)$ is $^{\#}$GPLC **-irresolute,then $g^{-1}(V)\in{}^{\#}$GPLC $^{**}(Y,\sigma)$
Since f: $(X,\tau)\rightarrow(Y,\sigma)$ is $^{\#}$GPLC **-irresolute, then $f^{-1}(g^{-1}(V))\in{}^{\#}$GPLC $^{**}(X,\tau)$
i.e $(g0f)^{-1}(V)\in{}^{\#}$GPLC $^{**}(X,\tau)$
Thus we get $(g0f)^{-1}(V)\in{}^{\#}$GPLC ** (X,τ) for every $V\in{}^{\#}$GPLC $^{**}(Z,\eta)$
Hence g0f :$(X,\tau)\rightarrow(Z,\eta)$ is $^{\#}$GPLC **-irresolute

(iv) Given f: (X,τ)→(Y,σ) is $^{\#}$GPLC -continuous and g: (Y,σ)→(Z,η) is continuous
To prove g0f :(X,τ)→(Z,η) is $^{\#}$GPLC -continuous
Let V be an open set of (Z, η)
Since g: (Y,σ)→(Z,η) is continuous,then $g^{-1}(V)$ is an open set of (Y,σ)
Since f: (X,τ)→(Y,σ) is $^{\#}$GPLC -continuous, $f^{-1}(g^{-1}(V)) \in {}^{\#}$GPLC (X,τ)
i.e $(g0f)^{-1}(V) \in {}^{\#}$GPLC (X,τ)
Thus we get $(g0f)^{-1}(V) \in {}^{\#}$GPLC (X,τ) for every open set V of (Z,η)
Hence g0f :(X,τ)→(Z,η) is $^{\#}$GPLC -continuous.

(v) Given f: (X,τ)→(Y,σ) is $^{\#}$GPLC *-continuous and g: (Y,σ)→(Z,η) is continuous
To prove g0f :(X,τ)→(Z,η) is $^{\#}$GPLC *-continuous
Let V be an open set of (Z, η)
Since g: (Y,σ)→(Z,η) is continuous,then $g^{-1}(V)$ is an open set of (Y,σ)
Since f: (X,τ)→(Y,σ) is G$^{\#}$PLC*-continuous, $f^{-1}(g^{-1}(V)) \in {}^{\#}$GPLC *(X,τ)
i.e $(g0f)^{-1}(V) \in {}^{\#}$GPLC *(X,τ)
Thus we get $(g0f)^{-1}(V) \in {}^{\#}$GPLC * (X,τ) for every open set V of (Z,η)
Hence g0f :(X,τ)→(Z,η) is $^{\#}$GPLC *-continuous.

(vi) Given f: (X,τ)→(Y,σ) is $^{\#}$GPLC **-continuous and g: (Y,σ)→(Z,η) is continuous
To prove g0f :(X,τ)→(Z,η) is $^{\#}$GPLC **-continuous
Let V be an open set of (Z, η)
Since g: (Y,σ)→(Z,η) is continuous,then $g^{-1}(V)$ is an open set of (Y,σ)
Since f: (X,τ)→(Y,σ) is $^{\#}$GPLC **-continuous, $f^{-1}(g^{-1}(V)) \in {}^{\#}$GPLC **(X,τ)
i.e $(g0f)^{-1}(V) \in {}^{\#}$GPLC **(X,τ)
Thus we get $(g0f)^{-1}(V) \in {}^{\#}$GPLC (X,τ) for every open set V of (Z,η)
Hence g0f :(X,τ)→(Z,η) is $^{\#}$GPLC **-continuous.

(vii) Given f: (X,τ)→(Y,σ) is $^{\#}$GPLC -irresolute and g: (Y,σ)→(Z,η) is $^{\#}$GPLC - continuous
To prove g0f :(X,τ)→(Z,η) is $^{\#}$GPLC -continuous
Let V be an open set of (Z, η)
Since g: (Y,σ)→(Z,η) is $^{\#}$GPLC - continuous,then $g^{-1}(V) \in {}^{\#}$GPLC (Y,σ)
Since f: (X,τ)→(Y,σ) is $^{\#}$GPLC -irresolute,then $f^{-1}(g^{-1}(V)) \in {}^{\#}$GPLC (X,τ)
i.e $(g0f)^{-1}(V) \in {}^{\#}$GPLC (X,τ)
Thus we get $(g0f)^{-1}(V) \in {}^{\#}$GPLC (X,τ) for every open set V of (Z,η)
Hence g0f :(X,τ)→(Z,η) is $^{\#}$GPLC -continuous.

(viii) Given f: (X,τ)→(Y,σ) is $^{\#}$GPLC *-irresolute and g: (Y,σ)→(Z,η) is $^{\#}$GPLC *- continuous
To prove g0f :(X,τ)→(Z,η) is $^{\#}$GPLC *-continuous
Let V be an open set of (Z, η)
Since g: (Y,σ)→(Z,η) is $^{\#}$GPLC *- continuous,then $g^{-1}(V) \in {}^{\#}$GPLC * (Y,σ)
Since f: (X,τ)→(Y,σ) is $^{\#}$GPLC *-irresolute,then $f^{-1}(g^{-1}(V)) \in {}^{\#}$GPLC *(X,τ)
i.e $(g0f)^{-1}(V) \in {}^{\#}$GPLC *(X,τ)
Thus we get $(g0f)^{-1}(V) \in {}^{\#}$GPLC * (X,τ) for every open set V of (Z,η)
Hence g0f :(X,τ)→(Z,η) is $^{\#}$GPLC *-continuous.

(ix) Given f: (X,τ)→(Y,σ) is $^{\#}$GPLC **-irresolute and g: (Y,σ)→(Z,η) is $^{\#}$GPLC **-continuous
To prove g0f :(X,τ)→(Z,η) is $^{\#}$GPLC **-continuous
Let V be an open set of (Z, η)
Since g: (Y,σ)→(Z,η) is $^{\#}$GPLC **- continuous,then $g^{-1}(V) \in {}^{\#}$GPLC ** (Y,σ)
Since f: (X,τ)→(Y,σ) is $^{\#}$GPLC **-irresolute,then $f^{-1}(g^{-1}(V)) \in {}^{\#}$GPLC **(X,τ)
i.e $(g0f)^{-1}(V) \in {}^{\#}$GPLC **(X,τ)
Thus we get $(g0f)^{-1}(V) \in {}^{\#}$GPLC ** (X,τ) for every open set V of (Z,η)
Hence g0f :(X,τ)→(Z,η) is $^{\#}$GPLC **-continuous.

3. CONCLUSIONS

From the Figure 1,we see that

i. Every GLC* is $^{\#}$GPLC, $^{\#}$GPLC* and $^{\#}$GPLC**.

ii. Every LC is $^{\#}$GPLC, $^{\#}$GPLC* and $^{\#}$GPLC**.

iii. Every G$^{\#}$LC [resp. G$^{\#}$LC* and G$^{\#}$LC**] is $^{\#}$GPLC, $^{\#}$GPLC* and $^{\#}$GPLC**.

iv. Every G$^{\wedge}$LC[resp.G$^{\wedge}$LC* and G$^{\wedge}$LC**] is $^{\#}$GPLC, $^{\#}$GPLC* and $^{\#}$GPLC**.

v. Every G*LC [resp. G*LC* and GL*C**] is $^{\#}$GPLC, $^{\#}$GPLC* and $^{\#}$GPLC**. But the converses are not true.

From this, we conclude $^{\#}$GPLC, $^{\#}$GPLC* and $^{\#}$GPLC** sets are stronger than GLC* , LC, G$^{\#}$LC [resp. G$^{\#}$LC* and G$^{\#}$LC**] , G$^{\wedge}$LC[resp.G$^{\wedge}$LC* and G$^{\wedge}$LC**], G*LC [resp. G*LC* and GL*C**] sets.

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