



A Comparative Case Study on the Social Networking of the Second Year Under Graduate Mathematics Students of the M.D.T. Hindu College, Tirunelveli, Tamil Nadu, India Using Graph Theoretic Parameters

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ABSTRACT

In this experimental case study, the social relationship of boys and girls studying second year under graduate Mathematics boys in The M.D.T Hindu College, Tirunelveli is compared using sociometry.

Keywords: Sociometry; In-degree; out-degree; Domination; Digraphs; In-domination; Out-domination

1. INTRODUCTION

A case study is an in-depth study of a person or an organized group of people especially as a model of medical, psychiatric, psychological or social phenomena. Social network theory in the social sciences began with the field of sociometry, which is a quantitative method for measuring social relationships. It is the method of expanding the number of business and /or social contacts of a group/person under study by making connections through individuals. It is a map of the relationships between the individuals, indicating the ways in which they are connected under various aspects such as social familiarizes ranging from casual acquaintance

to close familial bonds. It makes views on social relationship in terms of nodes and ties. Nodes are individual actors and ties are the relationships between the actors. There can be many kinds of ties between the nodes, depending on the relationships being studied.

2. EXPERIMENTAL STUDY

In this experimental case study, the social relationship of boys and girls studying second year undergraduate Mathematics in The M.D.T Hindu college, Tirunelveli is compared using sociometry.

2. 1. Network data

The population of this social study consists of 44 under graduate students of second year Mathematics of The M.D.T Hindu College, Tirunelveli. The under graduate class was co-educated. There are 25 girls and 19 boys in the class. The population is heterogeneous.

2. 2. About the college

The M.D.T. Hindu College was incepted in 1878. In 1997, the college extended its service by establishing unaided courses. It celebrated its Centenary in 1979 and Post Centenary Silver Jubilee in 2004. The B.Sc degree course and M.Sc degree course in Mathematics were started in the year 1957 and 1980 respectively. In 2006, the college was approved as a Research Centre in Mathematics by Manonmaniam Sudaranar University, Tirunelveli, Tamil Nadu, India.

2. 3. Method of study

Data are collected from the population using a printed questionnaire. Instructions regarding the questionnaire are given well in advance. The questionnaire consists of two sections. The first section consists of the personal data and the second section consists of questions on social network. In the personal data, the name of the student, sex and class are asked. A provision for allotting the confidential number for each student is also provided in that section. In the second section, four questions each consisting of two parts are asked. In the first part, the students are asked to answer yes/no. If the response is yes, then he/she is asked to write the name of the student in the second part.

In the first question they are asked if they have a close friend in their class. In the second, they are asked if they approach anybody in their class to clear their doubts. Whether they approach anybody in their class for monetary help is the third question. The fourth question is if they approach anybody in their class for advice. The four questions are numbered as 1, 2, 3 and 4.

Each student is given a copy of the printed questionnaire and they are asked to fill in the personal data and answer the questions within the time provided. Their responses are collected and then the 19 male students are randomly labelled with confidential numbers 1,2,3,.....19 and the 25 female students are randomly labelled with confidential numbers 1,2,3,.....25. Their response are recorded and tabulated separately.

2. 4. Definition

A digraph D is a non-empty finite set V of element called vertices together with a collection E of ordered pairs of distinct vertices called arcs. An oriented graph is a digraph with no symmetric pairs of arcs. Equivalently, an oriented graph is a digraph that can be obtained from a graph G by assigning one orientation to each edge of G .

Directed graphs are drawn based on the responses given. The vertices denote the confidential numbers and the directed arcs for each response show the relationship between two confidential numbers. x approaching y is denoted by $x \longrightarrow y$.

A directed walk in a graph is an alternating sequence of points and arcs, $v_1, x_1, \dots, v_n, x_n$ in which each arc x_i is $v_{i-1}v_i$ the length of such a walk is n , the number of occurrences of arcs in it. A closed walk has the same first and last points. A closed walk as the same first and last points. A path is a walk in which all points are disjoint. A cycle is a non-trivial closed walk with all points distinct (except the first and last). A digraph is unilaterally connected, or unilateral, if for any two points at least one is reachable from the other. A unilateral component is a maximal unilateral sub graph.

Table 1. Response of the individuals for the questions 1 to 4.

No.	BOYS		GIRLS	
	YES	NO	YES	NO
1	74%	26%	92%	8%
2	63%	37%	92%	8%
3	63%	37%	76%	24%
4	58%	42%	72%	28%

In the class, the percentage of girls having friends in their class is more when compared with the boys. The percentage of students approaching for monetary help, advice and for clearing their doubts is also considerably more in girls when compared with the boys.

2. 5. Reciprocity in the network

A measure of reciprocity is a number which gives the extent to which support it extended to others and received from others in a relationship. Reciprocity in social psychology refers to responding to a positive action with another positive action. It is considered as a strong determining factor of human behaviour. Let B_i and G_i denote the graph of directed arcs of boys and girls respectively for the question i , $1 \leq i \leq 4$.

Table 2. Details of reciprocity under various networks for both boys and girls

NETWORK	RECIPROCITY	
	BOYS	GIRLS
1	14	22
2	Nil	6
3	1	7
4	1	6

In the networks B_1 and G_1 , there are fourteen and twenty two reciprocal ties respectively which represents their close friend in their class.

In the B_2 and G_2 networks, there are zero and six reciprocal ties respectively which represents their approach to clear their doubts in the subject.

In the B_3 and G_3 networks, there are one and seven reciprocal ties respectively which represents their approach for monetary help in their class.

In the B_4 and G_4 networks, there are one and six reciprocal ties respectively which represents their approach for advice in their class.

In all the networks the percentage of reciprocity for girls is more when compared with the boys.

In a directed network, the in-degree (id) of a vertex is defined to be the number of arrows directed to the vertex and the out degree (od) of a vertex is the number of arrows which arises from the vertex.

Table 3. Maximum-in-degree and the number of vertices with maximum in- degree

NETWORK	MAXIMUM IN DEGREE		NUMBER OF VERTICES	
	BOYS	GIRLS	BOYS	GIRLS
1	5	5	2	3
2	7	5	2	2
3	5	4	2	1
4	3	6	2	1

2. 6. Definition

Let $D = (V, A)$ be a directed graph. A vertex $v \in V$ is called

- (i) An isolated vertex if $od(v) = id(v) = 0$

- (ii) A transmitter if $od(v) > 0$ and $id(v) = 0$
- (iii) A receiver if $od(v) = 0$ and $id(v) > 0$
- (iv) A carrier if $od(v) > 0$ and $id(v) > 0$

A receiver is one who receives from other persons whereas transmitter is one who tends to provide help without expecting anything in return. A carrier is one who receives and transmits.

2. 7. Definition

A subset S of $V(G)$ of a graph G is called a dominating set of G if every vertex in $V(G)\setminus S$ is adjacent to a vertex in S .

2. 8. Definition

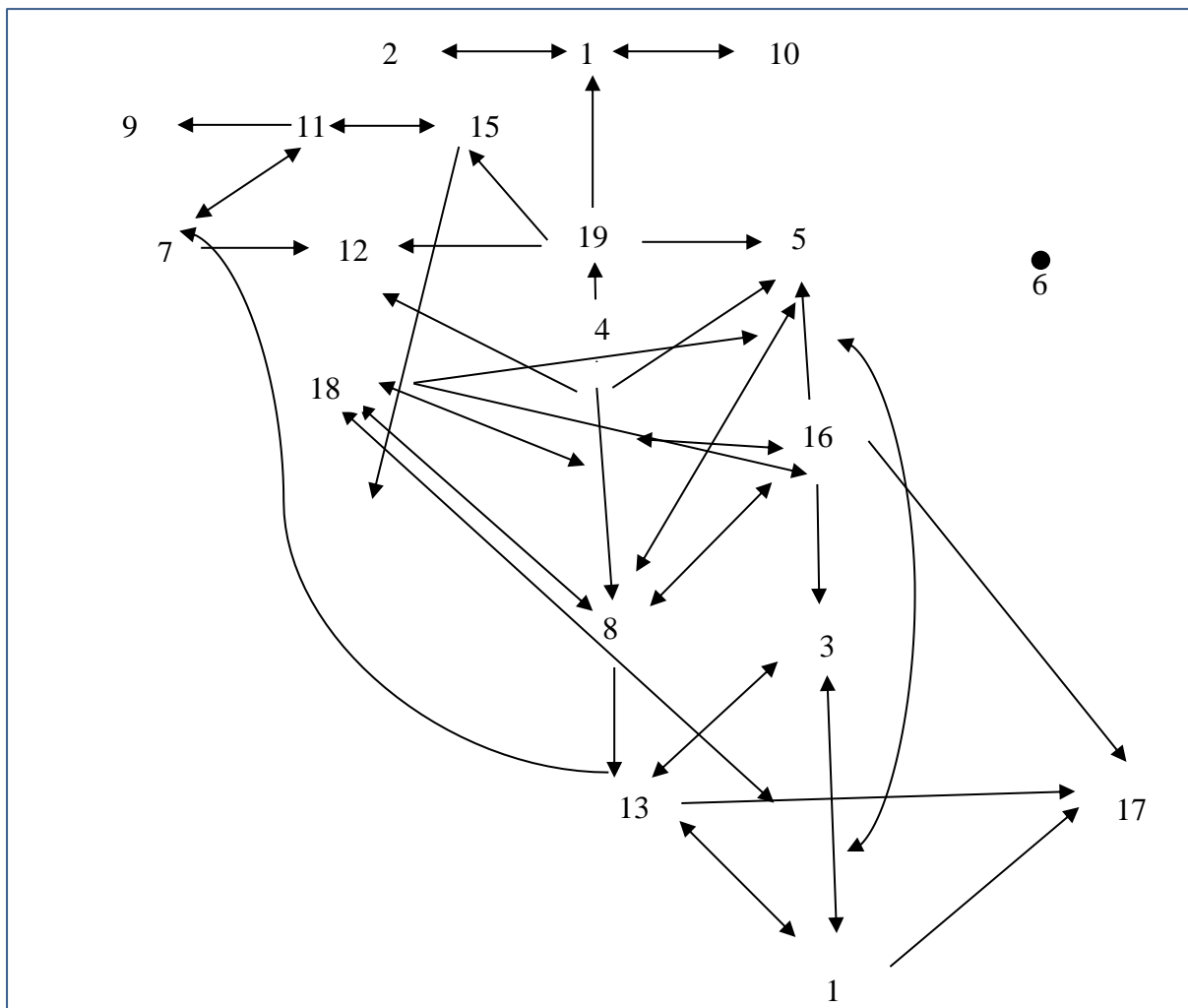


Figure 1. Network B_1

An out-dominating set of a digraph D is a set S of vertices of D such that every vertex of $D - S$ is adjacent from some vertex of S . The minimum cardinality of an out-domination

set for D is the out-domination number $\gamma^+(D)$ of D. The in-domination number $\gamma^-(D)$ is similarly defined.

Networks B_1 and G_1 shows the response of boys and girls for the question 1.

2. 9. Implication

In network B_1 , it is found that the vertices 3 and 13 have the maximum in-degree 5. The vertex 6 has no in-degree and out- degree. Hence the vertex 6 is isolated in the network B_1 . Clearly $S = \{13,17,1,11,12,9,5,6\}$ is an in- dominating set. Therefore $\gamma^-(B_1) = 8$.

2. 10 Interpretation

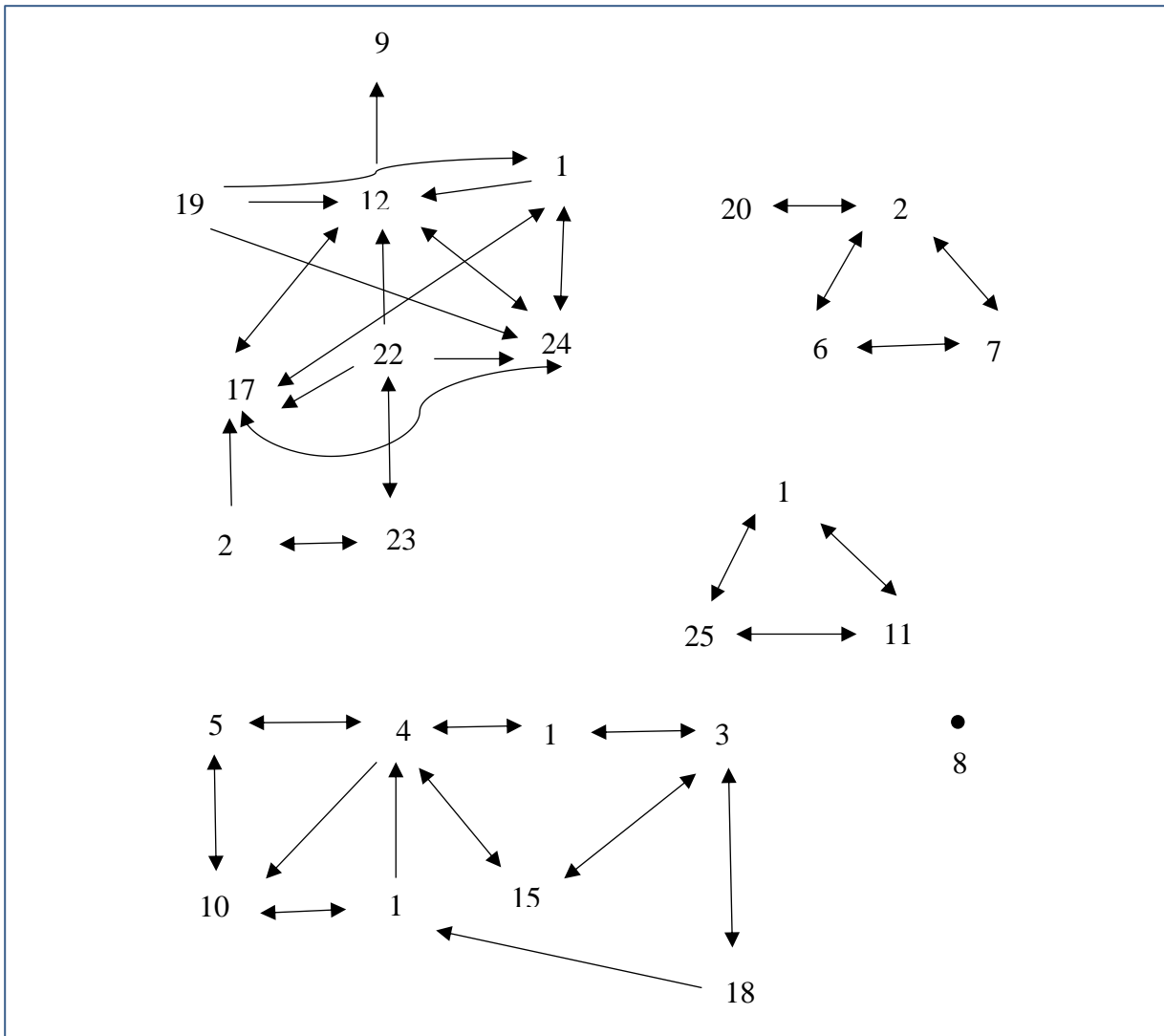


Figure 2. Network G_1

In the network B_1 , the boys with the confidential number 3 and 13 have more friends in their class. The confidential number 6 is found to be isolated. The conclusion got from this is

that he do not interact with his class mates. So it would be better to give a counselling to change his personality and make him interact with others. Since the in-domination number is 8, which is very large, proper guidance and counselling may be given to reduce the in-domination number.

2. 11 Implication

In network G_1 , it is found that the vertices 12, 17 and 24 have the maximum in-degree 5. The vertex 8 has no in-degree and out-degree. Clearly $S = \{1, 2, 3,4, 8, 9,12,23\}$ is an in-dominating set. Therefore $\gamma^-(G_1) = 8$.

2. 12. Interpretation

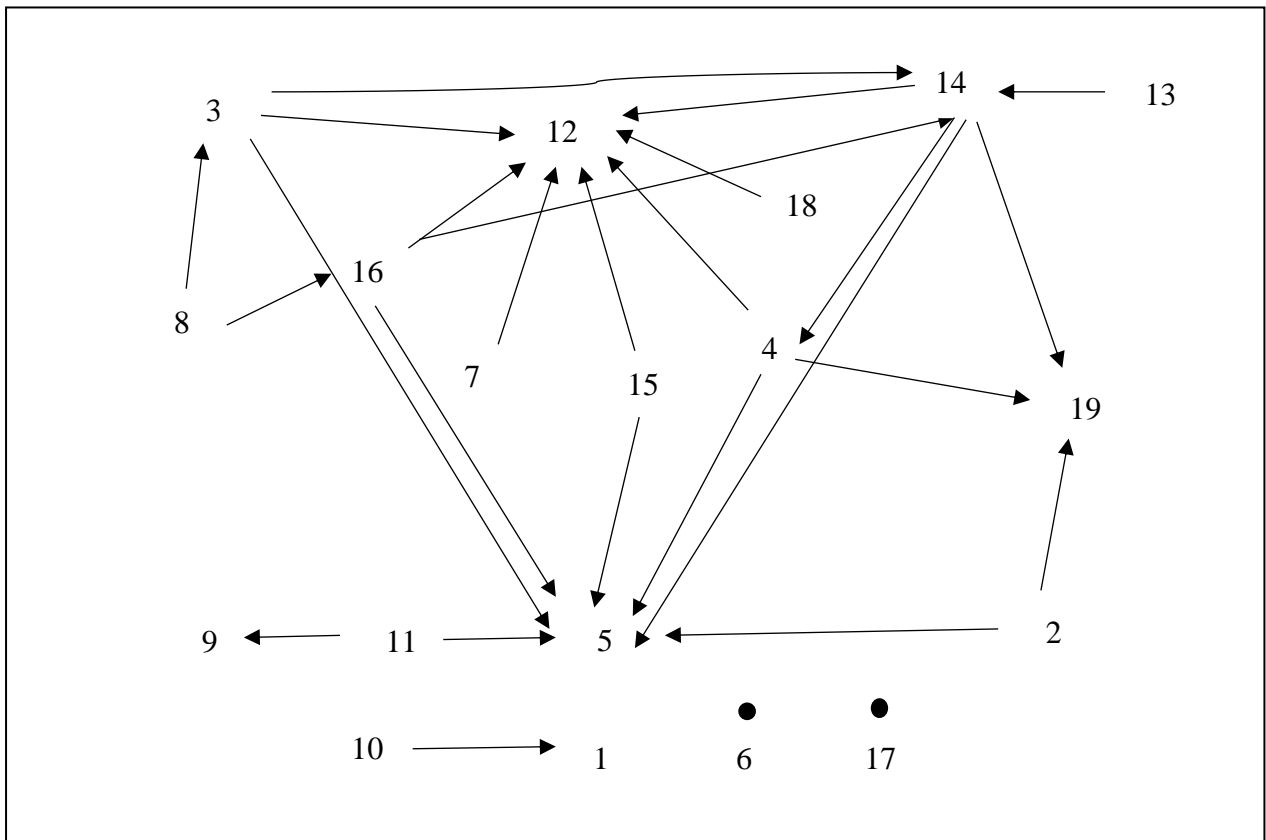


Figure 3. Network B_2

In the network G_1 , the confidential number 8 is found to be isolated. The conclusion got from this is that the confidential number 8 do not interact with her classmates. So, it is better to give a counselling to change her personality and make her interact with others. Even though the number of boys and girls vary, the in-domination number remains the same for both boys and girls.

The number of receivers is more in boys and the number of carriers is more in girls. It is interesting to note that the number of isolated vertices and transmitters are equal. Network B_2 and G_2 shows the response of boys and girls for the question 2.

Table 4. Distribution of respondents under various categories in the networks B_1 and G_1

QUESTION	ISOLATED		RECEIVER		TRANSMITTER		CARRIER	
	B_1	G_1	B_1	G_1	B_1	G_1	B_1	G_1
1	1	1	4	1	1	1	13	22

2. 13. Implication

In network B_2 , the vertices 5 and 12 have the maximum in-degree. Vertices 6 and 7 are isolated. Clearly $S = \{1, 6, 5, 8, 9, 12, 14, 17, 19\}$ is an in- dominating set. Hence $\gamma^-(B_2)$ is 9.

2. 14. Interpretation

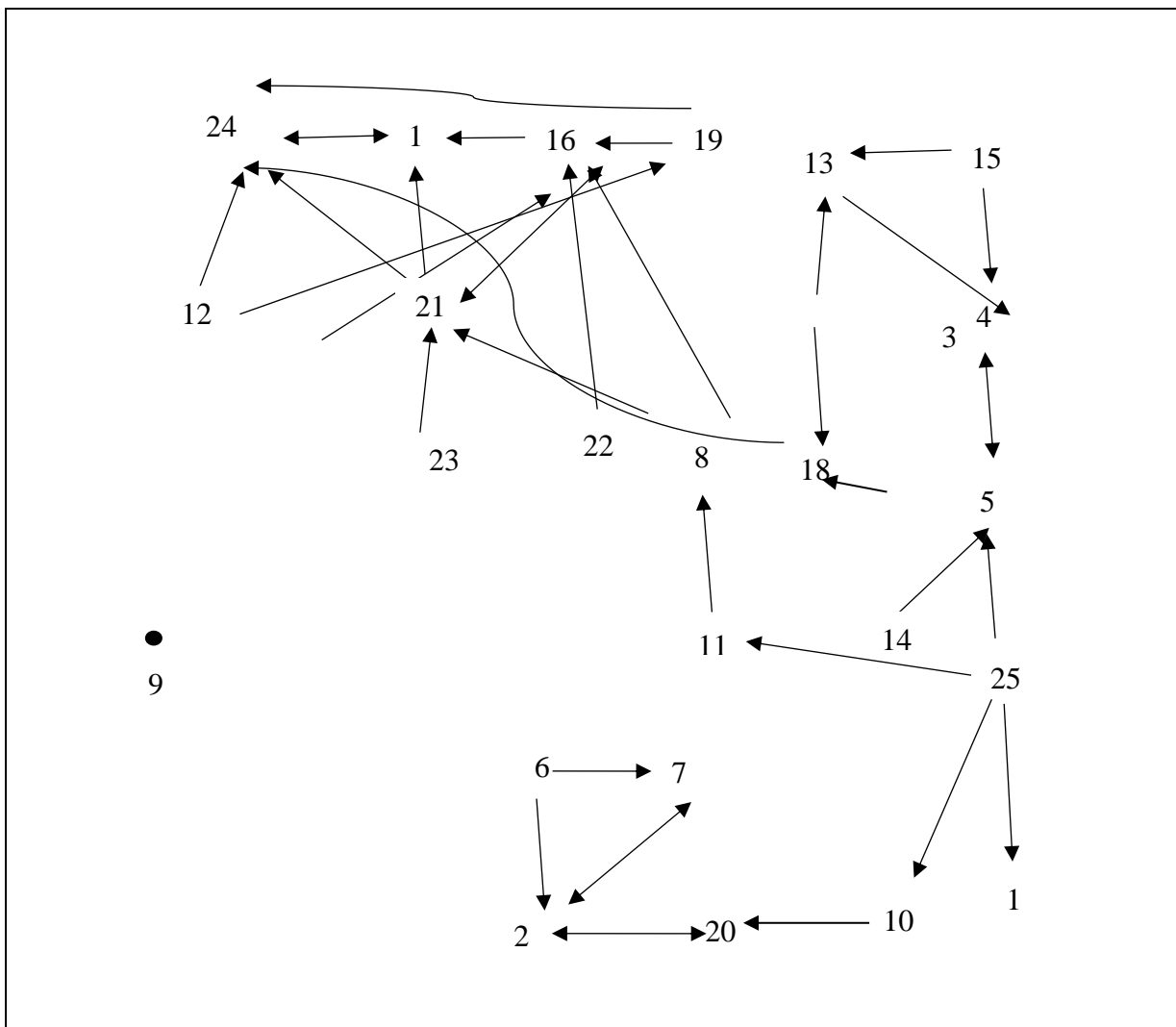


Figure 4. Network G_2

In the network B_2 , $\text{deg}(5) = \text{deg}(12) = 7$. Hence the confidential numbers 5 and 12 are approached by many students to clear their doubts. Hence the confidential numbers 5 and 12 are good in their studies. Since the domination number is 9, the class is more fragmented. The confidential numbers 6 and 7 do not approach anybody to clear their doubts. Those two boys may be given counselling, if needed.

2. 15. Implication

In network G_2 , the vertices 24 and 16 have the maximum in-degree 5. Vertex 16 has out degree only. Clearly, $S = \{1, 2, 5, 8, 9, 10, 13, 17, 21, 24\}$ is an in- dominating set. Hence $\gamma^-(G_2) = 10$.

2.16 Interpretation

In network G_2 , the confidential numbers 24 and 16 are approached by many students to clear their doubts. 24 and 16 are good in their studies. The dominating set for this network is $\{1, 2, 5, 8, 9, 10, 13, 17, 21, 24\}$ and so the domination number is 11. Hence this network is also more fragmented.

Table 5. Distribution of respondents under various categories in the network B_2 and G_2 .

QUESTION	ISOLATED		RECEIVER		TRANSMITTER		CARRIER	
	B_2	G_2	B_2	G_2	B_2	G_2	B_2	G_2
2	2	1	5	2	7	11	5	11

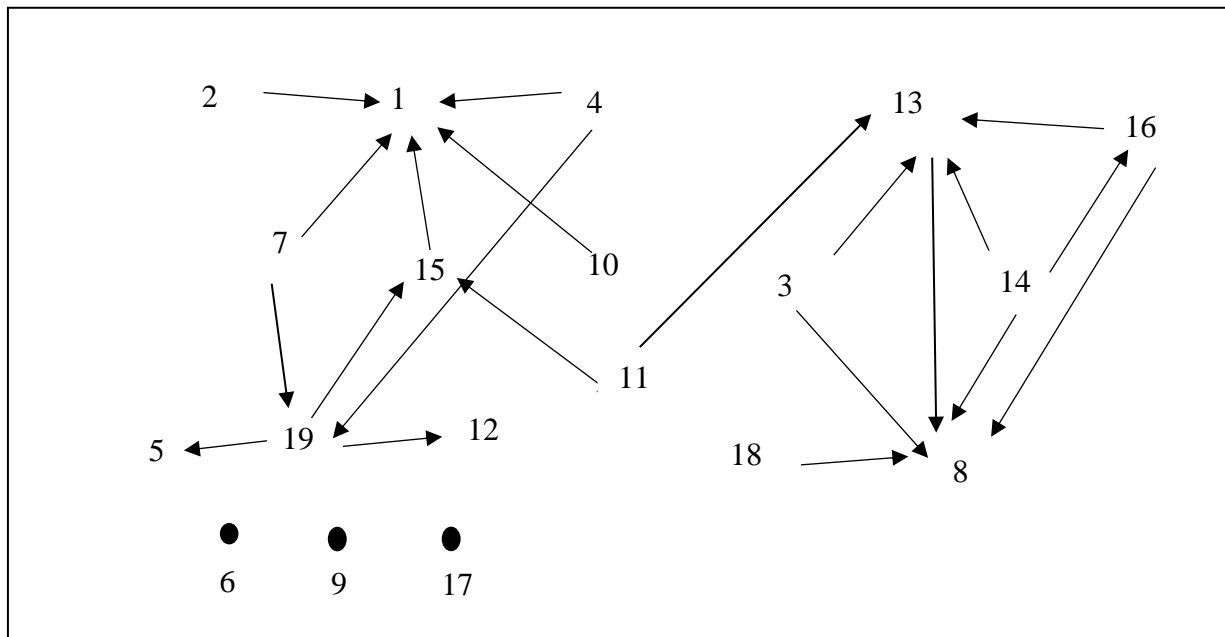


Figure 5. Network B_3

The number of isolated vertices and receivers is more in boys and the number of transmitters and carriers is more in girls. Networks B_3 and G_3 show the response of boys and girls for question 3.

2. 17. Implication

In network B_3 , the vertices 1 and 8 have the maximum in-degree. Vertex 17, 9 and 6 are isolated. Clearly $S = \{1, 5, 6, 8, 9, 12, 13, 17\}$ is an in- dominating set. Hence $\gamma^-(B_3) = 8$

2. 18. Interpretation

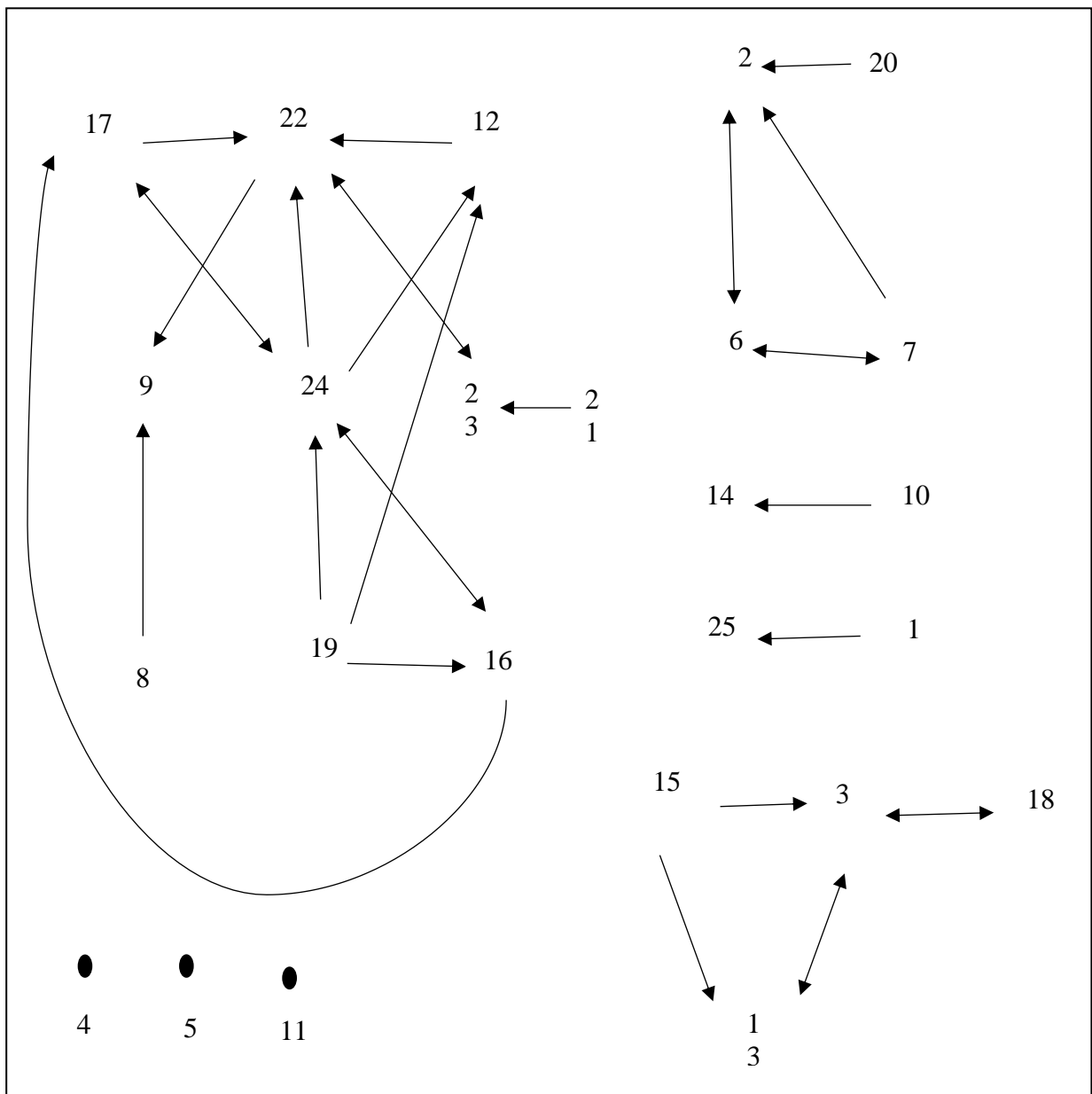


Figure 6. Network G_3

In network B_3 the confidential numbers 1 and 8 have more helping tendency. In the network B_3 , the confidential numbers 9, 17 and 6 do not seek the help of others. They may be self-sufficient.

2. 20. Interpretation

In the class, the confidential number 22 has more helping tendency. It is interesting to note that she in turn seek monetary help from 9. Girls with confidential numbers 4, 5 and 11 are isolated. They do not seek the help of others and they may be self-sufficient.

Table 6. Distribution of respondents under various categories in the networks B_3 and G_3

QUESTION	ISOLATED		RECEIVER		TRANSMITTER		CARRIER	
	B_3	G_3	B_3	G_3	B_3	G_3	B_3	G_3
3	3	3	4	3	7	7	5	12

The number of receivers is more in boys and the number of carriers is more in girls. It is very interesting to note that the number of isolated vertices and transmitters are equal. Networks B_4 and G_4 show the response of boys and girls for question 4.

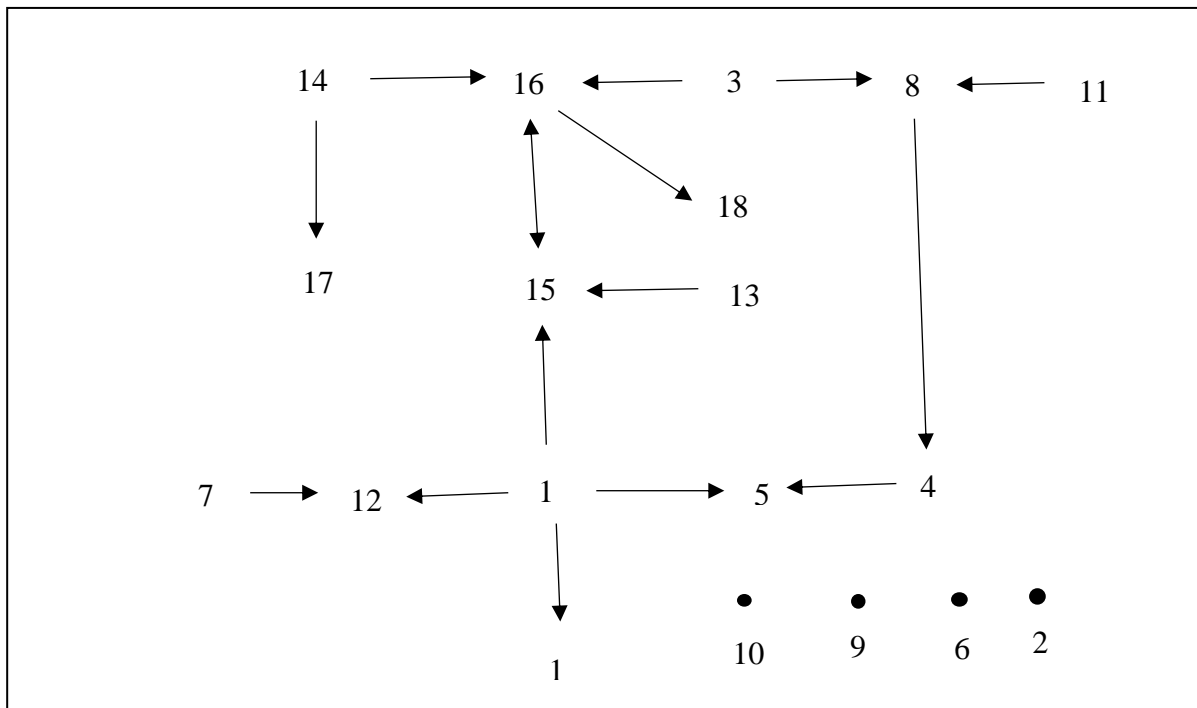


Figure 7. Network B_4

2. 21. Implication

In network B_4 , vertices 15 and 16 have the maximum in-degree 3. Vertices 2, 9, 6 and 10 are isolated. Clearly $S = \{15, 8, 12, 1, 17, 9, 2, 18, 5, 6, 10\}$ is an in-dominating set and hence $\gamma^-(B_4) = 11$.

2. 22. Interpretation

In the class, the confidential numbers 15 and 16 are approached by many students for advice. The conclusion got from the above study is that the confidential numbers 15 and 16 have a pleasing personality. The confidential numbers 2, 6, 9 and 10 do not like to approach others to solve their own problems. They do not like share their problems, if any, with others.

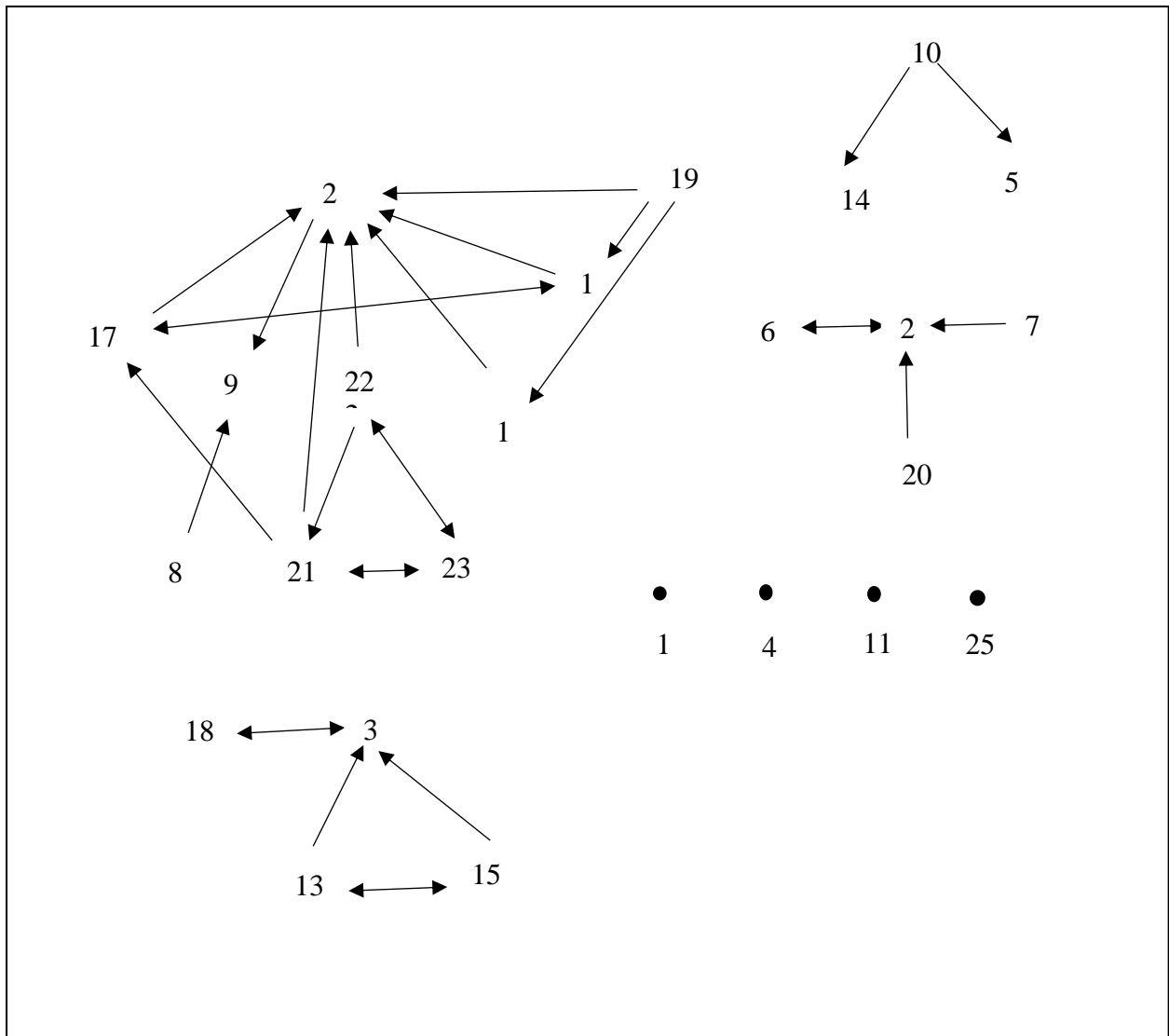


Figure 8. Network G_4

2. 23. Implication

In network G_4 , vertex 24 has the maximum in-degree. Vertices 1, 4, 11 and 25 are isolated. Clearly $S = \{1, 2, 3, 4, 5, 9, 11, 14, 24, 25, 23\}$ is an in- dominating set and so $\gamma^-(G_4) = 11$.

2. 24. Interpretation

In the class, the confidential number 24 is approached by many students for advice. The conclusion got from it is that that she has a pleasing personality. The confidential numbers 1, 4, 11 and 25 do not like to approach others to share their problems.

Table 7. Distribution of respondents under various categories in the networks B_4 and G_4

QUESTION	ISOLATED		RECEIVER		TRANSMITTER		CARRIER	
	B_4	G_4	B_4	G_4	B_4	G_4	B_4	G_4
4	4	4	5	3	6	5	4	13

The number of receivers and transmitters is more in boys and the number of carriers is more in girls. It is interesting to note that the number of isolated vertices is equal in the class.

3. CONCLUSIONS

This study gives an outline of the behaviour of students in various networks. The major roles played by some individuals in both the classes are exhibited. Some individuals found to be isolated in many networks. Reciprocity is low in some networks. As a follow-up action, proper guidance and counselling may be given to reduce the number of isolates. Efforts may be taken to increase the reciprocity. Further studies can be made on their approaches outside their classes.

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