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Classes of relativistic stars with quadratic equation of state

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ABSTRACT

In this paper, we found new exact solutions to the Einstein-Maxwell system of equations with charged anisotropic matter distribution considering quadratic equation of state. We specify the gravitational potential Z(x) that depends of a adjustable parameter n and that allow integrate analytical the field equations in order to calculate the energy density, the radial pressure, the anisotropy, charge density and the mass function for different values of n. The obtained solutions can be written in terms of elementary and polynomial functions.

***Keywords*:** Charge anisotropic matter distribution, quadratic equation of state, Radial pressure, Charge density, Gravitational potential, Field equations

1. INTRODUCTION

The study of the ultracompacts objects and the gravitational collapse is of fundamental importance in astrophysics and has attracted much since the formulation of the general theory of relativity. One of the fundamental problems in the general theory of relativity is finding exact solutions of the Einstein field equations [1,2]. Some solutions found fundamental applications in astrophysics, cosmology and more recently in the developments inspired by string theory [2]. Different mathematical formulations that allow to solve Einstein´s field equations have been used to describe the behaviour of objects submitted to strong gravitational fields known as neutron stars, quasars and white dwarfs [3-5].

In the construction of the first theoretical models of relativistic stars are important the works of Schwarzschild [6], Tolman [7], Oppenheimer and Volkoff [8]. Schwarzschild [6] found analytical solutions that allowed describing a star with uniform density, Tolman [7] developed a method to find solutions of static spheres of fluid and Oppenheimer and Volkoff [8] used Tolman's solutions to study the gravitational balance of neutron stars. It is important to mention Chandrasekhar's contributions [9] in the model production of white dwarfs in presence of relativistic effects and the works of Baade and Zwicky [10] who propose the concept of neutron stars and identify an astronomic dense objects known as supernovas.

The physics of ultrahigh densities is not well understood and many of the strange stars studies have been performed within the framework of the MIT bag model [11]. In this model, the strange matter equation of state has a simple linear form given by $p=\frac{1}{3}(\rho-4B)$ where ρ is the energy density, p is the isotropic pressure and B is the bag constant.

In theoretical works of realistic stellar models, is important include the pressure anisotropy [12-14]. Bowers and Liang [12] extensively discuss the effect of pressure anisotropy in general relativity. The existence of anisotropy within a star can be explained by the presence of a solid core, phase transitions, a type III super fluid, a pion condensation [15] or another physical phenomena as the presence of an electrical field [16].

The physics of ultrahigh densities is not well understood and many of the strange stars studies have been performed within the framework of the MIT-Bag model [11]. In this model, the strange matter equation of state has a simple linear form given by $p=\frac{1}{3}(\rho-4B)$ where ρ is the energy density, p is the isotropic pressure and B is the bag constant. Many researchers have used a great variety of mathematical techniques to try to obtain exact solutions for quark stars within the framework of MIT-Bag model: Komathiraj and Maharaj [11] found two new classes of exact solutions to the Einstein-Maxwell system of equations with a particular form of the gravitational potential and isotropic pressure. Malaver [17, 18] also has obtained some models for quark stars considering a potential gravitational that depends on an adjustable parameter.

Thirukkanesh and Maharaj [19] studied the behavior of compact relativistic objects with anisotropic pressure in the presence of the electromagnetic field. Maharaj et al. [20] generated new models for quark stars with charged anisotropic matter considering a linear equation of state. Thirukkanesh and Ragel [21] obtained new models for compact stars with quark matter. Sunzu et al. found new classes of solutions with specific forms for the measure of anisotropy [22].

With the use of Einstein´ field equations, important advances has been made to model the interior of a star. Feroze and Siddiqui [23, 24] and Malaver [25, 26] consider a quadratic equation of state for the matter distribution and specify particular forms for the gravitational potential and electric field intensity. Mafa Takisa and Maharaj [27] obtained new exact solutions to the Einstein-Maxwell system of equations with a polytropic equation of state. Thirukkanesh and Ragel [28] have obtained particular models of anisotropic fluids with polytropic equation of state which are consistent with the reported experimental observations. More recently, Malaver [29, 30] generated new exact solutions to the Einstein-Maxwell system considering Van der Waals modified equation of state with and without polytropical exponent. Raghoonundun and Hobill [31] found new analytical models for compact stars with the use of Tolman VII solution.

The main objective in this paper is to generate a new class for charged anisotropic matter with the quadratic equation of state that presents a relation between the energy density and the radial pressure in static spherically symmetric spacetime using a particular form for the metric potential $Z(x)$ that depends of an adjustable parameter. We have obtained two new classes of static spherically symmetrical models of charged matter without singularities in the charge distribution and the matter at the centre of the star. This article is organized as follows, in Section 2, we present Einstein´s field equations. In Section 3, we make a particular choice of gravitational potential $Z(x)$ that allows solving the field equations and we have obtained new models for charged anisotropic matter. In Section 4, a physical analysis of the new solutions is performed. Finally in Section 5, we conclude.

2. EINSTEIN FIELD EQUATIONS

We consider a spherically symmetric, static and homogeneous and anisotropic spacetime in Schwarzschild coordinates given by

$$ds^2 = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{1}$$

where $\nu(r)$ and $\lambda(r)$ are two arbitrary functions.

The Einstein field equations for the charged anisotropic matter are given by

$$\frac{1}{r^2}\left(1-e^{-2\lambda}\right)+\frac{2\lambda'}{r}e^{-2\lambda} = \rho \tag{2}$$

$$-\frac{1}{r^2}\left(1-e^{-2\lambda}\right)+\frac{2\nu'}{r}e^{-2\lambda} = p_r \tag{3}$$

$$e^{-2\lambda}\left(\nu''+\nu'^2+\frac{\nu'}{r}-\nu'\lambda'-\frac{\lambda'}{r}\right)=p_t \tag{4}$$

$$\sigma=\frac{1}{r^2}e^{-\lambda}(r^2E)' \tag{5}$$

where ρ is the energy density, p_r is the radial pressure, E is electric field intensity, p_t is the tangential pressure and primes denote differentiations with respect to r. Using the transformations, $x = cr^2$, $Z(x) = e^{-2\lambda(r)}$ and $A^2y^2(x) = e^{2\nu(r)}$ with arbitrary constants A and c>0, suggested by Durgapal and Bannerji [32], the Einstein field equations can be written as

$$\frac{1-Z}{x} - 2\dot{Z} = \frac{\rho}{c} + \frac{E^2}{2c} \tag{7}$$

$$4Z\frac{\dot{y}}{y} - \frac{1-Z}{x} = \frac{p_r}{c} - \frac{E^2}{2c} \tag{8}$$

$$4xZ\frac{\ddot{y}}{y} + (4Z + 2x\dot{Z})\frac{\dot{y}}{y} + \dot{Z} = \frac{p_t}{c} + \frac{E^2}{2c} \tag{9}$$

$$p_t = p_r + \Delta \tag{10}$$

$$\frac{\Delta}{c} = 4xZ\frac{\ddot{y}}{y} + \dot{Z}\left(1 + 2x\frac{\dot{y}}{y}\right) + \frac{1-Z}{x} - \frac{E^2}{c} \tag{11}$$

$$\sigma^2 = \frac{4cZ}{x}\left(x\dot{E} + E\right)^2 \tag{12}$$

σ is the charge density, $\Delta = p_t - p_r$ is the anisotropic factor and dots denote differentiation with respect to x. With the transformations of [32], the mass within a radius r of the sphere take the form

$$m(x) = \frac{1}{4c^{3/2}}\int_0^x \sqrt{x}\rho(x)dx \tag{13}$$

In this paper, we assume a quadratic equation of state relating the radial pressure to the energy density given by

$$p_r = \alpha\rho^2 \tag{14}$$

where α is an arbitrary constant.

2. THE NEW MODELS

In order to solve Einstein field equations, we have chosen specific forms for the gravitational potential Z and the electrical field intensity E. Following Feroze and Siddiqui [24] and Malaver [25] we have taken the forms

$$Z(x) = (1-ax)^n \tag{15}$$

$$E^2 = 2c(1-z) \tag{16}$$

where a is a real constant and n is the adjustable parameter. The potential is regular at the origin and well behaved in the interior of the sphere. We have considered the particular cases for n = 1, 2.

For case n = 1, using (15) and (16) in eq. (7), we obtain

$$\rho = ac(3-x) \tag{17}$$

Substituting (17) in eq.(14), the radial pressure can be written in the form

$$p_r = \alpha a^2 c^2 (3-x)^2 \tag{18}$$

Using (17) in (13), the expression of the mass function is

$$m(x) = \frac{a}{2\sqrt{c}} x^{3/2}\left(1-\frac{x}{5}\right) \tag{19}$$

With (17) and (18), eq.(8) becames

$$\frac{\dot{y}}{y} = \frac{\alpha a^2 c}{4(1-ax)} + \frac{a(1-x)}{4(1-ax)} \tag{20}$$

Integrating (20), we obtain

$$y(x) = c_1(-1+ax)^A e^{B(x)} \tag{21}$$

where c_1 is the constant of integration and

$$A = -\frac{9\alpha a^2 c - 6\alpha ac + \alpha c + a - 1}{4a},$$

$$B(x) = -\frac{\alpha ac}{8}x^2 + \left(\frac{3}{2}\alpha ac + \frac{1-\alpha c}{4}\right)x$$

The anisotropy factor is given by for

$$\Delta = 4x(1-ax)\frac{\ddot{y}}{y} - a\left(1+2x\frac{\dot{y}}{y}\right) + a - 2ax \tag{22}$$

The metric functions can be written as

$$e^{2\lambda(r)} = \frac{1}{1-ax} \tag{23}$$

$$e^{2\nu(r)} = A^2 c_1^{\;2} (-1+ax)^{2A} e^{2B(x)} \tag{24}$$

For the electrical field intensity we have

$$E^2 = 2acx \tag{25}$$

and the charge density is

$$\sigma^2 = 18ac^2(1-ax) \tag{26}$$

With n = 2, the expression for the energy density is

$$\rho = ac\left[ax^2 - (5a+2)x + 6\right] \tag{27}$$

replacing (27) in (14) we have for the radial pressure

$$p_r = \alpha a^2 c^2 \left[ax^2 - (5a+2)x + 6\right]^2 \tag{28}$$

With (27) in (13) , for the mass function we obtain

$$m(x) = \frac{a}{2\sqrt{c}} x^{3/2} \left[\frac{x^2}{7} - \frac{(5a+2)}{5} x + 2\right] \tag{29}$$

The electrical field intensity is given by for

$$E^2 = 2ac\left(2x - ax^2\right) \tag{30}$$

and the charge density is

$$\sigma^2 = \frac{8ac^2(1-ax)^2(3-2ax)^2}{(2-ax)} \tag{31}$$

Substituting (15), (28) and (30) in eq.(8) we have

$$\frac{\dot{y}}{y} = \frac{\alpha a^2 c\left[ax^2 - (5a+2)x + 6\right]^2}{4(1-ax)^2} + \frac{a^2x^2 - \left(a^2 + 2a\right)x + 2a}{4(1-ax)^2} \tag{32}$$

integrating (32)

$$y(x)=c_2(-1+ax)^C e^{D(x)} \tag{33}$$

c_2 is the constant of integration and

$$C=\frac{10\alpha c-10\alpha ac-1}{4}$$

$$D(x)=\alpha a^4cx^4-(15\alpha a^4c+4\alpha a^3c)x^3+(51\alpha a^3c+75\alpha a^4c+3a^2)x^2$$
$$(3\alpha ac-36\alpha a^2c-75\alpha a^3c-3a)x+6\alpha ac-3\alpha a^2c-3\alpha c-3a+3$$

The metric functions $e^{2\lambda(r)}$, $e^{2\nu(r)}$ and the anisotropy factor Δ can be written as:

$$e^{2\lambda(r)}=\frac{1}{(1-ax)^2} \tag{34}$$

$$e^{2\nu(r)}=A^2c_2{}^2(-1+ax)^{2C}e^{2D(x)} \tag{35}$$

$$\Delta=4x(1-ax)^2\frac{\ddot{y}}{y}-2a(1-ax)\left(1+2x\frac{\dot{y}}{y}\right)+2a^2x^2-(4a+a^2)x+2a \tag{36}$$

3. PHYSICAL FEATURES OF THE NEW MODELS

Any physically acceptable solutions must satisfy the following conditions [28,33]:

(i) Regularity of the gravitational potentials in the origin.
(ii) Radial pressure must be finite at the centre and it vanishes at the surface of the sphere.
(iii) p_r > 0 and ρ >0 in the origin.
(iv) Decrease of the energy density and the radial pressure with the increase of the radius.

With n=1, $e^{2\lambda(0)}=1$, $e^{2\nu(0)}=A^2c_1^2(-1)^{2A}$ and $\left(e^{2\lambda(r)}\right)'_{r=0}=\left(e^{2\nu(r)}\right)'_{r=0}=0$.

This demonstrates that the gravitational potential is regular in the origin. In the centre $r=0$, ρ and $p_r=9\alpha a^2c^2$, both are positive if a > 0. In the surface of the star r = R we have $p_r(r=R)=0$ and $R=\sqrt{\frac{3}{c}}$. From (19) and with the suggested transformations by

Durgapal and Bannerji [32], we obtain

$$m(r) = \frac{acr^3}{2}\left(1 - \frac{cr^2}{5}\right) \tag{37}$$

and the total mass of the star is

$$m(r = R) = \frac{3a}{5}\sqrt{\frac{3}{c}} \tag{38}$$

To maintain of causality, the radial speed of sound defined as $v_{sr}^2 = \frac{dp_r}{d\rho}$ should be within the limit $0 \le v_{sr}^2 \le 1$ in the interior of the star. In this model we have $v_{sr}^2 = \frac{dp_r}{d\rho} = 2\alpha ac(3 - x)$ and so we must impose the condition $0 \le 2\alpha ac\left(3 - cr^2\right) \le 1$

For n=2, $e^{2\lambda(0)} = 1$, $e^{2\nu(0)} = A^2 c_2^2 (-1)^{2C} e^{12\alpha ac - 6\alpha a^2 c - 6\alpha c - 6a + 6}$ and $\left(e^{2\lambda(r)}\right)'_{r=0} = \left(e^{2\nu(r)}\right)'_{r=0} = 0$ .Again the gravitational potential is regular in $r = 0$.

In the centre $\rho(0) = 6ac$ and $p_r(0) = 36\alpha a^2 c^2$. In the boundary of the star r = R, we have $p_r(r = R) = 0$ and $R = \frac{\sqrt{2ac\left(5a + 2 + \sqrt{25a^2 - 4a + 4}\right)}}{2ac}$.

From (29) and for the transformations of Durgapal and Bannerji [32],

$$m(r) = \frac{acr^3}{2}\left[\frac{ac^2 r^4}{7} - \frac{(5a + 2)}{5} cr^2 + 2\right] \tag{39}$$

and the total mass of the star is

$$m(r = R) = \frac{\left[2ac\left(5a + 2 + \sqrt{25a^2 - 4a + 4}\right)\right]^{3/2}}{16a^2c^2}\left[\frac{\left(5a + 2 + \sqrt{25a^2 - 4a + 4}\right)^2}{28a} + \frac{(5a + 2)\left(5a + 2 + \sqrt{25a^2 - 4a + 4}\right)}{10a} + 2\right] \tag{40}$$

For this case, the condition $0 \leq v_{sr}^2 \leq 1$ implies that

$$0 \leq 2\alpha a^2 c^3 r^4 - \left(10\alpha a^2 c^2 + 4\alpha a c^2\right) r^2 + 6 \leq 1$$

4. CONCLUSION

In this paper we have found a class of models for charged anisotropic matter with quadratic equation of state for the radial pressure where the gravitational potential Z depends on an adjustable parameter n. The new obtained model may be used to model relativistic stars in different astrophysical scenes. The anisotropy, electric field and charge density are well defined and not present singularities at the centre. In the new obtained models the gravitational potentials are regular at the origin r = 0 and well behaved. The radial pressure and energy density are regular and positive throughout the stellar interior. The mass function is an increasing function, continuous and finite.

We show as a modification of the parameter n of the gravitational potential affects the electrical field, charge density, the anisotropy factor and the mass of the stellar object. The models presented in this article may be useful in the description of compact objects with charge and configurations with anisotropic matter.

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