Available online at www.worldscientificnews.com

World Scientific News

WSN 50 (2016) 64-73 EISSN 2392-2192

Analytical models for compact stars with a linear equation of state

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ABSTRACT

In this paper, we found two new classes of solutions to the Einstein-Maxwell system of equations for compact stars assuming an anisotropic pressure and a linear equation of state for the matter distribution within the framework of MIT-Bag Model with a particular form of the metric function. The exact solutions can be written in terms of elementary function in presence of an electromagnetic field. All the obtained models have a singularity in the charge density but not admit singularities in the matter and metric functions at the centre.

***Keywords*:** Einstein-Maxwell system; linear equation of state; charge density; MIT-Bag Model; metric function; exact solution

1. INTRODUCTION

One of the fundamental problems in the general theory of relativity is finding exact solutions of the Einstein field equations [1,2]. Some solutions found fundamental applications in astrophysics, cosmology and more recently in the developments inspired by string theory [2]. Different mathematical formulations that allow to solve Einstein´s field equations have been used to describe the behavior of objects submitted to strong gravitational fields known as neutron stars, quasars and white dwarfs [3-5].

In the construction of the first theoretical models of relativistic stars are important the works of Schwarzschild [6], Tolman [7], Oppenheimer and Volkoff [8]. Schwarzschild [6] found analytical solutions that allowed describing a star with uniform density, Tolman [7] developed a method to find solutions of static spheres of fluid and Oppenheimer and Volkoff [8] used Tolman's solutions to study the gravitational balance of neutron stars. It is important to mention Chandrasekhar's contributions [9] in the model production of white dwarfs in presence of relativistic effects and the works of Baade and Zwicky [10] who propose the concept of neutron stars and identify a astronomic dense objects known as supernovas.

The physics of ultrahigh densities is not well understood and many of the strange stars studies have been performed within the framework of the MIT bag model [11].

In this model, the strange matter equation of state has a simple linear form given by $p=\frac{1}{3}(\rho-4B)$ where ρ is the energy density, p is the isotropic pressure and B is the bag constant. However, theoretical works of realistic stellar models [12-15] it has been suggested that superdense matter may be anisotropic, at least in some density ranges. The existence of anisotropy within a star can be explained by the presence of a solid core, phase transitions, a type III super fluid, a pion condensation [16] or another physical phenomena. In such systems, the radial pressure is different from the tangential pressure. This generalization has been very used in the study of the balance and collapse of compact spheres [17-20]. Many researchers have used a great variety of mathematical techniques to try to obtain exact solutions for quark stars within the framework of MIT bag model, since it has been demonstrated by Komathiraj and Maharaj [11], Malaver [21], Thirukkanesh and Maharaj [22] and Thirukkanesh and Ragel [23].

With the use of Einstein´ field equations, important advances has been made to model the interior of a star. Feroze and Siddiqui [24] and Malaver [25,26] consider a quadratic equation of state for the matter distribution and specify particular forms for the gravitational potential and electric field intensity. Mafa Takisa and Maharaj [27] obtained new exact solutions to the Einstein-Maxwell system of equations with a polytropic equation of state. Thirukkanesh and Ragel [28] have obtained particular models of anisotropic fluids with polytropic equation of state which are consistent with the reported experimental observations. More recently, Malaver [29,30] generated new exact solutions to the Einstein-Maxwell system considering Van der Waals modified equation of state with and without polytropical exponent. Raghoonundun and Hobill [31] found new analytical models for compact stars with the use of Tolman VII solution.

Our objective in this paper is to generate a new class for charged anisotropic matter with the barotropic equation of state that presents a linear relation between the energy density and the radial pressure in static spherically symmetric spacetime using particular forms for the metric function $y(x)$. We have obtained some new classes of static spherically symmetrical models of charged matter where the variation of metric function modifies the radial pressure, charge density and the mass of the compact objects. This article is organized as follows, in Section 2, we present Einstein´s field equations. In Section 3, we make a particular choice of metric potential $y(x)$ that allows solving the field equations and we have obtained new models for charged anisotropic matter. In Section 4, a physical analysis of the new solutions is performed. Finally in Section 5, we conclude.

2. EINSTEIN FIELD EQUATIONS

Consider a spherically symmetric four dimensional space time whose line element is given in Schwarzschild coordinates by

$$ds^2 = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{1}$$

Using the transformations, $x = cr^2$, $Z(x) = e^{-2\lambda(r)}$ and $A^2y^2(x) = e^{2\nu(r)}$ with arbitrary constants A and c, suggested by Durgapal and Bannerji [32], the Einstein field equations as given in (1) are

$$\frac{1-Z}{x} - 2\dot{Z} = \frac{\rho}{c} + \frac{E^2}{2c} \tag{2}$$

$$4Z\frac{\dot{y}}{y} - \frac{1-Z}{x} = \frac{p_r}{c} - \frac{E^2}{2c} \tag{3}$$

$$4xZ\frac{\ddot{y}}{y} + (4Z + 2x\dot{Z})\frac{\dot{y}}{y} + \dot{Z} = \frac{p_t}{c} + \frac{E^2}{2c} \tag{4}$$

$$p_t = p_r + \Delta \tag{5}$$

$$\sigma^2 = \frac{4cZ}{x}\left(x\dot{E} + E\right)^2 \tag{6}$$

where ρ is the energy density, p_r is the radial pressure, E is electric field intensity , σ is the charge density, $\Delta = p_t - p_r$ is the anisotropic factor, p_t is the tangential pressure and dot is the derivative with respect to x.

With the transformations of [32], the mass within a radius r of the sphere take the form

$$M(x) = \frac{1}{4c^{3/2}}\int_0^x \sqrt{x}\rho(x)dx \tag{7}$$

In this paper, we assume the following lineal equation of state within the framework of MIT-Bag Model

$$p_r = \frac{1}{3}\rho \tag{8}$$

3. THE MODELS

3. 1. Model 1

Following Komathiraj and Maharaj [11], we choose the metric function in the particular form $y(x)$ as

$$y(x)=(a+x) \tag{9}$$

where a is constant. In this paper, we have considered the form of the electrical field proposed for Feroze and Siddiqui [33]

$$E^2=\frac{2c(1-Z)}{x} \tag{10}$$

Substituting (10) in eq. (2) we obtain

$$\rho=2c\dot{Z} \tag{11}$$

Using (10) in eq. (3), the radial pressure can be written as

$$p_r=4cZ\frac{\dot{y}}{y} \tag{12}$$

With (8), the eq. (12) becomes

$$-\frac{2\dot{Z}}{3}=4Z\frac{\dot{y}}{y} \tag{13}$$

Replacing (9) in (13), we obtain the first order equation

$$-\frac{2\dot{Z}}{3}=\frac{4Z}{a+x} \tag{14}$$

Integrating (14), we have for the gravitational potential $Z(x)$

$$Z=\frac{1}{(x+a)^6} \tag{15}$$

With eq. (9) and eq. (15) , the metric functions $e^{2\lambda(r)}$ and $e^{2\nu(r)}$ can be written as

$$e^{2\lambda(r)}=(a+x)^6 \tag{16}$$

$$e^{2\nu(r)} = A^2 (a+x)^2 \tag{17}$$

for the energy density we have

$$\rho = \frac{12c}{(a+x)^7} \tag{18}$$

Replacing (18) in (8) , we obtain for the radial pressure

$$p_r = \frac{4c}{(a+x)^7} \tag{19}$$

Using (18) in (7) the expression of the mass function is

$$M(x) = \frac{\sqrt{x}}{\sqrt{c}(a+x)}\left[-\frac{1}{2(a+x)^5} + \frac{1}{20a(a+x)^4} + \frac{9}{a^2(a+x)^3} + \frac{21}{320a^3(a+x)^2} + \frac{21}{256a^4(a+x)} + \frac{63}{512a^5}\right] + \frac{63arctag\left(\frac{\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{c}a^5\sqrt{a}} \tag{20}$$

for the electric field intensity E^2

$$E^2 = \frac{2c(a^6 + 6a^5x + 15a^4x^2 + 20a^3x^3 + 15a^2x^4 + 6ax^5 + x^6 - 1)}{x(a+x)^6} \tag{21}$$

and for charge density

$$\sigma^2 = \frac{2c^2\left[5x - a - (a+x)^7\right]^2}{x^2(a+x)^{14}\left[(a+x)^6 - 1\right]} \tag{22}$$

The tangential pressure can be written as

$$p_t = \frac{c\left(-x^8 - 8ax^7 - 28a^2x^6 - 56a^3x^5 - 70a^4x^4 - 56a^5x^3 - 28a^6x^2 - 8a^7x - 13x^2 - a^8 + a^2\right)}{x(a+x)^8} \tag{23}$$

The metric for this model is

$$ds^2 = -A^2(a+x)^2 dt^2 + \frac{(a+x)^6 dx^2}{4xc} + \frac{x}{c}\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \tag{24}$$

3. 2. Model 2

Motivated for Sunzu et al [34], another new exact solution of the Einstein-Maxwell system of equations (2-6) is gives by the metric function

$$y(x) = \frac{1+ax}{1-ax} \tag{25}$$

With (25), equation (13) becomes

$$\dot{Z} = -\frac{12aZ}{1-a^2x^2} \tag{26}$$

Eq. (26) can be integrated to give the gravitational potential

$$Z = \frac{(1-ax)^6}{(1+ax)^6} \tag{27}$$

Therefore we can find the following analytical model

$$e^{2\lambda(r)} = \frac{(1+ax)^6}{(1-ax)^6} \tag{28}$$

$$e^{2\nu(r)} = A^2\frac{(1+ax)^2}{(1-ax)^2} \tag{29}$$

$$\rho = \frac{24ac(1-ax)^5}{(1+ax)^7} \tag{30}$$

$$p_r = \frac{8ac(1-ax)^5}{(1+ax)^7} \tag{31}$$

$$M(x)=\frac{\sqrt{x}}{\sqrt{c}(a+x)}\left[\begin{array}{l}\frac{81}{8}-\frac{149}{4(a+x)}+\frac{451}{5(a+x)^2}-\frac{642}{5(a+x)^3}\\+\frac{496}{5(a+x)^4}-\frac{32}{(1+ax)^5}\end{array}\right]-\frac{15}{8}\frac{arctag\left(\sqrt{ax}\right)}{\sqrt{ac}}$$

(32)

$$E^2=8ac\frac{\left(3+10a^2x^2+3a^4x^4\right)}{(1+ax)^6} \tag{33}$$

$$\sigma^2=\frac{8ac^2(1-ax)^6\left(18a^4x^4-20a^3x^3+40a^2x^2-12ax+6\right)^2}{x(a+x)^{14}\left(2+10a^2x^2+3a^4x^4\right)} \tag{34}$$

$$p_t=\frac{c\left[16a^2x(1-ax)^4+8a(1-ax)^5-12a(1-ax)^5\right]}{(1+ax)^7}-\frac{48ca^2x(1-ax)^4}{(1+ax)^8}-\frac{c\left(12a+40a^3x^2+12a^5x^4\right)}{(1+ax)^6}$$

(35)

The metric for this model is

$$ds^2=-A^2\frac{(1+ax)^2}{(1-ax)^2}dt^2+\frac{(a+x)^6dx^2}{4xc(1-ax)^6}+\frac{x}{c}\left(d\theta^2+\sin^2\theta d\varphi^2\right) \tag{36}$$

4. PHYSICAL ANALYSIS

Any physically acceptable solutions must satisfy the following conditions [28] :

(i) Regularity of the gravitational potentials in the origin.

(ii) Radial pressure must be finite at the centre.

(iii) $P_r > 0$ and $\rho > 0$ in the origin.

(iv) Monotonic decrease of the energy density and the radial pressure with increasing of radius

The presented models constitute another new family of solutions for a compact star with anisotropic pressure. The metric functions can be written in terms of polynomial and elementary functions and the variables energy density, radial pressure, charge density and tangential pressure also are represented analytical. For the model 1, the metric functions $e^{2\nu(r)}$ and $e^{2\lambda(r)}$ behaves well inside the star and have a finite value of $e^{2\nu(r)}=a^2A^2$ and $e^{2\lambda(r)}=a^6$

in r = 0. In this case $\left(e^{2\lambda(r)}\right)'_{r=0} = \left(e^{2\nu(r)}\right)'_{r=0} = 0$ in the origin. This analysis demonstrated that the gravitational potential is regular at the centre r=0. The energy density is positive throughout the interior of the star, regular at the centre with value $\rho = \frac{12c}{a^7}$.

The radial pressure p_r is regular at the centre with value $p_r = \frac{4c}{a^7}$.In this model, the electric field intensity, charge density and tangential pressure admit a singularity at the centre of the stellar object.

For the model 2, the functions $e^{2\nu(r)}$ and $e^{2\lambda(r)}$ acquire finite values at the centre and $\left(e^{2\lambda(r)}\right)'_{r=0} = \left(e^{2\nu(r)}\right)'_{r=0} = 0$ as in the model 1. The energy density, radial pressure and electric field take the values of $\rho = 24ac$, $p_r = 8ac$ and $E^2 = 24ac$ in r = 0, respectively. In all the new classes of models, the mass function is continuous and behaves well inside the star and the charge density σ has a singularity at the centre.

5. CONCLUSIONS

We have generated new exact solutions to the Einstein-Maxwell system of equations specifying the form of the metric function $y(x)$ and a linear equation of state which is relevant in the description of charged anisotropic matter. The relativistic solutions presented are physically reasonable. The first solution correspond to model with finite values for the energy density and the radial pressure at the origin but present a singularity at the centre of stellar object for the electric field, charge density and tangential pressure. In the second solution, the radial pressure and energy density are regular and positive throughout the stellar interior.

In the new obtained models the gravitational potentials are regular at the origin r=0 and well behaved. The charge density admits a singularity at the centre and the mass function is an increasing function, continuous and finite. The models presented in this article may be useful in the description of relativistic compact objects with charge, quark stars and configurations with anisotropic matter.

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(Received 30 May 2016; accepted 18 June 2016)