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Acoustic cylindrical cloak

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ABSTRACT

Using the analogy between anisotropic acoustic metamaterials with magnetic metamaterials in transverse magnetic mode, an electromagnetic wave of 2 GHz n transverse magnetic mode, at normal incidence propagating through a two dimensional, anisotropic, semi infinite, double negative, metamaterial slab of 800 × 800 cells, embedded in free space, for the ideal loss case was simulated by a radially dependent finite difference time dependent method to study an ideal acoustic cylindrical cloak. For the simulations multiple cycle m-n-m pulses generating Gaussian beams were used as sinusoidal hard line sources. The simulations for acoustic cylindrical cloaking by a reduced parameter model and a higher order parameter model are also presented. The cloaking behaviour is largely dependent upon the transformation and not on the thickness of the cloak. The radial dependent model and the higher order transformation model are suited for acoustic cloaking.

***Keywords*:** Acoustic metamaterials; electromagnetic metamaterials; negative index; anisotropic' cloaking, radially dependent FDTD method; reduced parameter model' higher order parameter model

1. INTRODUCTION

The purpose of a cloaking device is to define a region of space invisibly isolated from passing acoustic waves. This is accomplished by manipulating the paths traversed by sound through a novel acoustic material. Although the objects in the defined location are still present, the incident waves are guided around them without being affected by the object itself.

A new design paradigm called conformal mapping and coordinate transformation has inspired, a series of key explorations to manipulate, store and control the flow of energy, in form of either sound, elastic waves or light radiation. The Maxwell equations possess a special symmetry that is an important element of the equivalence between coordinate transformations and material properties for electromagnetic waves. Because of the coordinate invariance in Maxwell's equations, the space for light can be bent in almost arbitrary ways by providing a desired spatial distribution of electric permittivity and magnetic permeability. But they usually call for complicated medium with anisotropic and spatially varying permittivity and permeability to accomplish the desired functionality. Field equations that describe waves in most other systems do not have this same symmetry.

In two dimensions, acoustic and electromagnetic waves in isotropic media are exactly equivalent [1] and Cummer and Schurig have shown that this isomorphism holds for anistropic media as well. This equivalence implies that the two dimensional electromagnetic simulations of plane wave interaction with a cloaking shell are also solutions to the analogous acoustic cloaking problems [2]. The analogy between the acoustic metamaterial parameters and electromagnetic metamaterial parameters in transverse magnetic mode was used by Torrent and Dehesa [3]. Zhang developed experimental techniques to fabricate two dimensional acoustic cloaks and negative refractive lenses proposing unit cells for isotropic and anisotropic metamaterials [4]. A dispersive finite difference time domain (FDTD) scheme or a leap frog scheme has been used to determine the values of electric and magnetic fields [5]. A two dimensional electromagnetic cylindrical cloak proposed by Rahm et al. [6] with coordinate transformations developed using finite difference time domain method was used by Mishrikey [7] to investigate the cloaking behaviour. Electromagnetic two dimensional cylindrical cloaking in transverse electric mode was implemented by Dawood [8]. Due to the unrealistic nature of a radial dependent parameter model in electromagnetic cylindrical cloak, a nonmagnetic cloak with minimized scattering was proposed by Cai, et al with the use of a higher order transformation [9].

In electromagnetic metamaterials the simulations are mostly done in transverse electromagnetic mode due to natural existence of negative permittivity where focus is on for the creation of artificial negative permeability. In a previous work we have simulated acoustic isotropic metamaterials by using the analogy between acoustic and electromagnetic metamaterials in transverse magnetic mode [10]. In this paper we have extended this work to demonstrate the characteristics of acoustic cloaking by considering three coordinate transformations in a negative index semi infinite metamaterial slab embedded in free space for the ideal loss case for a sinusoidal "hard" line source at normal incidence. A two dimensional cylindrical ideal cloak was developed using radially dependent finite difference time dependent method. The radial dependent parameter model is unrealistic and hence a reduced parameter model is discussed and finally simulations were performed with a higher order transformation which introduces greater cloaking properties. The derivations and simulations were carried out for the electromagnetic metamaterial parameters to avoid confusions generated by some terms such as induced electric and magnetic currents and electric and magnetic plasma frequencies as the developed analogy cannot describe such terms in acoustics but can describe the final simulated results. The simulations were run on a domain size of 800×800 cells with cell size $\Delta x = \Delta y = \lambda / 150$, for electromagnetic wave frequency $2.0\,\mathrm{GHz}$ with Courant stability requirement with enough time steps so that the fields reached steady states. The inner and outer boundary radii of the cloak used are 0.1m and 0.2m respectively.

2. CYLINDRICAL CLOAK

In two dimensions, the acoustic waves in fluid and Maxwell equations for electromagnetic waves with one single mode have identical form under certain variable exchange. Acoustic wave is longitudinal wave and the pressure and the particle velocity are the parameters used to describe the wave. In an in viscid medium, the two dimensional time harmonic acoustic wave equations in cylindrical coordinate with z invariance are:

$$\begin{aligned}
\frac{\partial P}{r\partial \phi} &= -i\omega\rho_{\phi}u_{\phi} \\
\frac{\partial P}{\partial r} &= -i\omega\rho_{r}u_{r} \\
\frac{1}{r}\frac{\partial}{\partial r}(ru_{r})+\frac{1}{r}\frac{\partial u_{\phi}}{\partial \phi} &= -i\omega\beta P
\end{aligned} \tag{1}$$

where P is the scalar pressure, u_r, u_ϕ are particle velocity, ρ_r, ρ_ϕ are density along r and ϕ directions respectively, β is the compressibility. There is only a longitude wave in this anisotropic medium. In the same z invariant cylindrical coordinates, the Maxwell's equations for transverse magnetic (TM) mode are:

$$\begin{aligned}
\frac{1}{r}\frac{\partial E_z}{\partial \phi} &= i\omega\mu_r H_r \\
\frac{\partial E_z}{\partial r} &= -i\omega\mu_{\phi}H_{\phi} \\
\frac{1}{r}\frac{\partial}{\partial r}(rH_{\phi})-\frac{1}{r}\frac{\partial H_r}{\partial \phi} &= -i\omega\varepsilon_z E_z
\end{aligned} \tag{2}$$

The medium has anisotropic permittivity and permeability, but these tensors only has diagonal component. The two sets of equations given in equations 1 and 2 are equivalent under the following variable exchange:

$$P \leftrightarrow -E_z,\quad u_r \leftrightarrow H_{\phi},\quad u_{\phi} \leftrightarrow -H_r,\quad \rho_{\phi} \leftrightarrow \mu_r,\quad \rho_r \leftrightarrow \mu_{\phi},\quad \beta \leftrightarrow \varepsilon_z \tag{3}$$

In terms of boundary condition in electromagnetism, the normal component E_z and tangential component H_ϕ are continuous, while at a fluid interface, the normal component of particle velocity u_r and pressure P are continuous. Thus the boundary conditions are preserved under the variable exchange. This implies that the two-dimensional acoustic equations have the same form under the coordinate transformation as Maxwell's equations.

When light travels in a gradient index medium, where the refractive index changes as a function of position, the light ray follows a curved path determined by the nature of the gradient. The invariance of Maxwell's equations in a gradient medium enables to control the propagation path of electromagnetic waves by controlling the permittivity and permeability distribution in space. The Maxwell equations in free space are usually written in the virtual space in Cartesian coordinates and the Cartesian grid in the virtual space will deform under the transformation and this deformed grid shows ray trajectories in the new physical space. By providing the straightforward prescription for a material, the fields in the physical space which take up the distorted configuration can be obtained [4]. The invariance in Maxwell's equations can be proved by the rules of covariant and contravariant transformations [11]. In two dimensional cases, the idea is to make a coordinate transformation that takes a point in space and expands it into a cylinder such that the interior is invisible to an observer on the outside. So that neither radiation cannot get into a concealed volume nor any radiation can get out from it. A simple transformation that can realize this goal in an electromagnetic material is to compress all fields in the region $0 < r < R_2$ into the annual region $R_1 < r < R_2$. This transformation, in cylindrical coordinates is [4]:

$$r' = R_1 + r(R_2 - R_1)/R_2; \quad \theta' = \theta; \quad z' = z \tag{4}$$

leads to a prescription for the cloak in $R_1 < r < R_2$ as:

$$\begin{aligned} \frac{\mu_\phi}{\mu_0} &= \frac{r}{r-R_1}; \qquad \frac{\mu_r}{\mu_0} = \frac{r-R_1}{r} \\ \frac{\varepsilon_z}{\varepsilon_0} &= \left(\frac{R_2}{R_2-R_1}\right)^2 \frac{r-R_1}{r} \end{aligned} \tag{5}$$

By using material with above prescription, the transverse magnetic field will exclude all fields from the central region in the engineered space making the object together with the cloak invisible. As two dimensional acoustic equations have the same form invariance under coordinate transformation as Maxwell's equations, the equation 5 can be realized acoustically, if a gradient medium can be implemented with anisotropic density and bulk modulus as:

$$\begin{aligned} \frac{\rho_r}{\rho_b} &= \frac{r}{r-R_1}; \qquad \frac{\rho_\phi}{\rho_b} = \frac{r-R_1}{r} \\ \frac{\beta}{\beta_b} &= \left(\frac{R_2}{R_2-R_1}\right)^2 \frac{r-R_1}{r} \end{aligned} \tag{6}$$

where ρ_b, β_b are the density and compressibility of the background medium. R_1, R_2 are the radius of the inner and outer boundary of the two dimensional cloak [4].

3. FINITE DIFFERENCE TIME DEPENDENT METHOD FOR CLOAKING

The Yee finite difference time dependent (FDTD) algorithm only deals with frequency independent materials, so it is necessary to create a new FDTD scheme to handle frequency-dependent materials. A new radially dependent FDTD scheme was proposed by Rahm et.al [6] for modelling electromagnetic cloaking structures. Using the analogy proposed by Cummer and Schurig [2], that model was modified for a transverse magnetic mode in order to simulate acoustic cloaking. To avoid confusions the simulations were carried on for electromagnetic metamaterial parameters as the behaviour of the electromagnetic parameters in a metamaterial slab is exactly identical to the behaviour of their acoustic metamaterial counterparts.

In Drude metamaterial model, the Lossy Drude polarization and magnetization models are used in the frequency domain, and the radial dependent relative permeability of the medium $\mu_r(\omega)$ is :

$$\mu_r(\omega) = 1-\frac{\omega_p^2}{\omega^2 - j\omega\gamma} \tag{7}$$

where ω_p is the plasma frequency, γ is the collision frequency of material, $j=-i$ is the complex root. Since the material parameters are radially dependent, the finite different time domain scheme is used in rectangular coordinates, followed by a cylindrical to Cartesian coordinate transformation for the permeability tensor in the simulations:

$$\begin{bmatrix} \mu_{xx} & \mu_{xy} \\ \mu_{yx} & \mu_{yy} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \mu_r & 0 \\ 0 & \mu_\phi \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$\begin{bmatrix} \mu_{xx} & \mu_{xy} \\ \mu_{yx} & \mu_{yy} \end{bmatrix} = \begin{bmatrix} \mu_r\cos^2\theta + \mu_\phi\sin^2\theta & (\mu_r-\mu_\phi)\sin\theta\cos\theta \\ (\mu_r-\mu_\phi)\sin\theta\cos\theta & \mu_r\sin^2\theta + \mu_\phi\cos^2\theta \end{bmatrix} \tag{8}$$

The governing equations in Drude metamaterial model for the electromagnetic metamaterials are:

$$\boldsymbol{D} = \varepsilon_0\boldsymbol{E} + \boldsymbol{P} = \varepsilon_0\varepsilon_r\boldsymbol{E}$$

$$\boldsymbol{B} = \mu_0\boldsymbol{H} + \boldsymbol{M} = \mu_0\mu_r\boldsymbol{H}$$

(9)

where ε_0 and ε_r are electric permittivity of free space and relative permittivity of the medium and μ_0 and μ_r are magnetic permeability of free space and relative permeability of the medium. $\boldsymbol{M}$ is the magnetization and $\boldsymbol{P}$ is the electric polarization. Magnetic permeability and electric permittivity are only dependent on the frequency. If the induced electric and magnetic currents for the transverse magnetic mode are $\boldsymbol{J} = \partial\boldsymbol{P}/\partial t$ and $\boldsymbol{K} = \partial\boldsymbol{M}/\partial t$ respectively, then the field components E_z, H_x and H_y are:

$$E_z = \frac{1}{\varepsilon_0 \omega_{pe}^2}\left(\frac{\partial J_z}{\partial t} + \Gamma_e J_z\right)$$

$$H_x = \frac{1}{\mu_0 \omega_{pm}^2}\left(\frac{\partial K_x}{\partial t} + \Gamma_m K_x\right) \tag{10}$$

$$H_y = \frac{1}{\mu_0 \omega_{pm}^2}\left(\frac{\partial K_y}{\partial t} + \Gamma_m K_y\right)$$

Since the permittivity and permeability is highly anisotropic $\boldsymbol{J}$ and $\boldsymbol{K}$ are zero.

Yee's algorithm is based on the time-dependent Maxwell's curl equations and it couples the equations in order to solve for multiple field components simultaneously. The spatially staggered grid simplifies the contours involved in the curl equations, as it maintains the continuity of the tangential components of the electric and magnetic fields, as well as simplifying the implementation of the boundary conditions.

Yee's algorithm uses a fully explicit leapfrog scheme in time that also involved second-order central differences, so the field components are staggered temporally. Any three dimensional vector field component can be written as $F(i\Delta x, j\Delta y, k\Delta z, n\Delta t) = F_{i,j,k}^{n}$ where $\Delta x, \Delta y$ and Δz are the mesh sizes in the *x*, *y* or *z* direction respectively, and Δt is the time increment.

So, the second order finite difference for the first partial derivative of F in the x direction evaluated at time $t_n = n\Delta t$ is

$$\frac{\partial F}{\partial x}(i\Delta x, j\Delta y, k\Delta z, n\Delta t) = \frac{F_{(i+1/2),j,k}^{n} - F_{(i-1/2),j,k}^{n}}{\Delta x} \tag{11}$$

The Yee algorithm is a conditionally stable algorithm, and there exists a maximum allowable time step $\Delta t_{\max}$ in order for the algorithm to remain stable:

$$\Delta t_{\max} \le \frac{1}{c\sqrt{(1/\Delta x)^2 + (1/\Delta y)^2 + (1/\Delta z)^2}} \quad \Rightarrow \quad c\Delta t_{\max}\sqrt{(1/\Delta x)^2 + (1/\Delta y)^2 + (1/\Delta z)^2} \le 1 \tag{12}$$

where *c* is the speed of light in vacuum. In one dimension, the Courant number (*S*) is defined as, $S = c\Delta t / \Delta x$ and $S = 1$. If $\Delta x = \Delta y$ in two dimensional case, then in order that the algorithm to remain stable, $S = 1/\sqrt{2}$. Positions of the electric and magnetic field components about a cubic unit cell of the Yee lattice is shown in Figure 1. This staggered grid is used in FDTD.

Wave propagation from scattering or waveguides requires an unbounded domain. Since it is impossible to have an unbounded domain, the absorbing boundary conditions, an additional domain surrounding the computational domain is used. Though this external domain is connected to the computational domain boundary, the fields in this domain are computed separately where all tangential properties are preserved, and the fields are continuous across the boundary. The external region is designed as a lossy material, so that it really does not absorb the wave but severely dampen the wave as it enters that region by removing its power so quickly that there's nothing left to reflect back off the outer boundary [5].

Substituting equation 10 in 11 and simplifying with second order Taylor expansion lead to [2, 6]:

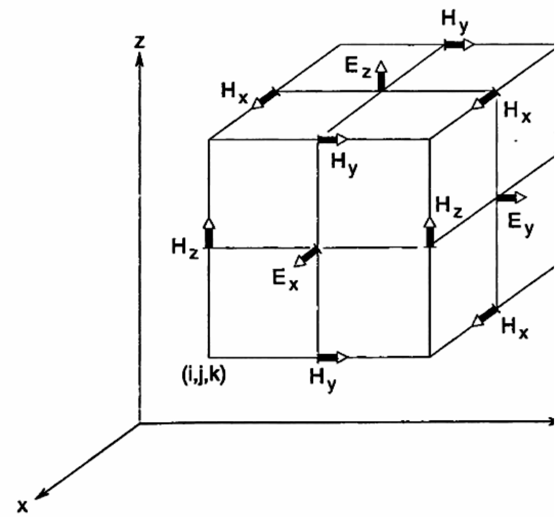

Figure 1. Positions of the electric and magnetic field components about a cubic unit cell of the Yee space lattice [5]

$$
\begin{aligned}
B_x\Big|_{i,j+\frac{1}{2}}^{n+\frac{1}{2}} &= B_x\Big|_{i,j+\frac{1}{2}}^{n-\frac{1}{2}} + \frac{\Delta t}{\Delta x}\left(-E_z\Big|_{i,j+1}^{n} + E_z\Big|_{i,j}^{n}\right) \\
B_y\Big|_{i+\frac{1}{2},j}^{n+\frac{1}{2}} &= B_y\Big|_{i+\frac{1}{2},j}^{n-\frac{1}{2}} + \frac{\Delta t}{\Delta x}\left(E_z\Big|_{i+1,j}^{n} - E_z\Big|_{i,j}^{n}\right) \\
D_z\Big|_{i,j}^{n+1} &= D_z\Big|_{i,j}^{n} + \frac{\Delta t}{\Delta x}\left(H_y\Big|_{i+\frac{1}{2},j}^{n+\frac{1}{2}} - H_y\Big|_{i-\frac{1}{2},j}^{n+\frac{1}{2}} - H_x\Big|_{i,j+\frac{1}{2}}^{n+\frac{1}{2}} + H_x\Big|_{i,j-\frac{1}{2}}^{n+\frac{1}{2}}\right)
\end{aligned}
\tag{13}
$$

To simulated metamaterial cloaking for electromagnetic waves in transverse magnetic mode (TM), the radial dependent model used by Dawood [8] was developed for the FDTD scheme. For the transverse magnetic mode, the field components E_z, H_x and H_y are:

$$
\begin{aligned}
E_z\Big|_{i,j}^{n+1} &= a_e(D_z\Big|_{i,j}^{n+1} - 2D_z\Big|_{i,j}^{n} + D_z\Big|_{i,j}^{n-1}) + b_e(D_z\Big|_{i,j}^{n+1} - D_z\Big|_{i,j}^{n-1}) \\
&\quad + c_e(2E_z\Big|_{i,j}^{n} - E_z\Big|_{i,j}^{n-1}) + d_e(2E_z\Big|_{i,j}^{n} + E_z\Big|_{i,j}^{n-1}) + e_e E_z\Big|_{i,j}^{n-1} \\
H_x\Big|_{i,j+\frac{1}{2}}^{n+\frac{1}{2}} &= a_m(B_x\Big|_{i,j+\frac{1}{2}}^{n+\frac{1}{2}} - 2B_x\Big|_{i,j+\frac{1}{2}}^{n-\frac{1}{2}} + B_x\Big|_{i,j+\frac{1}{2}}^{n-\frac{3}{2}}) + b_m\left(B_x\Big|_{i,j+\frac{1}{2}}^{n+\frac{1}{2}} - B_x\Big|_{i,j+\frac{1}{2}}^{n-\frac{3}{2}}\right) \\
&\quad + c_m\left(2H_x\Big|_{i,j+\frac{1}{2}}^{n-\frac{1}{2}} - H_x\Big|_{i,j+\frac{1}{2}}^{n-\frac{3}{2}}\right) + d_m\left(2H_x\Big|_{i,j+\frac{1}{2}}^{n-\frac{1}{2}} + H_x\Big|_{i,j+\frac{1}{2}}^{n-\frac{3}{2}}\right) + e_e H_x\Big|_{i,j+\frac{1}{2}}^{n-\frac{3}{2}}
\end{aligned}
\tag{14}
$$

$$H_y\Big|_{i+\frac{1}{2},j}^{n+\frac{1}{2}} = a_m\left(B_y\Big|_{i+\frac{1}{2},j}^{n+\frac{1}{2}} - 2B_y\Big|_{i+\frac{1}{2},j}^{n-\frac{1}{2}} + B_y\Big|_{i+\frac{1}{2},j}^{n-\frac{3}{2}}\right) + b_m\left(B_y\Big|_{i+\frac{1}{2},j}^{n+\frac{1}{2}} - B_y\Big|_{i+\frac{1}{2},j}^{n-\frac{3}{2}}\right)$$
$$+c_m\left(2H_y\Big|_{i+\frac{1}{2},j}^{n-\frac{1}{2}} - H_y\Big|_{i+\frac{1}{2},j}^{n-\frac{3}{2}}\right) + d_m\left(2H_y\Big|_{i+\frac{1}{2},j}^{n-\frac{1}{2}} + H_y\Big|_{i+\frac{1}{2},j}^{n-\frac{3}{2}}\right) + e_e H_y\Big|_{i+\frac{1}{2},j}^{n-\frac{3}{2}}$$

where

$$a_e = a_m = \frac{4}{g};\quad b_e = b_m = \frac{\beta}{g};\quad c_e = a_e\varepsilon_0,\quad c_m = a_m\mu_0;\quad d_e = -\frac{c_e\alpha}{4},\quad d_m = -\frac{c_m\alpha}{4};$$

$$e_e = \frac{c_e b_e \gamma_e}{4\gamma},\quad e_m = \frac{c_m b_m \gamma_m}{4\gamma},\quad \beta = \gamma(2\Delta t),\quad \alpha = \omega_P^2(\Delta t)^2,\quad g = \varepsilon_0\left(4+\alpha+\beta\right).$$

γ_e and γ_m are the electric and magnetic damping (collision) frequencies respectively.

The three different mathematical models used by Elander [11] in transverse electric mode for electromagnetic metamaterial cloaking were modified to be used in transverse magnetic mode.

The two dimensional FDTD simulation with cell size $\Delta x = \Delta y = \lambda/150$, where λ is the wavelength of the operating frequency $f = 2.0\,\text{GHz}$. The time step meets the usual Courant stability requirement, so the time step $\Delta t = \Delta x/\sqrt{2}c$, where c is the speed of light. A semi infinite double negative (DNG) medium or negative index medium in which both the relative permittivity and permeability are negative i.e., $\varepsilon_r < 0$, $\mu_r < 0$ at a certain frequency embedded in free space was considered.

Then $n_i = -\sqrt{\varepsilon_r\mu_r}$, the negative sign is taken to satisfy the causality. A sinusoidal hard line source at normal incidence was used, where the whole column of the field E_z is updated with the source continually in order to propagate the wave through the space.

4. IDEAL RADIALLY DEPENDENT CLOAK

For the ideal lossless case, $\gamma = 0$ was assumed. In this case for all of the simulations, an ideal cloak was considered, with the parameters $\varepsilon_r = \varepsilon_\phi = \mu_z = 0$. The material parameters read:

$$\mu_\phi = \frac{r}{r-R_1};\quad \mu_r = \frac{r-R_1}{r};\quad \varepsilon_z = \left(\frac{R_2}{R_2-R_1}\right)^2\frac{r-R_1}{r} \tag{15}$$

where μ_ϕ, μ_r are angular and radial relative permeability values and ε_z is the relative permittivity of medium. Using the radial dependent parameter model given in equation 5, the real time simulation of E_z was performed for inner and outer radius $R_1 = 0.1\,\text{m}$ and $R_2 = 0.2\,\text{m}$ for the domain size of 800×800 cells. For each instant of time, the snapshots of the electric field intensity in z direction in the simulation region were taken. Figure 2, 3 and 4 show the snapshots taken for the simulations 800, 1000 and 1800 time steps respectively. The highest positive values are colored red and the highest negative values are blue. Since E_z is analogous to the magnitude of pressure *P*, these frames describe the behavior of an acoustic wave inside an acoustic metamaterial cloak. From the snapshots taken for the ideal cloak simulation it is evident that the metamaterial act as a perfect cloak as there are no reflections or interferences in the incident wave region. The hidden object is nicely cloaked. But a radial dependent model where all the parameters vary radially is unrealistic and not practically attainable.

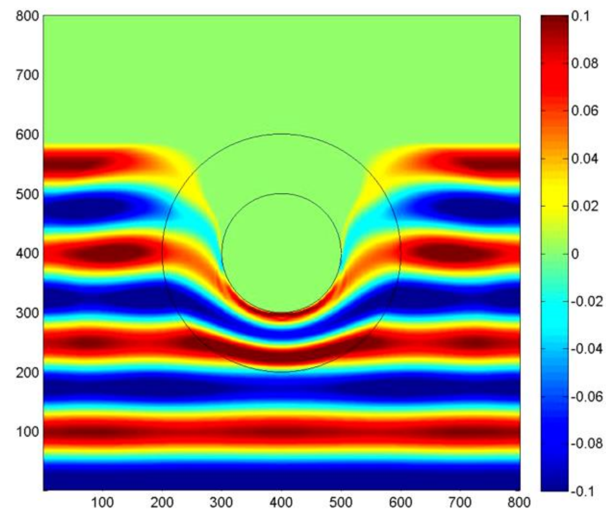

Figure 2. Electric field intensity distribution in z direction for electromagnetic slab or magnitude of pressure distribution for acoustic slab at 800 time step. Black lines indicate inner and outer surfaces.

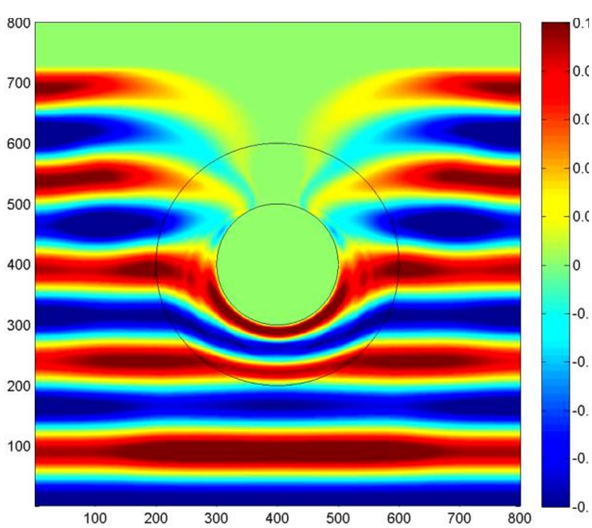

Figure 3. Electric field intensity distribution in z direction for electromagnetic slab or magnitude of pressure distribution for acoustic slab at 1000 time step. Black lines indicate inner and outer surfaces.

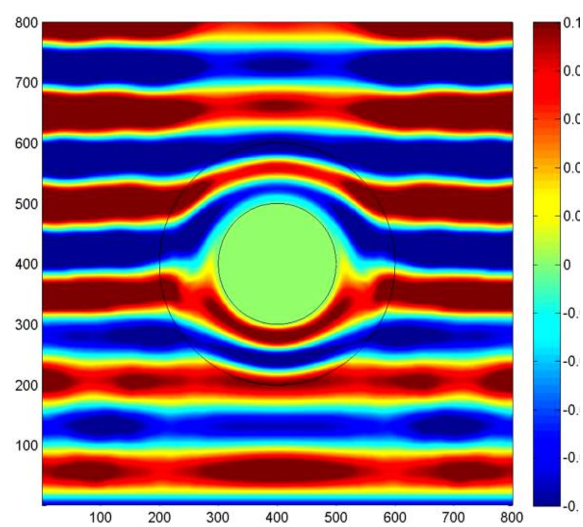

Figure 4. Electric field intensity distribution in z direction for electromagnetic slab or magnitude of pressure distribution for acoustic slab at 1800 time step. Black lines indicate inner and outer surfaces.

5. SIMPLIFIED CLOAK

Since all the parameters in an ideal cloak depend on the radius it is unrealizable. Therefore a reduced set of parameters was realized in the ideal cloak by setting $\varepsilon_r = \varepsilon_\phi = \mu_z = 0$ and the relative permittivity of medium $\varepsilon_z = 1$:

$$\varepsilon_z = \left(\frac{R_2}{R_2 - R_1}\right)^2 \frac{r - R_1}{r} = 1$$

$$\mu_\phi = \frac{r}{r - R_1} = \left(\frac{R_2}{R_2 - R_1}\right)^2 \tag{16}$$

$$\mu_r = \mu_r \varepsilon_z = \left(\frac{r - R_1}{r}\right)^2 \left(\frac{R_2}{R_2 - R_1}\right)^2$$

The real time simulation of both E_z and H_x were performed for $R_1 = 0.1\,\text{m}$ and $R_2 = 0.2\,\text{m}$ for the domain size of 800×800 cells. These are shown in figures 5 and 6 for 1400 and 1800 time steps respectively. The highest positive values are colored red and the highest negative values are blue. Since the electric field intensity E_z is analogous to the magnitude of the pressure P these frames describe the behavior of an acoustic wave inside an acoustic metamaterial cloak. These figures show the reduced parameter model does not present a good cloaking behaviour as there are some ripples in the incident wave region in electric field intensity distribution in z direction or in the pressure distribution, indicating reflections. This is because the impedance matching at the outer boundary of the cloak is no longer satisfied. Even though this model is realistic it does not hide the object in the cloak properly. The simulation performed for different set of inner and outer radii, namely for $R_1 = 0.125\,\text{m}$ and $R_2 = 0.250\,\text{m}$ for 2200 time steps is presented in figure 7. With the increase thickness of the cloak, although the number of ripples in the transmitted region for electric field intensity E_z or the pressure P is reduced, there are still

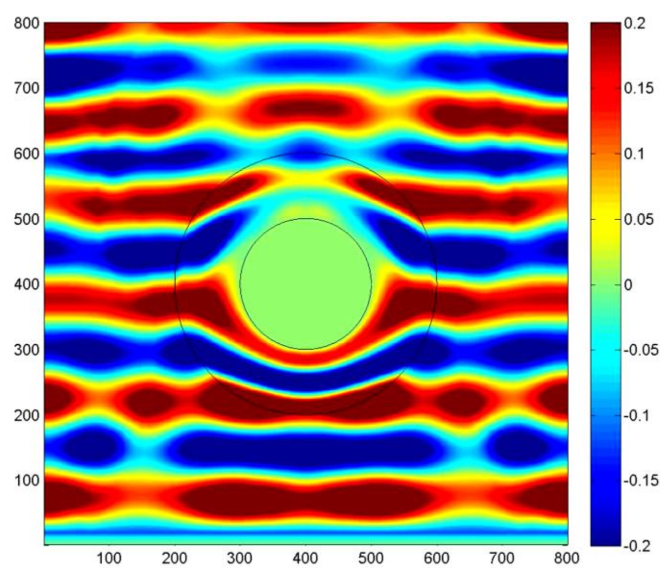

Figure 5. Electric field intensity distribution in z direction for electromagnetic slab or magnitude of pressure distribution for acoustic slab of $R_1 = 0.1\,\text{m}$ and $R_2 = 0.2\,\text{m}$ at 1400 time step. Black lines indicate inner and outer surfaces.

some ripples present in the incident wave region. Varying the thickness beyond the thickness of the shielding object is not useful as in this case, the cloak occupies a substantial amount of space than the object.

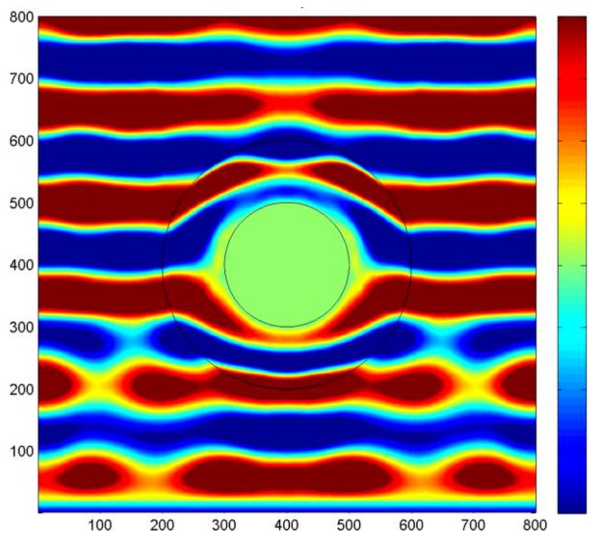

Figure 6. Electric field intensity distribution in z direction for electromagnetic slab or magnitude of pressure distribution for acoustic slab of $R_1 = 0.1\,\text{m}$ and $R_2 = 0.2\,\text{m}$ at 1800 time step. Black lines indicate inner and outer surfaces.

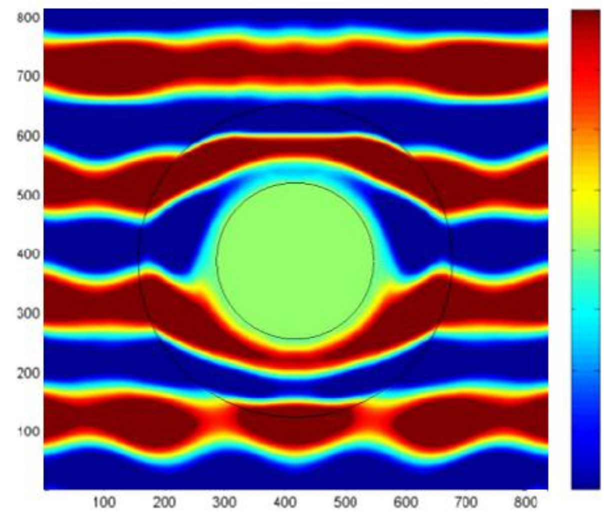

Figure 7. Electric field intensity distribution in z direction for electromagnetic slab or magnitude of pressure distribution for acoustic slab of $R_1 = 0.125\,\text{m}$ and $R_2 = 0.225\,\text{m}$ at 2200 time step. Black lines indicate inner and outer surfaces.

6. HIGER ORDER TRANSFORMATION

To reduce the scattering in the simplified model, a high order transformation proposed by Cai, et al. [9] for transverse electric mode was modified for the transverse magnetic mode. The ideal situation is for $\varepsilon_r = \varepsilon_\phi = \mu_z = 0$ and the basic constraint is that r must lie between inner and outer radii boundaries $R_1 < r < R_2$. The material parameters for transverse magnetic mode read as:

$$\mu_r = \left(\frac{r'}{r}\right)^2; \qquad \mu_\phi = \left(\frac{\partial g(r')}{\partial r'}\right)^{-2}; \qquad \varepsilon_z = 1 \tag{17}$$

where,

$$r = g(r') = \left(\frac{R_1}{R_2}\frac{r'}{R_2 - 2} + 1\right) r' + R_1 \tag{18}$$

According to Cai, et al. [9], in the transformation, the constraint should satisfy $R_1 / R_2 < 0.5$. Therefore inner and outer radii of the cylinder were taken as $R_1 = 0.1\,\mathrm{m}$ and $R_2 = 0.2\,\mathrm{m}$. Simulations were done with respect to r', for $0 \le r' \le R_2$. These values are transformed back to r by the inverse transformation $\mu_r = (r'/r)^2$. The numerical simulations were done using Matlab® 2012a for the domain size of 800×800 cells and the z component of the electric field intensity in the simulation region is shown in figures 8 and 9 for 1600 and 1800 time steps respectively. The highest positive values are coloured red and the highest negative values are blue. These frames describe the behaviour of pressure P of an acoustic wave inside an acoustic cloak. The higher order transformation indicates a better cloaking behaviour and it is almost perfect as the unrealistic radial dependent model. There are no visible ripples or interferences in the incident wave region of the cloak. And hence it shields the object from the incident waves properly.

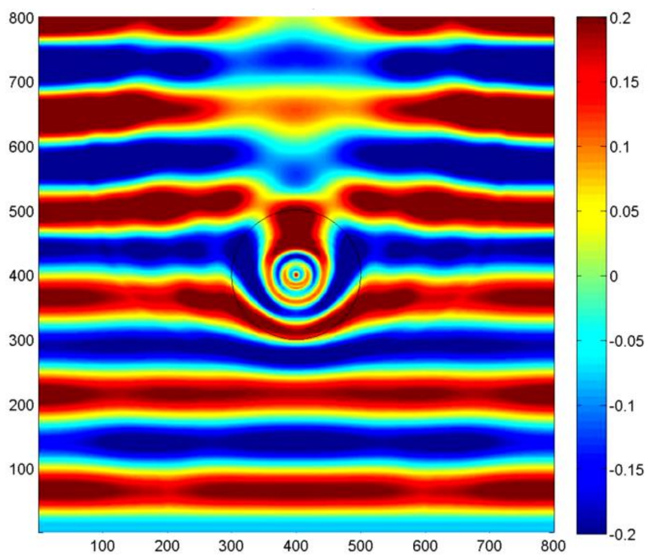

Figure 8. Electric field intensity distribution in z direction for electromagnetic slab or magnitude of pressure distribution for acoustic slab of $R_1 = 0.1\mathrm{m}$ and $R_2 = 0.2\,\mathrm{m}$ at 1600 time step. Black lines indicate inner and outer surfaces.

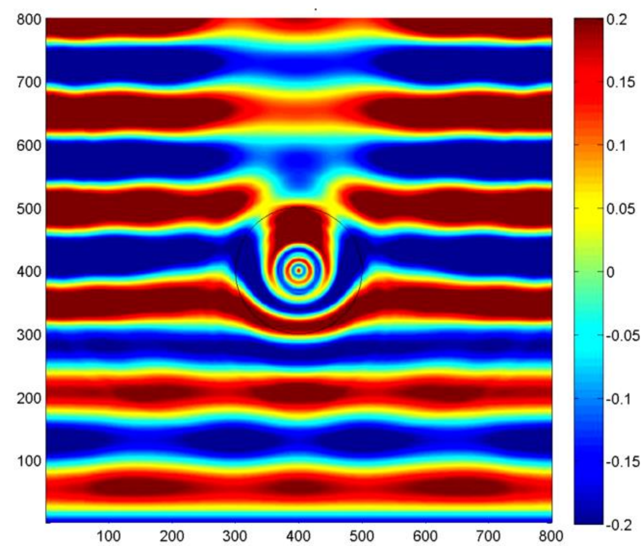

Figure 9. Electric field intensity distribution in z direction for electromagnetic slab or magnitude of pressure distribution for acoustic slab of $R_1 = 0.1\mathrm{m}$ and $R_2 = 0.2\mathrm{m}$ at 1800 time step. Black lines indicate inner and outer surfaces.

7. CONCLSIONS

Using the analogy between acoustic and electromagnetic metamaterials in two dimensions for semi-infinite double negative material in the transverse magnetic mode, that is the intensity of E_z is analogous to the variation of the magnitude of pressure P in acoustics, the simulations were performed for transverse magnetic mode to observe the behaviour of pressure in acoustic cloaking. The FDTD method was adopted to describe a two dimensional cylindrical acoustic cloak and the results were simulated for three transformation models. The radial dependent model has no ripples in the incident waves.

So there are no reflections occurring from it and has nicely guided incident waves around the subject and the cloaking behaviour can be observed. Since all its parameters depend on the radius it is unrealistic. With the reduced parameter model there are considerable amount of ripples in the E_z field or pressure, indicating reflections from the cloak. Even the increased thickness of cloak produced considerable amount of ripples. So the reduced material parameter model is not good for an acoustic cloak even though it is a realistic model.

The higher order transformation model showed no observable ripples in z component of the electric intensity or pressure field for the incident waves. The thickness of the cloak is at its maximum for this model with the constraint $2R_1 < R_2$. The higher order transformation model is suited for acoustic cloaking behaviour. The cloaking behaviour is largely dependent upon the transformation which has been used and not on the thickness of the cloak as increasing the thickness of the cloak is limited due to practical aspects.

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