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# Influence of Variable Electroconductivity and Radiation on MHD Couette Flow

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**ABSTRACT**

A study of Couette flow in cylindrical coordinate under the influence electroconductivity and magnetic field was carried out. Approximate solution of the governing equations and its analysis show decrease in temperature as a result of increase in radiation and Prandtl number. The study also reveals that increase in grashof number, Reynolds number, Prandtl number and electroconductivity result in an increase in velocity distribution while magnetic field increase, lead to a decrease in velocity. The shear stresses of the outer and inner cylinders at the wall of the plates and the heat flux is also determined.

***Keywords*: **MHD; Electrconductivity; Prandtl number; Grashof number; Reynolds Number; Radiation; Couette flow

## 1. INTRODUCTION

Maurice Couette is credited to have made extensive contribution to the study of fluid flow phenomena, in particular, flow of fluid brought about by the relative movement of the plates. A typical example is the earth and the atmosphere, where the earth rotates relative to the atmosphere. The flow and direction of such fluids in the geometry of Couette is important in important in engineering designs, and in combustion engines just to mention few. To be specific, Couette, studied the flow of fluid with its motion brought about by the relative movement of two parallel plates or surfaces or where one of the plates is moving laterally in its own plane. The study was later named after him and defined as a steady laminar flow of viscous incompressible fluid between two infinite parallel plates separated by a distance (h). The two plates moving relative to each other that cause the flow of fluid in between them could be flat, parallel or two concentric cylinders of varying radii. Ngiangia and Wonu [1], examined the stability of Couette flow in a porous medium and deduced that radiation and porosity has destabilizing effect on the stability of Couette flow. Ngiangia [2] also studied the Couette-Poiseuille flow and showed that at high wave number and high Reynolds number regime, instability is set in. Mebine [3] and Bodosa and Borkakati [4] also considered the effect of Magnetic field and other parameters in the study of Couette flow while Ngiangia et al [5] considered the onset of instability of Couette-Poiseuille flow in a porous medium and opined that instability is set in early in the presence of the parameters under consideration. Orukari et al [6], included viscous dissipation to the work of Ngiangia [2] and deduced interesting results. Oladele et al [7], discussed the effect of Eckert number on flow of radiating fluid between concentric cylinders and made far reaching deductions on the velocity and temperature flow fields. Conrenflos et al [8], considered experimental and numerical study of Couette-Poiseuille flow using data available for developing and developed flows by varying the Reynolds number regime and made interesting findings, particularly as it relates to Couette flow. Ngiangia and Orukari [11], showed that porosity is a potent factor in the analysis of Couette-Poiseuille flow. From the results of the works cited so far, study of Couette flow in two concentric cylinders of varying radii is few and electroconductivity which enhances MHD in this flow configuration is new, hence the study.

$\rho$ = Fluid density
$u'$ = fluid velocity
P = fluid pressure
$\mu$ = absolute viscosity
g = acceleration due to gravity
t = dimensionless time
T = fluid temperature
C = dimensionless fluid concentration
a = thermal diffusivity
$q_z$ = radiative term
$r'$ = coordinate
$\sigma_\infty$ = constant fluid electroconductivity
$\sigma_c$ = Stefan Boltzmann constant
$\sigma_0$ = dimensionless fluid electroconductivity
$C_p$ = specific heat at constant pressure
$T_o$ = initial temperature
$\beta_T$ = coefficient of volume expansion for temperature
$T_\infty$ = reserviour temperature

$\wedge$ = Planck's function
$\alpha_{K^*}$ = absorption coefficient
$\kappa^*$ = frequency of radiation
$\rho_\infty$ = reserviour density
$\theta$ = dimensionless temperature
$Gr$ = free convection parameter due to temperature (Grashof number)
$t'$ = time
r = dimensionless coordinate
$\upsilon$ = kinematic viscosity
$C_\infty$ = charateristics fluid concentration
$R$ = dimensionless radiation term
Re = Reynolds number
u = dimensionless fluid velocity
U = cylindrical plate velocity
$k_T$ = thermal conductivity

## 2. MATHEMATICAL FORMULATION OF THE PHYSICAL PROBLEM

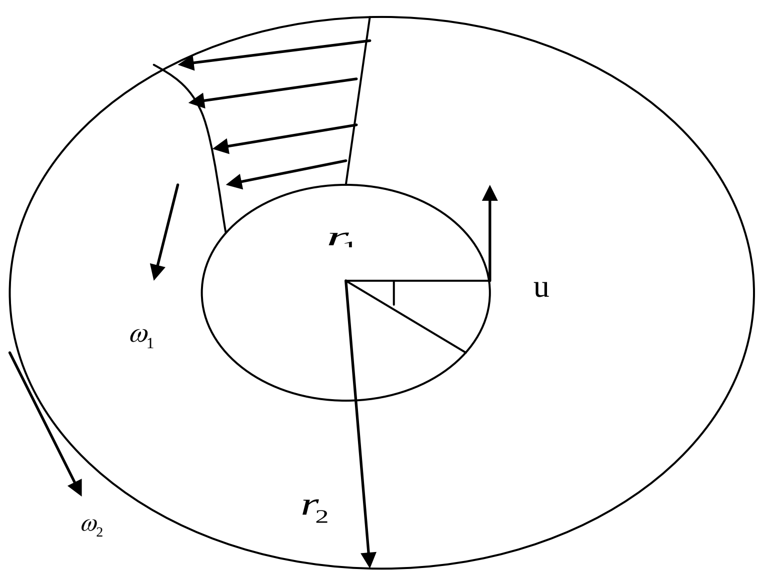


**Figure 1.** Geometry of the problem.

We consider infinitely long, two concentric cylinders of radii $r_1$ and $r_2$ rotating with constant angular velocity $\omega_1$ and $\omega_2$. Let there be viscous incompressible fluid in the annular space. The cylinders induce a steady axisymmetric tangential motion in the fluid. Let z axis be chosen along the axis of the cylinders. Since the motion is only tangential at constant density and the flow is axially symmetric, the governing hydrodynamic equations take the form

$$\frac{1}{r'}\frac{\partial}{\partial r'}\left(r'u\right)=0 \tag{1}$$

$$\frac{\partial u'}{\partial t'}=\left[\frac{\partial^2 u'}{\partial r'^2}+\frac{1}{r'}\frac{\partial u'}{\partial r'}-\frac{u'}{r^2}\right]+\beta g\left(T-T_0\right)-\frac{\sigma_c B_0^2 u'}{\rho}-\frac{\sigma_\infty u'}{U} \tag{2}$$

where following Boricic et al [9], the fluid electro conductivity is assumed to be of the form $\sigma_\infty\left(1-\frac{u'}{U}\right)$ but for physical exigency and mathematical amenability, it is approximated to the form in (2)

$$\frac{\partial T}{\partial t'}=-a\left[\frac{\partial^2 T}{\partial r'^2}+\frac{1}{r'}\frac{\partial T}{\partial r'}\right]-\frac{1}{\rho Cp}\frac{\partial q_r}{\partial r} \tag{3}$$

$$\frac{\partial^2 q_z}{\partial y'^2}-3\sigma^2 q_z-16\sigma T_\infty^3\frac{\partial T}{\partial y'}=0 \tag{4}$$

For optically thin medium with relatively low density in the spirit of Cogley et al [10], equation (4) reduces to

$$\frac{\partial q_z}{\partial y'}=4\delta^2\left(T-T_\infty\right) \tag{5}$$

where $\delta^2=\int_0^\infty(\alpha_{K^*}\frac{\partial\wedge}{\partial T})dk^*$

Subject: to the initial and boundary conditions

$$u'=r_1'\omega_1 \text{ at } r'=r_1' \tag{6}$$

$$u'=r_2'\omega_2 \text{ at } r'=r_2' \tag{7}$$

$$T=1 \text{ at } r'=0 \tag{8}$$

$$T=0 \text{ at } r'=\infty \tag{9}$$

## 3. DIMENSIONLESS VARIABLES

For dimensional homogeneity of the governing fluid equations, we substitute the following dimensionless quantities or expressions of the fluid variables

$$u = \frac{u't}{r'}, M = \frac{\sigma B_0^2 r'^2}{\rho\mu\upsilon}, R = \frac{4\delta^2 \rho_\infty C_\infty r'^2}{\rho C_p \upsilon}, \sigma_0 = \frac{\sigma_\infty t'}{\sigma u' r'}$$

$$\Pr^{-1} = \frac{\upsilon}{a}, \theta = \frac{T - T_o}{T}, U' = \frac{U}{u}, \mathrm{Re}^{-1} = \frac{\mu}{u'r'\rho}$$

$$t = \frac{u'r'}{t'}, Gr = \frac{g\beta(T - T_o)r'^3}{u'^3}$$

into equations (1) – (9) and we have

$$\frac{1}{r}\frac{\partial}{\partial r}(ru) = 0 \tag{10}$$

$$\frac{\partial u}{\partial t} = \mathrm{Re}^{-1}\left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2}\right] + Gr\theta - Mu - \sigma_0 u \tag{11}$$

$$\frac{\partial \theta}{\partial t} = -\Pr^{-1}\left[\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r}\frac{\partial \theta}{\partial r}\right] - R\theta \tag{12}$$

Subject: to the initial and boundary conditions

$$u = r_1\omega_1 \text{ at } r = r_1 \tag{13}$$

$$u = r_2\omega_2 \text{ at } r = r_2 \tag{14}$$

$$\theta = 1 \text{ at } r = 0 \tag{15}$$

$$\theta = 0 \text{ at } r = \infty \tag{16}$$

## 4. METHOD OF SOLUTION

We seek solution to equations (11) and (12) of the form

$$u = u_1(r)e^{-nt} \tag{17}$$

$$\theta = \theta_1(r)e^{-nt} \tag{18}$$

where: n is a constant.

Subject to

$$u = r_1\omega_1 e^{nt} \text{ at } r = e^{nt} r_1 \tag{19}$$

$$u = r_2\omega_2 e^{nt} \text{ at } r = e^{nt} r_2 \tag{20}$$

$$\theta = e^{nt} \text{ at } r = 0 \tag{21}$$

$$\theta = 0 \text{ at } r = e^{nt}\infty \tag{22}$$

If we put equation (18) into equation (12) and simplify, we obtain

$$\frac{d^2\theta_1}{dr^2} - \frac{1}{r}\frac{d\theta_1}{dr} + \Pr(R-n)\theta_1 = 0 \tag{23}$$

The series solution of equation (23) after the imposition of the boundary conditions of equations (21) and (22) is

$$\theta_1(r) = \frac{3}{3-\beta}\left(r - \frac{\beta}{3}r^3 + ---\right) \tag{24}$$

where $\beta = \Pr(R-n)$

Similarly, equations (17) and (24) is put into equation (11) and simplify, we get,

$$\frac{d^2u_1}{dr^2} + \frac{1}{r}\frac{du_1}{dr} - \frac{1}{r^2}u_1 + \mathrm{Re}(n - M - \sigma_0)u_1 = \mathrm{Re}\,Gr\frac{3}{3-\beta}\left(r - \frac{\beta}{3}r^3 + ......\right) \tag{25}$$

The solution of equation (25) subject to equations (19) and (20) is

$$u = a_0\left(1 - \frac{\beta_2}{3}r^2 + \frac{\beta_2^2}{141}r^4 + ....\right) + a_1\left(r - \frac{\beta_2}{17}r^3 + \frac{\beta_2^2}{1258}r^5 + ......\right) + \left(\frac{3+10\beta}{3\beta_2}\right)r - \frac{\beta}{3}r^3 + \frac{3\mathrm{Re}\,Gr}{15-5\beta}r^2 \tag{26}$$

$$a_0 = \frac{\varphi_1\theta_{33} - \theta_{11}\varphi_2}{\theta_{33}\alpha_{22} - \theta_{11}\alpha_{11}}$$

$$a_1 = \frac{\varphi_2\theta_{33} - \alpha_{11}\varphi_1}{\alpha_{22}\theta_{33} - \theta_{11}\alpha_{11}}$$

where

$$\theta_{33} = 1 - \frac{\beta_2}{3} r_1^3 + \frac{\beta_2^2}{141} r_1^3 + .......$$

$$\theta_{11} = r_1 - \frac{\beta_2}{17} r_1^3 + \frac{\beta_2^2}{128} r_1^5 + ........$$

$$\theta_{22} = \left(\frac{3+10\beta}{3\beta_2}\right) r_1 - \frac{\beta}{3} r_1^3 + \frac{3\mathrm{Re}\,Gr}{15-5\beta} r_1^2$$

$$\alpha_{11} = \left(1 - \frac{\beta_2}{3} r_2^2 + \frac{\beta_2^2}{141} r_2^4 + ....\right)$$

$$\alpha_{22} = \left(r_2 - \frac{\beta_2}{17} r_2^3 + \frac{\beta_2^2}{1258} r_2^5 + ......\right)$$

$$\alpha_{33} = \left(\frac{3+10\beta}{3\beta_2}\right) r_2 - \frac{\beta}{3} r_2^3 + \frac{3\mathrm{Re}\,Gr}{15-5\beta} r_2^2$$

$$\beta_2 = \mathrm{Re}(n - M - \sigma_0)$$

$$\varphi_1 = r_1\omega_1 e^{nt} - \theta_{22}$$

$$\varphi_2 = r_2\omega_2 e^{nt} - \theta_{33}$$

The shear stress at the walls is given by

$$\zeta_r = \mu\left(\frac{du}{dr} - \frac{u}{r}\right) \tag{27}$$

For the walls of the outer and inner cylinders respectively, the shear stresses are given as

$$(\zeta_r)_{r=r_2} \tag{28}$$

and

$$\left(\zeta_r\right)_{r=r_1} \tag{29}$$

The rate of heat transfer between the walls and the fluid is given as

$$q_a - k_T\left(\frac{\partial\theta}{\partial r} - \frac{\theta}{r}\right) \tag{30}$$

The relations of equations (28) and (29) also follows.

## 5. RESULTS AND DISCUSSION

In order to get physical insight and numerical validation of the problem, an approximate value of constant angular velocity $\left(\omega_1 = 1.5, \omega_2 = 2.0\right)$ and constant radii $\left(r_1 = 0.25, r_2 = 0.5\right)$ as well as n = 0.2 is chosen. The values of other parameters made use of are

$$\mathrm{Re} = 10{,}00{,}20.00{,}30.00.40.00{,}50.00$$
$$Gr = 4.0{,}6.0{,}8.0{,}10.0{,}12.0$$
$$M = 1.0{,}2.0{,}3.0{,}4.0{,}5.0$$
$$\sigma_0 = 0.7{,}1.4{,}2.1{,}2.8{,}3.5$$
$$\mathrm{Pr} = 0.31{,}0.41{,}0.51{,}0.61{,}0.71$$
$$R = 1.0{,}1.5{,}2.0{,}2.5{,}3.0$$
$$t = 1$$

Increase in Prandtl number decreases the temperature profile of the fluid as shown in Figure 2. In Figure 3, it is shown that increase in radiation decrease the temperature. Increase in velocity brings about a corresponding increase in prandtl number as depicted in Figure 4. Increase in radiation results in a decrease in velocity as shown in Figure 5. Increase in Grashof number (Gr > 0) cools the temperature of the plate thereby heating the fluid which causes an increase in velocity distribution as in Figure 6. Increase in Reynolds number causes a corresponding increase in the velocity profile of the fluid and shown in Figure 7. While similar a result is also observed in Figure 9. Magnetic parameter is a resistive force, hence its increase brings about a decrease in the velocity profile of the fluid as depicted in Figure 8.

## 6. CONCLUSION

The effect of electroconductivity on Couette flow in cylindrical geometry is new and its effect complement that of the Grashof number. The approximation of electroconductivity parameter is for mathematical amenability in the flow configuration.

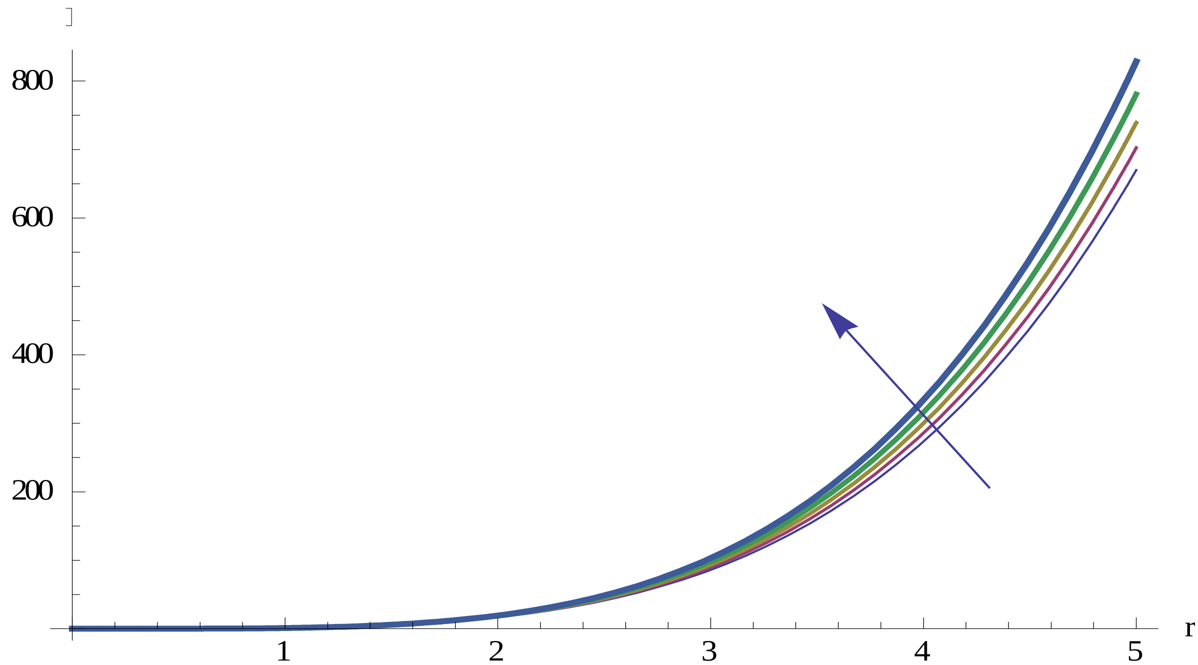


**Figure 2.** Temperature profile $\theta$ against boundary layer $r$ for varying Prandtl number Pr

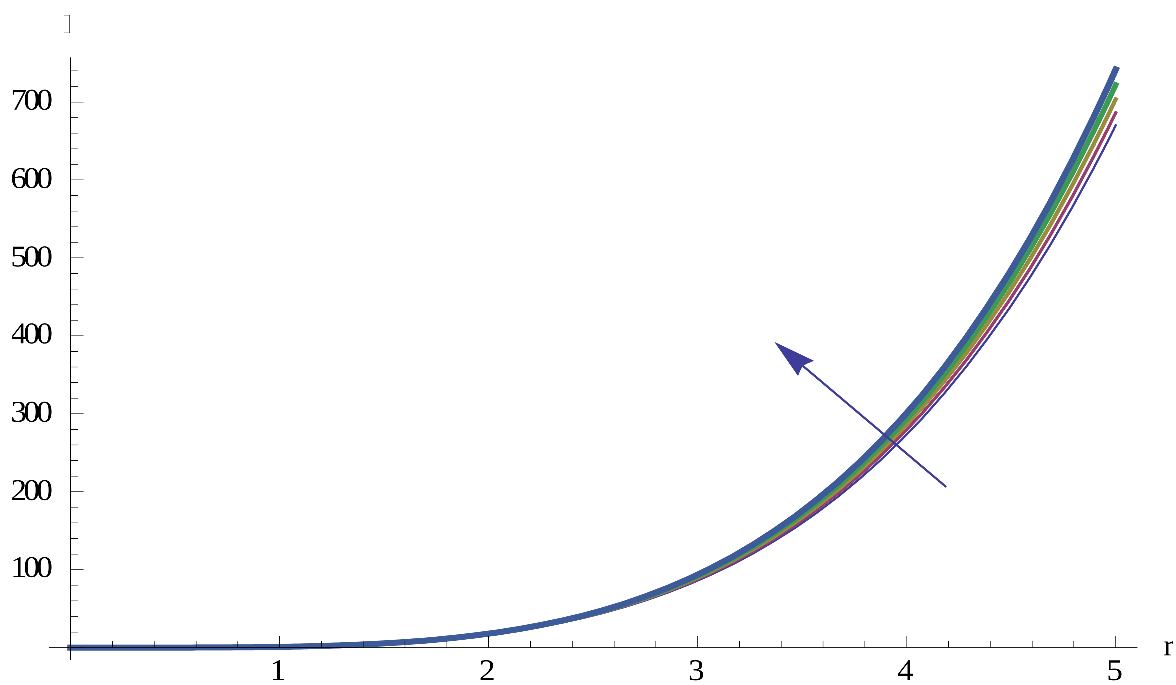


**Figure 3.** Temperature profile $u$ against boundary layer $r$ for varying Radiation $R$

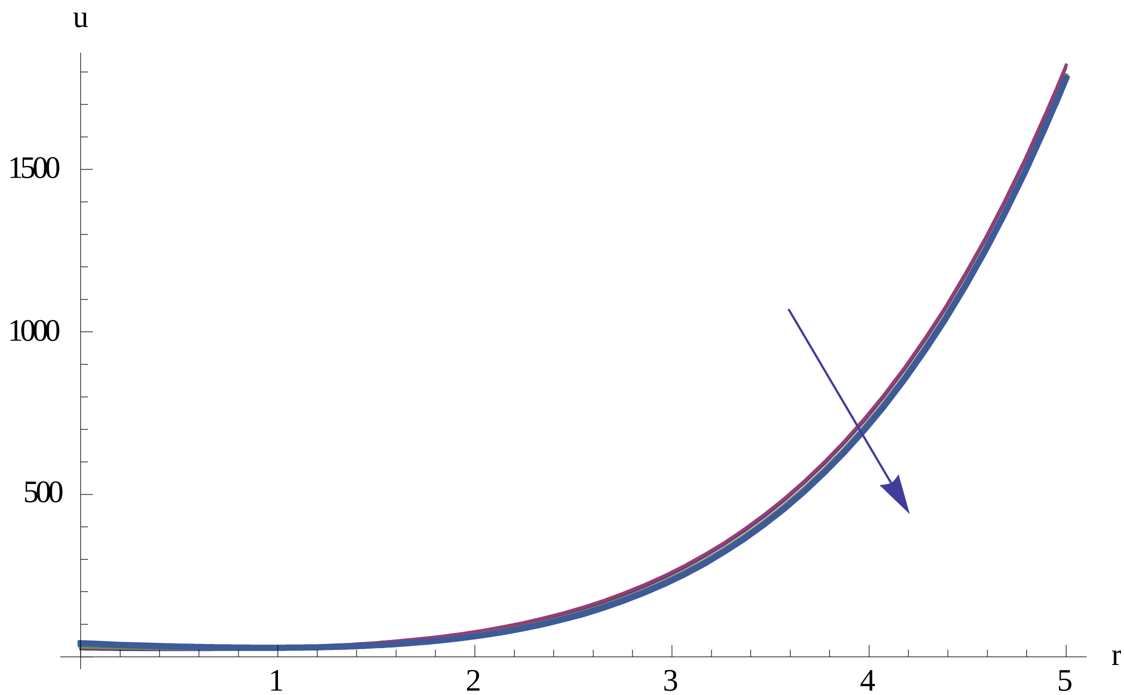


**Figure 4.** Velocity profile $u$ against boundary layer $r$ for varying Prandtl number Pr

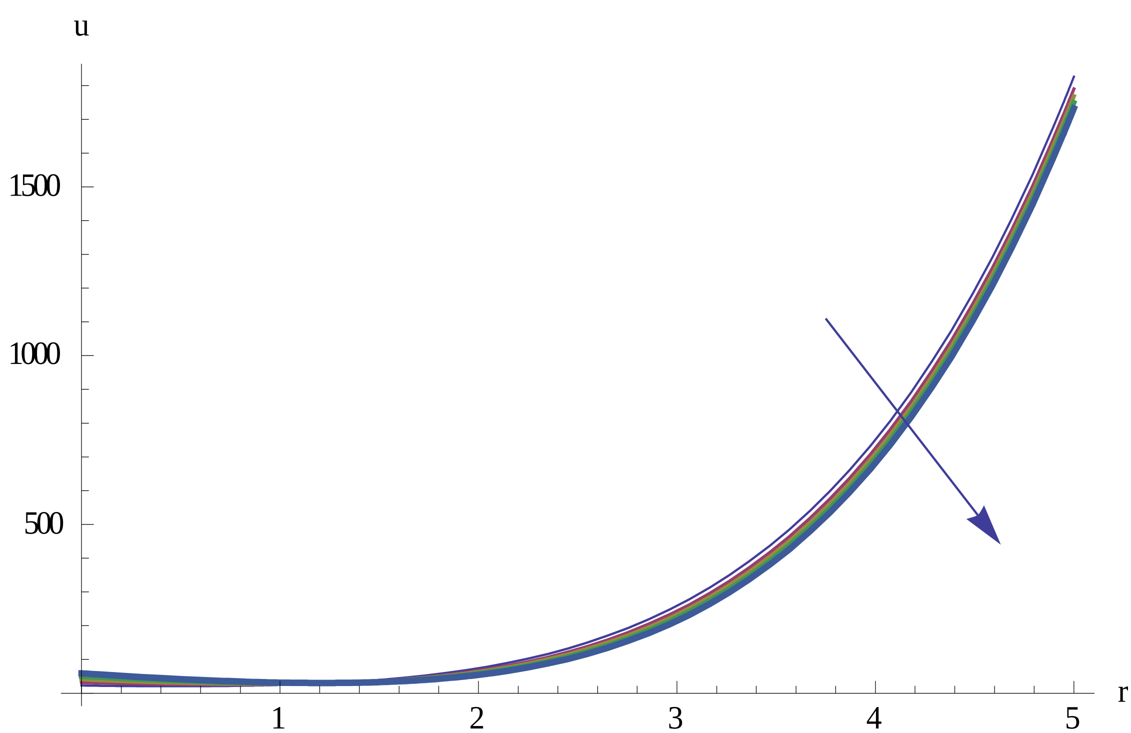


**Figure 5.** Velocity profile $u$ against boundary layer $r$ for varying Radiation $R$

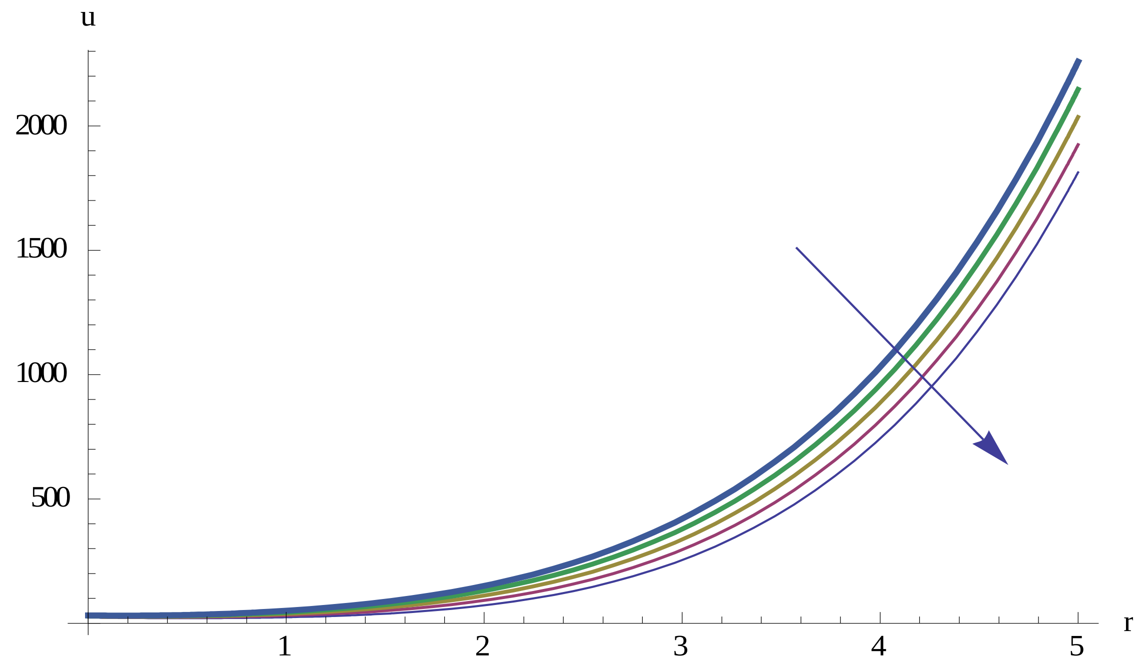


**Figure 6.** Velocity profile $u$ against boundary layer $y$ for varying Grashof number $Gr$

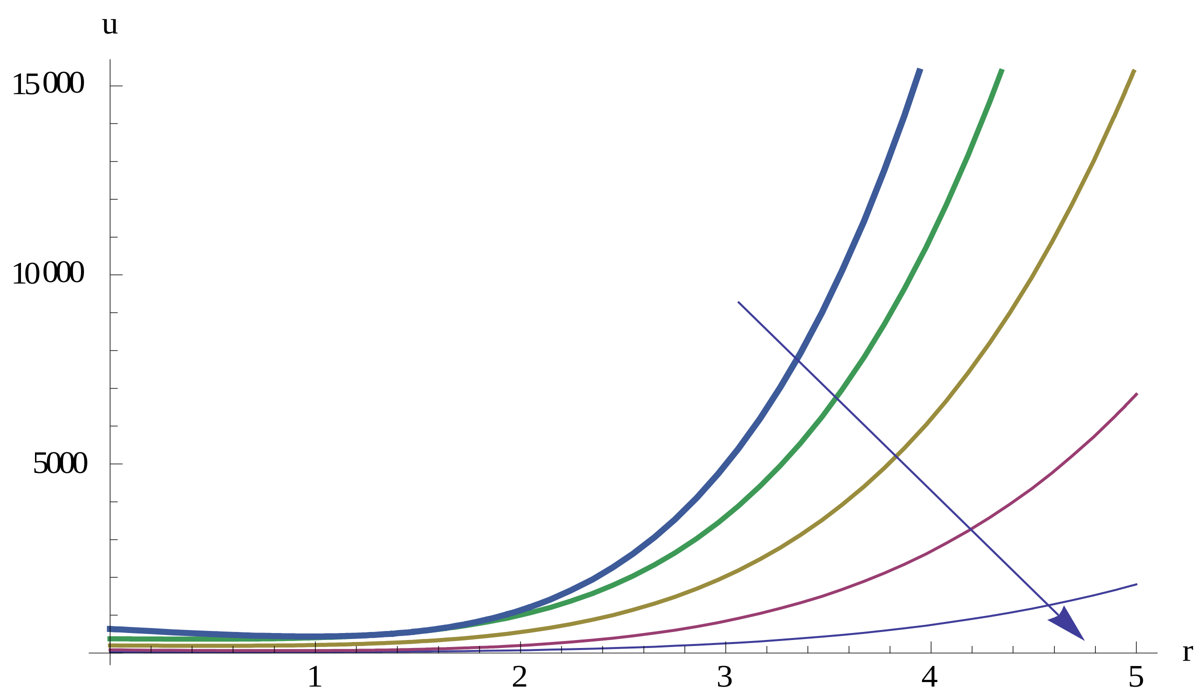


**Figure 7.** Velocity profile $u$ against boundary layer $y$ for varying Reynolds number Re

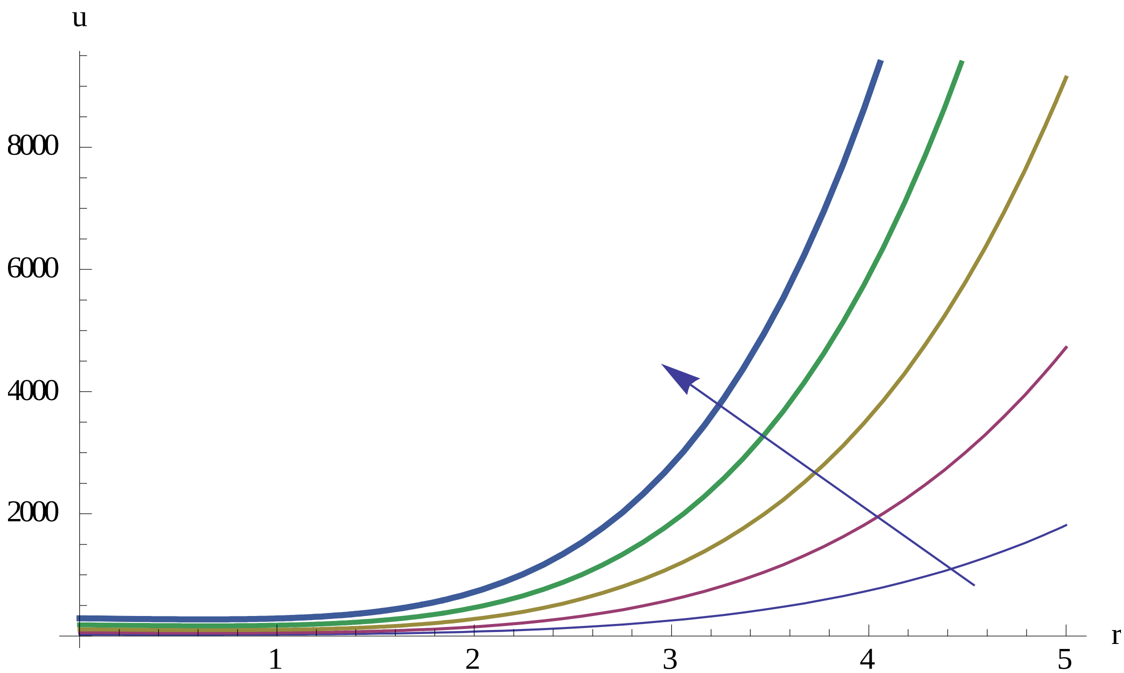


**Figure 8.** Velocity profile $u$ against boundary layer $y$ for varying Hartmann number ($M$)

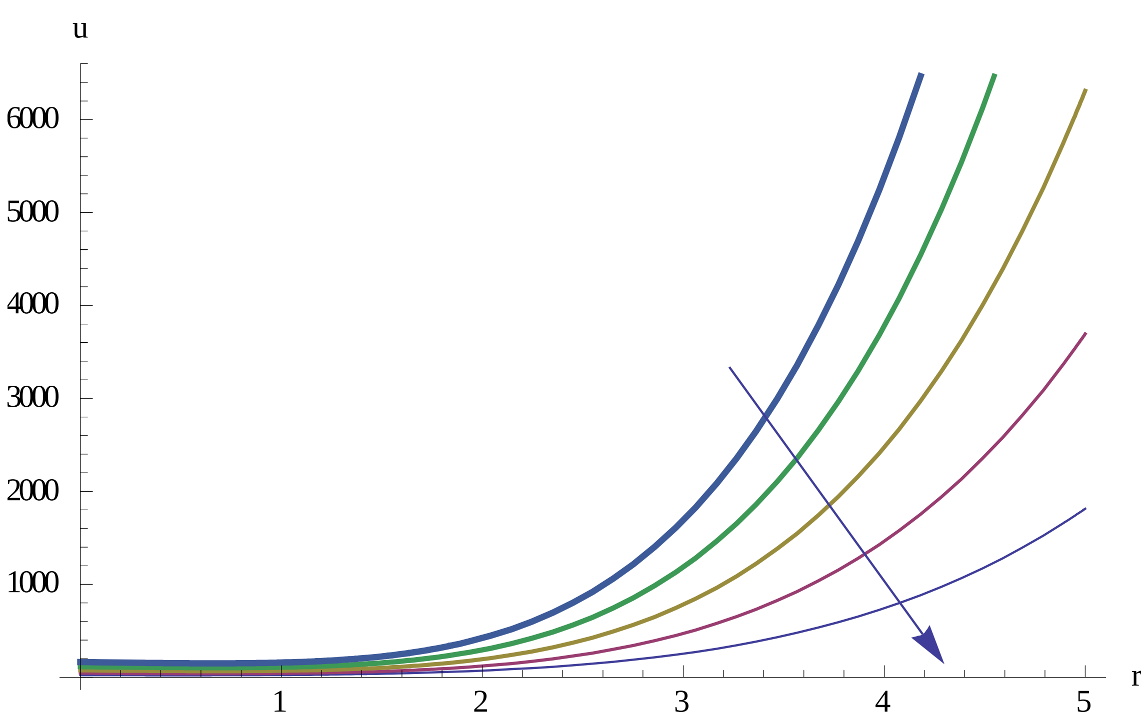


**Figure 9.** Velocity profile $u$ against boundary layer $y$ for varying electronegativity $\sigma_0$

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