Condensed Geometry II: New Constraints, Temporal Confinement Phase and Structure and Interpretation of Space and Time

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ABSTRACT

In this paper we basically intend to deductively study aspects and physical behavior of time in the Quantum Mechanical and Loop Quantum Gravity regime and the physical interpretations therein. The implicit gauge invariance of the vortical time is deduced. A new form of the Hamiltonian constraint is produced and the corresponding Vector or Diffeomorphism constraint is also introduced, the Gauss constraint being found to be the same as that in the original. The new consequences are discussed. The paper presents fresh new ideas and interpretations as well as perspectives on the spatial as well as the now introduced temporal aspects of Loop Quantum Gravity. In this we basically intend to deductively study aspects and physical behavior of Time in the Quantum Mechanical and Loop quantum gravity regime and the physical interpretations therein. In what follows, we provide a systematic mathematico-physical treatment of the vortical aspect of quantum gravity or a test quantum system, relative to space, as the time for that system. The implicit gauge invariance of the vortical time is deduced. A general theory of implicit and explicit gauge invariance is proposed. The gauging out of the acausality gauge degree of freedom is deduced and the causal direction of time is shown to nucleate out as a self ordered criticality due to a “tweaking” field which is shown to be a form of $U(1)$ magnetic phase transition. This magnetic phase arising in the Kodama holomorphic wave functional universe, arises naturally due to the Ashtekar “magnetic” field in general. Also the wave-particle duality of gravity/ geometry as a quantum system is interpreted in terms of the physical role played by time. The long standing problem of the interpretation of the quantum mechanical time-inverse temperature has apriori been resolved with a heuristic interpretation. As an epilogue, the Pauli theorem on time as an operator has been discussed along with the standard proof.
1. INTRODUCTION

It was deduced statistically [1] by us that there need not be either a Singular Big Bang event in the past of all events in standard General Relativistic (GR) cosmology nor in quantum cosmologies. In fact, the Big Bounce appearing in the Loop Quantum Cosmology (LQC) is also a phenomenon arising from the quantization of the traditional phase space, of the ADM-Wheeler-DeWitt theory recast in the Ashtekar formalism, called the mini/midi-super space. Such a modern canonical GR quantization of an old model – the mini-super space – results in a technically and aesthetically superior but physically short sighted cosmology. The right thing to do and which was done so was to consider the statistical thermodynamic behavior of the geometric particles as one goes backwards in time, since the temperature goes on increasing and the rigidly interwoven spin networks actually come loose and the resulting configuration of geometric particles starts to become liquidy at first and finally gaseous at the so-called initial point of the universe. This is the trans-Planckian regime. As we showed in that paper, the gas of geometric particles dubbed the grannulons would condense as the temperature decreased with time as the order parameter and a quantum universe would emerge. This is the causal picture opposite of the backward extrapolated picture that we reasoned out above.

We plan to discuss the physical properties and aspects of time here and try to resolve long standing problems on the physical interpretations concerning time in quantum mechanics and quantum gravity. Here, we reprove the time-vorticity relation derived in [1] to make our discussion clearer. In what follows later, we provide a systematic mathematico-physical treatment of the vortical aspect of quantum gravity or a test quantum system, relative to space, as the time for that system. This is proved as a theorem. A general theory of implicit and explicit gauge invariance will be proposed elsewhere. The gauging out of the acausality gauge degree of freedom is deduced and the causal direction of time is shown to nucleate out as a self-ordered criticality due to a “tweaking” field which is shown to be a form of U(1) magnetic phase transition. This magnetic phase arising in the Kodama holomorphic wave
functional universe, arises naturally due to the Ashtekar “magnetic” field in general as well. The causal balls dubbed on the lines of the QCD glueballs, are shown to exist much below the Planck Energy scale, inspite of being quantum gravitational objects. Lastly, the Pauli theorem on time as an operator has been discussed along with the proof. and a plan to construct a gauge invariant operator has been proposed to hint a further paper [9] on this topic.

2. THE PROBLEM OF TIME AND QUANTUM GRAVITY

The problem of time and its association with space has been a longstanding problem in theoretical physics. It carries itself forward into the considerations of quantum gravity. The nature of time presents itself in the basic interpretative and physical as well as mathematical considerations of the problem of quantum gravity. The physics behind this lies in the fact that just like in the case of the quarks in QCD, the observer and all energy-momentum are confined in spacetime to move forward in time. This is the essence of the problems that arise in the interpretations of the solutions of the quantum Einstein equations also known more traditionally as the Wheeler-DeWitt equations. The observer is no longer making quantum measurement processes on any arbitrary quantum system but gravity itself and since gravity pervades the entire universe, the observer is himself the part of the quantum system and is affected by time within the system of gravity; he is also evolving with the time and at a rate different from that of the rest of quantum system of gravity depending upon his dynamical state and the relativistic considerations within. The confinement problem is more or less the same. The only difference is the size of the universe compared to the size of the hadron or the nucleus. Quantum gravity encompasses exactly that concept. The properties of the very large embodied in Einstein’s General Relativity are to be brought on the same platform as those of the very small embodied in Quantum Gauge Field Theory. Concrete steps have to be taken in this direction and the proper treatment of the physics of time in this paper is one such step.

2.1. TIME AND BACKGROUND INDEPENDENT VORTICITY

Theorem: Let be the time and \( \Gamma \) be the circulation due to the vorticity associated with the quantum system; then, for the analytical continuity between the wavefunctions, each
separately containing the canonical conjugate pairs of energy-time and mass-vorticity, leads to a linear relation between time and the circulation of the precise form given by

\[ t = \frac{r}{c^2}. \]

**Proof:**

Consider the configuration space \( C_{GR} \) of General Relativity whose cotangent bundle is the Hamiltonian phase space of the \( SL(2, \mathbb{C}) \) Yang-Mills gravity gauge. Here, for every ground state variable \( |\psi(0)\rangle = |n, R(0)\rangle \) in the abstract vector space or the Dirac space, we have a wave function in the (Hilbert space) diffeomorphism invariant square integrable cross-sections of the vector bundle associated with the quantum system, here gravity – a system with finite number of degrees of freedom. Now, we have a simple familiar wave function \( \psi(x) = \langle \chi | \psi \rangle = e^{i (x \varphi - E t)} \) for any quantum system embedded in gravity. For our purpose for this quantum system embedded in gravity, postulate the existence of an azimuthal 1-form – a bra: \( \langle \varphi | \), whose inner product with the corresponding ket, \( |\psi\rangle \), yields the corresponding angular wave function or spin function \( \psi(\varphi) = \langle \varphi | \psi \rangle = e^{i (\varphi \varphi - m t)} \). Now this form we postulate as the equivalent form for the quantum mechanical wave function. We further postulate that every field phenomenon can be modeled into a spin system (see [2] and references therein) and thereby viewed as a collective phenomenon. This means that gravity also may be viewed as an Ising model of light cones centered at nodes.

For our purpose we will consider the constraint surface of our quantum mechanical matter system imbedded in constraint surface of the gravity in the Hamiltonian phase space as it is the Hamiltonian constraint that generates the dynamics. Fix now an isomorphism, \( \varphi \), between the frame relative to which the quantum mechanical matter system is stationary and that of gravity i.e., the frame of the stationary observer in which the quantum system is dynamical. This isomorphism is the relative or Lorentzian velocity of the quantum system relative to the inertial frame of reference of our observer. The isomorphism allows for the velocities of both, the system’s frame and the inertial frame, to be converted into each other via Einstein’s velocity addition theorem. This velocity \( \varphi \) can also be taken as the generalized velocity in the configuration space of general relativity under certain special circumstances. Incidentally this dynamics is generated by the action of the Hamiltonian constraint. This
provides for a time independent Hamiltonian formulation of the Einstein velocity addition theorem. The relativistic nature of the dynamics is therefore conserved in the Poisson algebra of the Lagrange-Hamilton isomorphisms taken above. The constraint functionals generate canonical transformations which result in the rotation of the isomorphism indices. The isomorphisms are time-void generators of dynamics in 3-space. This is how the dynamics can be measured in the temporal gauge. The geodesic of the quantum system is coarse graining/classical limit of the eigenvalue of the Thiemann length operator and moves causally in classical continuum. As the quantum system moves with a relative velocity \( \mathbf{v} \) and thereby traces a geodesic in the continuum, the generalized velocity of this quantum system traces a trajectory on the constraint surface. Thus the transition of the quantum system from a state \( |\psi(0)\rangle = |n, R(0)\rangle \), to another state \( |\psi(t)\rangle = |n, R(t)\rangle \), is given by the transition amplitude expressed as the time-void path integral between two points on the constraint surface (the flows in time are in any case generated by the Hamilton constraint).†

Define a vorticity, \( \mathbf{\omega} \) of \( \mathbf{v} \), in a time-void \( d = 3 \) space — a fluid of future light cones, as

\[
\mathbf{\omega} := \nabla \times \mathbf{v}
\]  

where, the differential form convention has been used as in [1]. Now, fix an automorphism, \( \mathfrak{u} \), of the constraint surface of gravity representing the bulk of the surface states/light cone fluid giving the collective dynamical behavior of the space — its velocity of flow. In other words, fix an isomorphism between two quantum fundamental observers’ frames. This velocity also is time-void. For canonical vorticity, \( \mathbf{\omega} \), we have

\[
\Gamma := \int_\mathcal{S} \mathbf{\omega} \cdot d\mathbf{S} = \oint_\mathcal{C} \mathbf{v} \cdot dl
\]  

where, \( \Gamma \), the circuit integral is the circulation which is also time void.

The analytical continuity of the functions \( \psi(x) \) and \( \psi(\varphi) \) tells us that \( pq \) corresponds to \( \int_\mathcal{S} \varphi \) and similarly \( Et \) corresponds to \( m\Gamma \) so that we have by the Einstein theorem of inertia of energy that

\[
Et = \frac{E}{c^2} \Gamma
\]

Thus time arises naturally as a kind of vorticity in the temporally gauged Hamiltonian quantum dynamics, although gravity doesn’t arise until curvature is involved; or

\[
t = \frac{\Gamma}{c^2}.
\]
This proves our above theorem. Now, for an inviscid liquid, with canonical time $\theta$ measuring its motion in the configuration space of general relativity, the vorticity, $\omega$, satisfies the equation

$$\frac{\partial \omega}{\partial \theta} = \text{curl} \ u \times \omega.$$  \hspace{1cm} (5)

Eq (5) tells us that the vortex lines are dragged with the fluid moving a velocity $u$. Thus, we perceive spacetime rather than space and time. The Landau-Raychaudhuri equation tells us that vorticity causes expansion and shear causes contraction in spacetime.

2.2. THE BACKGROUND INDEPENDENT “EIKONAL CONSTRAINT” AND THE IMPlicit TEMPORAL VORTEX GAUGE

Let $u := u(x, y)$ be the function describing the wavefront of a pulse of light. Then we fix the “Eikonal Constraint” given by:

$$c^2 (\nabla_x u + \nabla_y u)^2 - 1 = 0$$  \hspace{1cm} (6)

to the background independent version ($c = 1$), as

$$\sum_{i,a} (E^i_a u(A))^2 - 1 = 0$$  \hspace{1cm} (7)

which constrains the dynamics of quantum gravity to a background free (BF) causally based one. Here, the $E^i_a$s are the momenta canonically conjugate to the Ashtekar variables $-A^i_a$s $u(A)$ is the Monge-Ashtekar wave(-front) function. The Eikonal Constraint brings in a Monge’s cone constraint on the quantum dynamics of Loop Quantum Gravity thereby rendering a tachyon condensation by virtue of which tachyonic velocities, viz., faster than light travel is naturally removed. By the eq (5) we see that time is bound to the light cones, in terms of causality. Now whereas,

$$\Gamma = \oint \nabla \cdot dl = \oint (\nabla \times \omega) \cdot dl$$  \hspace{1cm} (8)

to be say, positive, we have $\Gamma = -\oint \omega \cdot dl$, so that vorticity line integral can change sign over contour direction. Change in the direction of the contour however, that acausality is the inherent direction of time and this cannot be gauged out since it is then no longer an
unphysical gauge degree of freedom of gravity. But this we all know; what is new that we now know is that acausality can merely be gauged out as follows:

\[ \Gamma = -\oint \mathbf{w} \cdot d\mathbf{l} \implies t < t_0 \text{ (for some } t_0 > 0) \] for \( \mathbf{w} = -\nabla \times \mathbf{v} \). Define then \( \mathbf{v}' = \mathbf{v} + \nabla \Lambda \).

In so doing, \( \nabla \times \mathbf{v}' = \nabla \times \mathbf{v} \) as \( \nabla \times \nabla \Lambda = 0 \). This shows that for suitable \( \Lambda \), \( -\nabla \times \mathbf{v} = \nabla \times (-\mathbf{v}) \) so that

\[ \nabla \times \mathbf{v}' = \nabla \times (-\mathbf{v} + \nabla \Lambda) \ni \nabla \Lambda > |\mathbf{v}'| \implies \mathbf{w} > 0 \implies \Gamma > 0 \implies t > 0 \forall t > t_0. \]

2.3. THE NEW FORM OF THE HAMILTONIAN CONSTRAINT

The recent attacks on the Master Constraint of Thiemann [10] has led the author to produce a humble alternative. The Hamiltonian constraint should not be mistaken for the zero-energy Schrödinger equation. The truly general relativistic Hamiltonian constraint conforming to the quantum principles is the general form of the zero-energy Dirac equation. Remember, the Dirac equation reads:

\[ E = \mathbf{p} \cdot \mathbf{a} + m_0 \beta \] (A)

A little dabbling gives the new Dirac-Hamiltonian constraint, as

\[ \widetilde{\mathcal{C}} := \varepsilon^{ijk} \mathbf{E}_j \mathbf{R}_{a,jk} + \tilde{m}_0 \mathbf{R}^0 = 0 \] (B)

We define this form of the Hamiltonian constraint so that it annihilates the apparently fuzzy states or unphysical events, i.e., the improbable light cones and brings about a reduction of state as proposed by Penrose. Together, with the Eikonal constraint, it rids one of the acausality and fuzziness to yield definitive solutions. In so doing, it also endows a mass to the event, by means of the second term in the above Hamiltonian constraint, which is appropriated in its exact magnitude by the Diffeomorphism and the Gauss constraints. We provide a function of the Dirac-Hamiltonian constraint as follows: there exist two continua, separated by the gap \( 2m_0 \). Now, the two continua are of positive energy and negative respectively. A past light cone exists in the real continuum i.e., the former of the two. Given a wavefunction \( \psi \), \( \exists \) a unitary operator \( \mathbf{U} \ni \mathbf{U} \psi = \psi_0 \), where \( \psi_0 \) is the wavefunction at an initial time \( t_0 \). Now, the past light cone existing in the positive continuum makes an event fuzzy. By the action of the new Hamiltonian constraint, a proper future light cone jumps from the "negative" energy state to the positive continuum and makes a proper event to occur leaving a
proper causal history in the negative energy continuum and giving an expectation value of an associated measured Dirac (gauge invariant) observable and bringing about a reduction of state just as Penrose predicted. Thus, the new Hamiltonian constraint is in a sense, an annihilator of states in the sense of Penrose. This is a mathematical explanation of the action of the new Hamiltonian constraint. What is the physical process behind this mathematical explanation? It is this: Two light cones, a past one and a future one, come together and make a "present" event. That, the past as well as the future light cones equally well determine an event in the present. They both have equal "influence" on the event. Thus, the future also has influence on the present. This is a prediction of the theory presented in this paper and may be experimentally determined by testing the curves of say atomic transitions over a long time and taking the readings for different times. Then the readings of present events should have had some effect on the past readings. The test will be more accurate if one has a set of graphs each for a different time interval. Since the new Hamiltonian constraint is a zero energy Dirac equation, the process is unidirectional in that, events do not decay: the past and future light cones do not separate from each other at temperatures less than the Planck temperatures. The second term in the Dirac-Hamiltonian constraint is the primordial Higgs term defined as

\[ r G_{ab} = m_0 K^0. \]

Thus, we get a direct conclusion that the existence of the Higgs condensate (vacuum) validates the existence of the vacuum of gravity. What this vacuum configuration is for canonical gravity as well as for loop quantum supergravity and supersymmetric spin networks will be dealt with elsewhere more rigorously. From the work of Smolin [11], if strings are indeed the perturbations of evolving spin networks, then superstrings should likewise arise from supersymmetric spin networks. Then, the work of Candelas et. al. [12] suggests that the vacuum configuration of supersymmetric spin networks are \( D = 6 \) Calabi-Yau manifolds which stems from the holonomy group \( SU(3) \). From the non-abelian \( SU(2) \) holonomy, it is not too ambitious to predict that the vacuum configuration should be Calabi-Yau 4-folds, say the K3 surfaces. In particular, in the context of quantum field theory of gravitation, it is a virtual particle of Planck mass that impresses the action of the above defined Hamiltonian constraint as will be showed below. Unlike the torsion-free formalism of Ashtekar, this newly defined Hamiltonian constraint ascribes gravitational moment to every event i.e, to every point of quantum geometry and thus brings about an onset of torsion in gravity. Next, since the total Hamiltonian of gravity contains the Vector constraint also
known as the diffeomorphism constraint which generates flows on the space-like hyper surfaces (constant time- hyper surfaces) or in our formalism, generates interactions between the loops interpreted as the sections of the light cones by the space-like hyper surfaces, we propose the following construction for the Diffeomorphism (vector) constraint consistent with our above construction of the Hamiltonian constraint, as

\[ \tilde{\gamma}_i := \tilde{E}^k_{a} \tilde{R}^a_{ik} = 0 \]  

(C)

The above equations are numbered (A), (B) and (C) to signify their importance separate from the rest of the paper. It was Thiemann who demonstrated that with the use of the Barbero real connection, the physics of LQG is simplified considerably. So for simplicity, we consider a real field instead of a complex one for the time being to physically consistently introduce a primordial vacuum for canonical gravity formulation i.e., a spin network vacuum.

Lastly, comes the problem of regularization if the New Hamiltonian constraint acts at the event itself. By the annihilation of the states at the event, it induces ultraviolet divergences in the past and infrared divergences. The Thiemann trick comes to the rescue as follows: The expression (B) can be rewritten as

\[ C := \{ \int d^3x \sqrt{\det E} A^k \} \kappa + \tilde{m}_i \tilde{R}^i = 0 \]  

(D)

which in the quantum limit becomes an operator equation just like eq(B) as

\[ C := [\tilde{\gamma}_i, \tilde{A}^k] \tilde{R}^a_{ai} + \tilde{m}_0 \tilde{R}^i = 0 \]  

(E)

The volume operator acts as a UV as well as IR regulator and thus makes past as well as the future finite. If one considers an event post the t = 0 event then there are absolutely no UV divergences and as such the new Hamiltonian constraint rules out the singularity involving all spacetime, in short: a Big Bang Singularity. Thus, in this paper, we have systematically eliminated the existence of a spacetime singularity. The above equations are numbered (A), (B) and (C) to signify their importance separate from the rest of the paper. It was Thiemann who demonstrated that with the use of the Barbero real connection, the physics of LQG is simplified considerably. So for simplicity, we consider a real field instead of a complex one for the time being to physically consistently introduce a primordial vacuum for canonical gravity formulation i.e., a spin network vacuum.
2.4. TIME AS A COLLECTIVE PHENOMENON AND GAUGE FIELD THEORY FOR DIRECTION OF TIME

Now, we have a causality field or time field for vortex degree of freedom. For the acausality and causality being equiprobable, the time on its own has no meaning and thus has zero effect. But due to the vortex liberation in a topological phase transition (discussed in [1]), depending upon the direction of vorticity, i.e., $\omega < \omega > 0$, one of acausality or causality is gauged out as unphysical. By the CPT theorem, there is a possibility of a connection of this gauging out of acausality degree of freedom with the abundance of matter over antimatter. Thus, by the same token, there can be an acausal universe with abundance of antimatter over matter. The two universes could coexist separated by a causality-acausality domain wall which brings to the following discussion: At the liberation of the vortices which is time, a topological phase transition which causes this liberation makes a disordered phase to transit to another disordered phase. However, this disordered phase will have more order with respect to the orientation of the future light cones. This new disordered phase will be the domain wall with a temporal thickness of $t_0$ (i.e., from $t = 0$ to $t = t_0$). Proper analysis of time scales shows that $t_0 \approx 10^{-43}$ s. For the X-Y or rather the global $U(1)$symmetry that was broken at say the Planck time, we have over that temporal thickness, a dilaton mode (a Goldstone mode in conformal field theory). The quantum of the time-vortex field is a vector boson that is initially massless but upon Spontaneous Symmetry Breaking (SSB) gauges out acausality and in so doing eats up the dilaton and becomes heavy through the Higgs mechanism. The Lagrangian for this process may be written out for a complex scalar field as follows:

$$\mathcal{L} := \partial^\alpha \phi^\dagger \partial_\alpha \phi - V(\phi)$$ (9.1)

where

$$V(\phi) := \frac{1}{4} \lambda (\phi^\dagger \phi - v^2)^2$$ (9.2)

In a way, the causality transition is a confinement phase transition for an ensemble of a gas of randomly "oriented" future light cones. The future light cones get causally connected and are confined to point in a certain direction called the past to future direction. Above the transition temperature, we have a deconfinement, i.e., just before, Planck’s time there exist only uncertain future light cones oriented randomly. The rest is discussed in Appendix A at
the end of the paper. The vector “meson” now acquires Planck mass $m_{pl} \approx 10^{-6} \text{g}$, as the interaction occurs at $\tau_{pl}$, i.e., $E_{pl}$. Initially, it interacted between future light cones but after SSB, it now interacts between light cones with history and energy-momentum sources. Thus, the vector meson being so heavy causes future light cones to bend towards massive objects. Thus, we have quantum gravitational proposition for the primordial existence of light cones independent of light pulses, photons etc., and the quantum gravitational reason for the bending of light around massive objects and gravitational lensing. The future light cones are bent towards even the quantum systems such as atoms, molecules and even elementary particles which leads to the deformation of quantum events near such quantum objects. Here, we get a motivation and nourishment for a proposal for a dust/ fluid of light cones as a standard of spacetime. This will be dealt with elsewhere. The gauging out of the acausality degree of freedom at the Planck time can be interpreted as a spontaneous $U(1)$ “magnetization” of time. For the vortex-time as an order parameter, there exists an order parameter force with which one can tweak, in our case time. There corresponds also, an order parameter susceptibility—here, the vortex-time susceptibility, $\chi_t$. Now, the relation between these three quantities is given by

$$t = \chi_t F$$

(9.3)

The minimal tweaking corresponds to $t = 10^{-43} \text{s}$ and the vortical order parameter force is $F_{pl} = \frac{e^4}{g} \approx 12 \times 10^{43} \text{N}$. Plugging in these values in the above relation we get $\chi_t = 8.333 \times 10^{-88} \text{N}^{-1}\text{s}$. Thus, we see that the ability to tweak the time takes a lot of force since the susceptibility is so tiny. The rich set of calculations for the spontaneous magnetization of time is not considered here. It is similar to the one given by Yang [3]. The $SU(2)$ group is where one considers the quantum dynamics of quantum (geometry) space. There time arises in the form of vortices but get magnetized in $U(1)$ only after a domain wall jump of $\tau_{pl}$ thickness (refer [1] for a preview or a prelude if the current discussion is sounding vague as the mathematical content is contained in ref. [1]). Due to the $SU(2) \otimes U(1)$ nature and the Weyl curvature hypothesis, $C_{abcd} = 0$ for $t = 0$ to $\tau_{pl}$, a dilaton is “consumed” by the three $SU(2)$ vector bosons, interacting in the $SU(2) \otimes U(1)$ mode only between the primordial future light cones but which, becomes heavy upon eating the conformal Goldstone mode, by the Higgs mechanism with masses around $m_{pl} = 10^{-6} \text{g}$ at $\tau_{pl}$. This gauges out
the acausality degree of freedom and is the physical reason for spontaneous magnetization of
time at $\tau_{Pl}$ to a completely causal universe. The magnetization is delivered by thye
Ehresmann $U(1)$ connection. The corresponding magnetic field $B^i_a = \frac{\delta S}{\delta A^i_a}$. Next, the $U(1)$
magnetization yields a Wu-Yang monopole related to the $U(1)$ magnetization of time. This is
a physically real existent object. The spin 1 vector bosons can be found to interact with the
primordial future light cones and between these and the masses so that events appear to be
influenced by heavy masses or in our conventional language – gravity. These vector “mesons”
post the Planck epoch, are the mediators of a short range interaction that weaves the causal
tapestry of spacetime. While the magnetization of time is $U(1)$, the SSB and the
corresponding gauging out of the acausality degree of freedom is a $SU(2)$ local gauge
transformation. The spontaneous symmetry breaking (SSB) leads the isotropy of time in the
sub-Planckian domain to the one of a future oriented time with a consequent expanding
spacetime phase. We need not calculate the associated Higgs phenomenon for $SU(2)$ as it is
standard technique of the Yang-Mills gauge theory which is usually borrowed in the canonical
$SU(2)$ gravity.

The combination of the Ashtekar and the Ehresmann connections is given by
$G^i_{ab} := A^i_a \otimes A^E_{b}$, a "sub-connection" of the Garrett-Lisi-Schreiber super-connection as
follows. The basic $SU(2) \otimes SU(2)$ symmetry of a temporally oriented spacetime established
in our discussion is isomorphic to $SO(3,1)$ upto a $\mathbb{Z}_2$ grading. Thus the $\mathbb{Z}_2$ forms a kernel of
the isomorphism between the Lorentz group and the non-abelian Lie group $SU(2) \otimes SU(2)$
Thus Wick rotating we have $SO(4) = SU(2) \otimes SU(2) / \mathbb{Z}_2$. The corresponding algebra fits
exactly into the $so(3,1)$ subalgebra of $e_8$. Thus, the Ashtekar-Einstein spacetime theory of
quantum gravity fits exactly into the Garrett-Lisi $\textit{E}_8$ structure - an exceptionally simple theory
of everything.

The dilaton field is responsible for the acausality gauge. The acausality is the gauge
degree of freedom associated with the dilaton field. The $SU(2) \otimes U(1)$ conformal gauge
symmetry is broken and the three gauge degrees of freedom associated with the three spatial
degrees of freedom. The rich mathematical representation theory and the actual calculations
of this beautiful theory are available in the standard literature (see for example Yang [3]) but
are not completely BF. In [7] these calculations have been explicitly shown corresponding to
the afore elucidated theory. Due to limited space we concentrate upon the main thread of the theory viz., the quantal interpretation of time.

Now the mass of the causal ball. We have seen above that the mass of the "causalons" is approximately $10^{-6} \, gm$. So, the mass of a causal ball is the reduced mass of the three: $\sim 3.3334 \times 10^{-7} \, kg$, equivalent energy being $\sim 1.8750375 \times 10^{14} \, TeV$. So the mass gap estimate for the SU(2) gauge theory of gravitation is $\Delta \sim 1.8750375 \times 10^{14} \, TeV/c^2$

or $\Delta \sim 1.8750375 \times 10^{17} \, GeV/c^2$

This is much smaller than the Planck energy. Thus, these entities – the causal balls existed in the Standard Model even after the Planck epoch. This signifies something significant and of extreme importance in particle physics and cosmology both, classical and quantum.

2.5. DUAL INTERPRETATION OF TIME

The vortical and inverse temperature aspects of time arise from the wave-particle duality of quantum (geometry) gravity respectively. But we are very well familiar with the particle aspect of quantum geometry. If the surface states i.e., the grannulons and the bulk states or the nodes of the spin networks are the particles of quantum geometry and lead to the inverse temperature-time then what is waving? What is waving is the probability of the quantum geometric states and consequently the vortical time. The grannulons due to the waving probability are fuzzy and are everywhere and consequently a grannulon can appear in more than one “place” at the same instant. Same goes for the nodes of the spin networks. Thus, vortical time can appear simultaneously everywhere with different probabilities and consequently there should be a spread of vortical time throughout quantum spin foam.

How is the vortical aspect of time, a quantum/semi-classical property arising from the wave nature of quantum (geometry) gravity? Note firstly that the two aspects of time should carry over from quantum gravity to semi-classical geometry and finally to quantum mechanics. These two aspects are as abovementioned, vorticity and inverse temperature. Now, for a quantum mechanical system, the inverse temperature-time relationship comes not
just from the analytical continuity between the Boltzmann factor and the wave function, but also from the fact that a quantum system itself as observed and upon which measurements are performed, is a statistical average or expectation value universe copy arising from an ensemble of copies of the system. The temperature is the average kinetic energy of each of these copies. The time $t = (iT)^{-1}$ is the time arising from such an ensemble. The vorticity is justified above as in [1] in quantum gravity, but has to carry over to quantum mechanics. And it so does as follows: the work of Madelung [4] and independently of that by Korn [5] on the analogies between the hydrodynamic model and the Schrödinger equation based quantum mechanics yields similarities. Let us do the analysis, math and finally physics of this theory with our own interpretations. Actually, the similarity of the Schrödinger wave equation and its implications with the equations of hydrodynamical flow formed the basis of another attempt to interpret quantum mechanics. We consider this as a fluid of copies of the quantum system under investigation. We therefore repeat some of the calculations done by Madelung but with our interpretations.

Starting with the Schrödinger equation,

$$\nabla^2 \psi - \frac{2m}{\hbar^2} V - \frac{2im}{\hbar} \frac{\partial \psi}{\partial t} = 0$$  \hspace{1cm} (10)

let

$$\psi = \alpha e^{i\beta}$$  \hspace{1cm} (11)

with $\alpha, \beta \in \mathbb{R}$, we can obtain for the purely imaginary part of (10)

$$\nabla \cdot (\alpha^2 \nabla \varphi) + \frac{\partial \alpha^2}{\partial t} = 0$$  \hspace{1cm} (12)

where

$$\varphi = -\frac{\hbar}{m} \beta.$$  \hspace{1cm} (13)

Equation (12) has the structure of the hydrodynamical equation of continuity viz.,

$$\nabla \cdot (\sigma \nu) + \frac{\partial \sigma}{\partial t} = 0.$$  \hspace{1cm} (14)

On the basis of this analogy, we interpret $\alpha^2$ as the density and $\varphi$ as the velocity potential as velocity, $\nu = -\nabla \varphi$, of a hydrodynamic flow process which is subject to the additional condition expressed by the real part of (10), that is, in terms of $\varphi$. 
\[ \frac{\partial \varphi}{\partial t} + \frac{1}{2} (\nabla \varphi)^2 + \frac{V}{m} - \frac{\nabla^2 \varphi \hbar^2}{\alpha 2m^2} = 0 \]  (15)

Now Euler’s hydrodynamic equation

\[ F - \frac{1}{\sigma} \nabla p = \frac{1}{2} \nabla v^2 + (\nabla \times v \times v) + \frac{\partial v}{\partial t} \]  (16)

where

\[ F = -\nabla U \]  (17)

is the force per unit mass, \( U \) the potential per unit mass, and \( p \) the pressure, can be written for irrotational motions (i.e., if a velocity potential exists) in the simpler form

\[ \nabla \left[ \frac{\partial \varphi}{\partial t} + \frac{1}{2} (\nabla \varphi)^2 + U + \frac{p}{\sigma} \right] = 0 \]  (18)

where

\[ \nabla p = \frac{1}{\sigma} \nabla P \]  (19)

If, therefore, the negative term in (15) is identified with the force function of the inner forces of the continuum, \( \int dp/\sigma \), the motion described by Schrödinger’s equation appears as an irrotational hydrodynamical flow subjected to the action of conservative forces.

In the case of Schrödinger’s time-independent equation as we know,

\[ \nabla^2 \psi_0 + \frac{2m}{\hbar^2} (E - V) \psi_0 = 0 \]  (20)

and its solution

\[ \psi = \psi_0 \exp \left( \frac{iEt}{\hbar} \right) \]  (21)

so that, clearly

\[ \frac{\partial \alpha}{\partial t} = 0 \text{ and } \frac{\partial \varphi}{\partial t} = -E/m. \]  (22)

Hence, eq (15) implies

\[ E = \frac{m}{2} (\nabla \varphi)^2 + V - \frac{\nabla^2 \alpha \hbar^2}{\alpha 2m} \]  (23)

The last negative term in eq (23) is the Bohm quantum potential in the hidden-variable theory of Bohm. In our theory, this may not turn out to be the same. An
eigenfunction of eq (20) thus represents, despite its time factor, a stationary flow pattern \(\partial v/\partial t = 0\) and, with \(\sigma = \alpha^2 = \rho/m\) corresponding to the normalization \(\int \sigma \, dt = 1\), the total energy

\[
E = \int d\tau \left( \frac{p}{2} v^2 + \sigma V - \sqrt{\sigma} \nabla^2 \sigma \frac{h^2}{2m} \right)
\]

turns out to be the space integral of a kinetic and potential energy density just as in the case of classical mechanics of continuous media. Check that if the motion of an inviscid fluid is initially irrotational then it remains irrotational, provided that the body forces are conservative and pressure is a function of density only. From this follows the Helmholtz vorticity theorem that if the body forces are conservative and density \(\rho\) is a function of pressure \(p\) only, then

\[
\frac{\partial}{\partial t} \left( \frac{w}{\rho} \right) = \left( \frac{w}{\rho} \cdot \nabla \right) v
\]

where \(w\) is the vorticity defined in eq (5).

Two things are noteworthy here;

First, the stationary flow pattern with vortex dynamics in arises when the vortex conditions are satisfied. This arises in any Navier-Stokes equations given by eq (16) or the easier Euler equations. Second, the energy integral equation when concretized in the kinematical Hilbert space context for canonical quantum General Relativity (QGR) will deliver a simple set of Diffeomorphism and Hamiltonian constraints.

A more rigorous canonical approach using the Hamilton-Jacobi equation of Classical Mechanics leading to the Schrödinger equation and the corresponding Hydrodynamic model based on the work of Isakson [6] and its concretization to canonical gravity in terms of a fluid of light cones to yield a new standard for spacetime and a new Hamiltonian constraint operator will be found in [8].

Finally this hydrodynamic model that we have considered is not the one by Madelung i.e., not of continuously distributed electricity with a mass density proportional to the charge density, or any other physically incorrect interpretations and constructions made then. This hydrodynamic model is a virtual one of copies of the system. A flow in time of the test quantum system is generated when a quantum measurement is made. The reduction of state
causes the vortex to expand the fluid. The difference in the wavelengths being large, the wavelets in the fluid cancel rapidly and we get a localized system flowing in time by means of a push given by the vorticity (of the quantum) time itself. The copies of the system interact with each other continuously and produce kinetic energy. The average kinetic energy of the systems in the fluid ascertain the temperature of the system. This allows one to build a partition function, as

\[ Z = \text{Tr} \exp(-\beta \mathcal{H}) \]  \hspace{1cm} (26)

which, when compared with eq (21) as follows compare just the kernel eigenfunction in eq (21) and the Boltzmann factor in eq (26), by analytical continuity yields \( t = (iT)^{-1} \). We have thus demonstrated a direct relation between the vortex-time and the imaginary inverse-temperature time. The correspondence is that of wave-particle duality. The average kinetic energy arises due to the particle nature of the copies of the quantum system of which the fluid is composed. The wave nature of the individual systems compose the waves in the fluid. These deliver the vorticity or vortex-time. The wave-particle duality of the quantum system yield the complementarity of the vortex-inverse temperature times. Thus the duality relation between the temporal vorticity and inverse-temperature follows.

2. 6. THE PAULI THEOREM ON TIME

We now consider the Pauli theorem on time, it states that if the operator for time, \( \hat{T} \), and that for the Hamiltonian, \( \hat{H} \), satisfy the commutator relation

\[ [\hat{T}, \hat{H}] = i\hbar \hat{1}, \]  \hspace{1cm} (27)

such that \( \hat{H} \) is semibounded or discrete, then \( \hat{T} \) cannot be a self-adjoint operator.

The proof of this is by the method of contradiction i.e., reduction ad absurdum as follows: let \( \hat{T} \) be self-adjoint satisfying eq (26). As such, \( \forall r \in \mathbb{R}, \exists \) a unitary operator \( \hat{U}_r = \exp(-ir\hat{T}) \). Now, let \( \varphi_E \) be an eigen vector of \( \hat{H} \) with eigen value \( E \), then the canonical conjugacy relation (26) implies that \( \hat{T} \) is a generator of energy shifts so that

\[ \hat{H} \hat{U}_r \varphi_E = (E + r) \varphi_{E+r} \]  implies that \( \hat{H} \) has a continuous spectrum over the entire real line since in the beginning itself \( r \) was taken to be arbitrary real number. Thus, we have a clear contradiction over our assumption of the semiboundedness or discreteness of \( \hat{H} \). Q.E.D.
Thus, we conclude our paper mentioning that a canonical gauge invariant (Dirac) operator of time is under construction in the context of Quantum General Relativity (QGR). The BF nature of canonical QGR is exploited there.

2.7. APPENDIX

At the liberation of the vortices which is time, a topological phase transition which causes this liberation makes a disordered phase to transit to another disordered phase. However, this disordered phase will have more order with respect to the orientation of the future light cones (see [1] and references therein). This new disordered phase will be the domain wall with a temporal thickness of $t_0$ (i.e., from $t = 0$ to $t = t_0$). Proper analysis of time scales shows that $t_0 = t_{pl} \sim 10^{-43}$ s. For the X-Y or rather the global $U(1)$ symmetry that was broken at say the Planck time, we have over that temporal thickness, a dilaton mode (a Goldstone mode in conformal field theory). The quantum of the time-vortex field is a vector boson that is initially massless but upon Spontaneous Symmetry Breaking (SSB) gauges out acausality and in so doing eats up the dilaton and becomes heavy through the Higgs mechanism. The Lagrangian for this process may be written out for a complex scalar field as follows:

\[ \mathcal{L} := \partial^a \varphi^+ \partial_a \varphi - V(\varphi) \]  \hspace{1cm} (A.1)

where

\[ V(\varphi) := \frac{1}{4} \lambda (\varphi^+ \varphi - v^2)^2 \]  \hspace{1cm} (A.2)

The vacuum field configurations are

\[ \varphi(A) = v \exp(i\alpha) \]  \hspace{1cm} (A.3)

where $A \equiv A^i_a$ is the Ashtekar connection and $\alpha$ is an arbitrary angle.

There is the familiar theorem known as the Derrick's theorem which asserts that with scalar fields only, there are no finite-energy, time-independent solitons that are localized in more than one dimension. The aim is to arrive at the monopole delivered by the topological phase transitions which liberate the time-vortices. The first step is to get solitons, in the given $U(1)$ theory, that are localized in more dimensions than one. For this we introduce gauge
fields. Notice that the Lagrangian of eq(A.1) has, as we had taken up in our above discussion, a global U(1) symmetry; gauging this symmetry the Lagrangian becomes

\[ \mathcal{L} := D^a \varphi^a D_a \varphi - V(\varphi) - \frac{1}{4} F_{ab}^i (A) F^{ab}_i (A) \]  

(A.4)

where

\[ D_a \varphi = \partial_a \varphi - ig A_a \varphi \]  

(A.5)

and \( V(\varphi) \) is still given by eq(A.2). The gauge symmetry is therefore spontaneously broken, and the mass of the vector particle is \( m_v = g \nu \). The mass of the scalar particle is \( m_s = \lambda^{1/2} \nu \). The rest of the theory follows from conventional QFT and we do get a BF monopole. The only difference between the conventional QFT and our theoretical calculations is that the Wightman axioms are generalized to the context of diffeomorphism invariance instead of the Poincaré invariance which is the subgroup of the former; also the Hilbert space is now allotted the Gelfand-Neumark-Segal (GNS) construction, i.e., we now have the following axiom involving the GNS Hilbert space:

\[ \exists a \text{ unitary and continuous representation } U: \mathcal{D} \rightarrow \mathcal{B}(\mathcal{H}_{\text{GNS}}) \text{ of the diffeomorphism group } \mathcal{D} \text{ on a GNS Hilbert space } \mathcal{H}_{\text{GNS}}. \]

3. CONCLUSIONS

It is worthy to note that the problems faced by the interpretations of quantum mechanics and measurement come with a new face in the quantum theory of gravitation. A complete rethinking into the foundational issues is warranted. Such, a radical thinking, interpretation or even conceptualization is not even remotely considered in covariant perturbation theories claiming quantization of gravity or even simultaneous unification including the currently fashionable (super) string theory. These, ideals from the old school of thought also have to be taken into consideration into the development of a physical theory ambitious enough to encompass everything and not just develop abstract mathematical structures and esoteric objects forcefully arising in such endeavors. The theory of quantum gravity or of “everything” should be an exercise in epistemological and philosophical thought also and not just a foray in abstract mathematical basin. However mathematical the laws of physics become, the character of physical law changes with every mathematical or physical change and modification. As such physics resulting thereby will look like a cut and paste theory with several tapings,
adhesives and finally, the resultant leakages in the form of inconsistencies and interpretative ambiguities.

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References


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