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Characteristics of thermal explosions on Newtonian and non-Newtonian fluids

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ABSTRACT

An analytical study of thermal explosion on Newtonian and non-Newtonian fluid is carried out. Approximate solutions to the governing equations and analysis of the study in the Newtonian case, showed that an increase in Frank-Kamenestkii parameter and that of the Semenov parameter decreases the minimum temperature of the reacting fluid thereby delaying the initiation of thermal explosions while additional heat source brings about early initiation of thermal explosions. The same observation was deduced in the non-Newtonian case but in varying degrees.

Keywords: Newtonian fluids; non-Newtonian fluid; Thermal explosions' Frank-Kamenestkii parameter; Semenov parameter

1. INTRODUCTION

Recent studies show that internal combustion and thermal explosions are increasingly becoming popular and its qualitative treatment were in agreement with experimental investigations, The thermal theory of ignition came into existence on the premise of breakdown in thermal equilibrium.

The breakdown is as a result of heat loss due to conduction no longer keep pace with the heat released due to exothermic reaction. The interest in combustion and thermal explosion are not only relevant to the development of physics of combustion and explosion but also to fundamental branches of knowledge such as chemical kinetics and theory of heat transfer. As a result of the imbalance in the thermal explosion processes, addition of heat source which result to either initiate early ignition point or delay it has been investigated theoretically and experimentally.

Adegbie *et al* in [1], studied thermal explosion in a combustible gas mixture containing fuel droplets with Arrhenius power law model and deduced interesting results, [2-10] also carried out similar studies and reported closely related results. Ajadi and Gol'dshtein, in [11], studied the thermal explosion characteristics in the presence of an additional heat source and opined the contributions of the heat release due to the forces of internal friction on the Frank-Kamenestkii parameter and Semenov parameter.

Ayeni *et al* [12], Lamidi and Ayeni [13] and Lamidi *et al* [14], all reported in their studies that decrease in temperature is caused by increase in the Frank-Kamenestkii parameter. Ngiangia *et al* [15], took a critical look at the approximation of flow behaviour index to Newtonian fluids in reaction pathway and deduced that Frank-Kamenestkii parameter and Semenov parameter decrease the minimum temperature of the reaction, thereby slowing down the initiation of thermal explosion. Recently, Ngiangia [16], analyzed the effect of Frank-Kamenestkii parameter on Newtonian fluids and non-Newtonian fluids and showed that for Newtonian and non-Newtonian fluids cases, a decrease in temperature is observed as a result of increase in Frank-Kamenestkii parameter. Our aim in this study is to extend the work of Ngiangia [16] by incorporating the effect of heat due to chemical reaction and Semenov parameter. This in our view will broaden the heat characteristics of fluid flow.

2. NOMENCLATURE

A = reactant
B = product
T= temperature of the reacting fluid
t= reaction time
k = thermal conductivity
x' = space coordinate
ψ = consistency index
u = fluid velocity
α = flow behaviour index (power law exponent)
χ = heat transfer coefficient
S = surface area of fluids
V = volume of fluids
T_0 = characteristic temperature
Q = heat released or absorbed
C = reactant concentration
R = universal fluid constant
k_0 = constant

m = numerical exponent
E = activation energy
$\rho =$ fluid density
P = fluid pressure
$\delta =$ Frank-Kamenestkii parater
$\psi =$ Semenov parameter
$\theta =$ dimensionless temperature
x = dimensionless space coordinate
$\rho' =$ dimensionless fluid density
$\beta =$ dimensionless thermal conductivity
$\mu =$ dimensionless heat absorbed or released
$\lambda =$ dimensionless activation energy
$t' =$ dimensionless time
q = quantity of heat in the reaction

3. FORMALISMS

The mathematical statement of the study suggests that the velocity gradient is a function of temperature and the power law exponent varies. Using these facts and assuming that the reaction taking place in the region under study is one-step and irreversible

$$A \rightarrow B \tag{1}$$

The simplified energy equation takes the form

$$\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial x'^2} + \varphi\left(\frac{\partial u}{\partial x'}\right)^{\alpha} - \chi\frac{S}{V}(T - T_0) \tag{2}$$

and the heat released by the chemical reaction is expressed by the Arrhenius law and obey the characteristics of Newtonian and non-Newtonian fluids

$$q = Q^{k_0\left(\frac{T}{T_0}\right)^m} C\exp\left(-\frac{E}{RT}\right) \tag{3}$$

with the boundary conditions $T(1) = 0,\ T(-1) = 1$ (4)

It has been established by Hughes and Brighton [17] that

$$\varphi\left(-\frac{\partial u}{\partial x}\right)^{\alpha} = -\frac{x}{2}\left(\frac{\partial p}{\partial x}\right) \tag{5}$$

$$-\frac{\partial u}{\partial x}=\frac{1}{2^{\alpha}\varphi}\left(-\frac{\partial p}{\partial x}\right)^{-\alpha}x^{-\alpha} \qquad (6)$$

The equation of state for an ideal fluid is given by

$$p=\rho RT \qquad (7)$$

Using equations (7), (6) and (3), we can rewrite equation (2) as

$$\frac{\partial T}{\partial t}=k\frac{\partial^2 T}{\partial x^2}+\frac{1}{2^{\alpha}\varphi}\left(\frac{\partial T}{\partial x}\right)^{-\alpha}(\rho Rx)^{-\alpha}-\chi\frac{S}{V}(T-T_0)+Q^{k_0\left(\frac{T}{T}\right)^m}C\exp\left(-\frac{E}{RT}\right) \qquad (8)$$

4. DIMENSIONLESS VARIABLES

The following dimensionless quantities have been used

$$\lambda=\frac{ER}{kQ},\rho'=\frac{\rho u^2 x}{p},R'=\frac{Rpx^2}{T_0},\psi=\frac{\chi SRT_0^2}{VQkCE}\exp\left(\frac{-E}{RT_0}\right),$$
$$t'=\frac{t}{t_0},\theta=\frac{T}{T_0},\delta=\frac{\varphi}{k},\beta=kRT_0,\mu=\frac{QC}{E\rho},x=\frac{x'}{ut}$$

and equation (8) rewritten as

$$\frac{1}{\beta}\frac{\partial\theta}{\partial t'}=\frac{\partial^2\theta}{\partial x^2}+\frac{1}{2\beta\delta}\left(\frac{\partial\theta}{\partial x}\right)^{-\alpha}(\rho'R'x)^{-\alpha}-\psi\theta+\mu R'^{k_0\theta^m}\exp\lambda\theta \qquad (9)$$

with the boundary conditions $\theta(1)=0,\theta(-1)=1$ (10)

5. METHOD OF SOLUTION

Case 1: for Newtonian fluids, $\alpha=1$, If we approximate $\left(\frac{\partial\theta}{\partial x}\right)^{-1}$ by ignoring powers of it greater than unity using Taylor series expansion about 0, steady state and the approximation of the heat term by taking logarithm of the term as well as simplifying, we get

$$x\frac{\partial^2\theta}{\partial x^2}-\frac{1}{2\beta\delta\rho'R'}\frac{\partial\theta}{\partial x}+x[-\psi+(\lambda_1+\lambda)]\theta=0 \qquad (11)$$

where $\lambda_1 = k_0 \log_e \mu R'$

To solve equation (11), we employ the Frobenius series solution method of the form

$$\theta = \sum_{i=0}^{\infty} a_i x^{i+c} \tag{12}$$

We substitute equation (12) into equation (11) and simplify, we get

$$\sum_{i=0}^{\infty}\left[a_i(i+c)(i+c-1) - \frac{1}{2\beta\delta\rho' R'} a_i(i+c)\right]x^{i+c-1} + \left[-\psi + (\lambda_1 + \lambda)\right]\sum_{i=0}^{\infty} a_i x^{i+c+1} = 0 \tag{13}$$

We multiply equation (13) by x^{1-c} and for the resulting equation to be satisfied for all x, the coefficient of each power of x in the equation must be zero (i = 0), hence

$$\left[c(c-1) - \frac{1}{2\beta\delta\rho' R'}\right]a_0 = 0 \tag{14}$$

Since $a_0 \neq 0$, we obtain

$$c = 1 \pm \frac{\sqrt{\dfrac{\beta\delta\rho' R' + 2}{\beta\delta\rho' R'}}}{2} \tag{15}$$

If we return to equation (11), make the exponents of x equal and simplify, we obtain

$$a_{i+1} = -\frac{\left[-\psi + (\lambda_1 + \lambda)\right]a_i}{\left[(i+c+2)(i+c+1) - \dfrac{1}{2\beta\delta\rho' R'}(i+3)\right]} \qquad i \geq 0 \tag{16}$$

Hence equation (16) is given as

$$a_{i+1} = -\frac{\left[-\psi + (\lambda_1 + \lambda)\right]a_i}{\left[\left(i + 1 + \dfrac{\sqrt{\dfrac{\beta\delta\rho' R' + 2}{\beta\delta\rho' R'}}}{2} + 2\right)\left(i + 1 + \dfrac{\sqrt{\dfrac{\beta\delta\rho' R' + 2}{\beta\delta\rho' R'}}}{2} + 1\right) - \dfrac{1}{2\beta\delta\rho' R'}(i+3)\right]} \tag{17}$$

and

$$a_{i+1} = -\frac{\left[-\psi + \left(\lambda_1 + \lambda\right)\right]a_i}{\left[\left(i+1-\frac{\sqrt{\frac{\beta\delta\rho' R'+2}{\beta\delta\rho' R'}}}{2}+2\right)\left(i+1-\frac{\sqrt{\frac{\beta\delta\rho' R'+2}{\beta\delta\rho' R'}}}{2}+1\right)-\frac{1}{2\beta\delta\rho' R'}\left(i+3\right)\right]} \quad (18)$$

We expand equation (12) to terminate at $i=1$ for brevity and apply the boundary conditions in equation (10), we get

$$a_0 = -a_1 = -a_2 = -a_3 \quad (19)$$

If we terminate the series of equation (12) at $i=2$, we get the complete solution as

$$\theta(x) = a_{0+}\left(x^{c_1} + x^{1+c_1} + x^{2+c_1}\right) + a_{0-}\left(x^{c_2} + x^{1+c_2} + x^{2+c_2}\right) \quad (20)$$

where $a_{0+} = -\frac{\left[\left(3+\frac{\sqrt{\frac{\beta\delta\rho' R'+2}{\beta\delta\rho' R'}}}{2}\right)\left(2+\frac{\sqrt{\frac{\beta\delta\rho' R'+2}{\beta\delta\rho' R'}}}{2}\right)-\frac{3}{2\beta\delta\rho' R'}\right]}{\left[-\psi + \left(\lambda_1 + \lambda\right)\right]}$

$$a_{0-} = -\frac{\left[\left(3-\frac{\sqrt{\frac{\beta\delta\rho' R'+2}{\beta\delta\rho' R'}}}{2}\right)\left(2-\frac{\sqrt{\frac{\beta\delta\rho' R'+2}{\beta\delta\rho' R'}}}{2}\right)-\frac{3}{2\beta\delta\rho' R'}\right]}{\left[-\psi + \left(\lambda_1 + \lambda\right)\right]}$$

$$c_1 = 1 + \frac{\sqrt{\frac{\beta\delta\rho' R'+2}{\beta\delta\rho' R'}}}{2} \qquad c_2 = 1 - \frac{\sqrt{\frac{\beta\delta\rho' R'+2}{\beta\delta\rho' R'}}}{2}$$

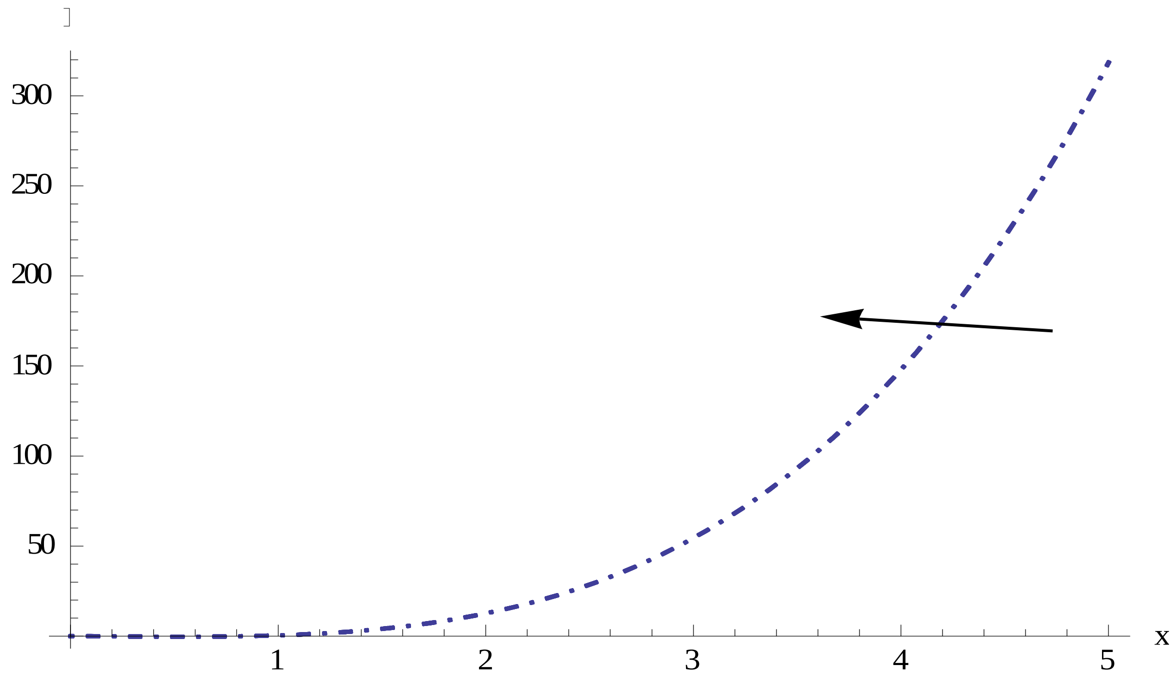

Figure 1. Showing increase of Frank-Kamenestkii parameter (δ) on temperature of Newtonian fluids.

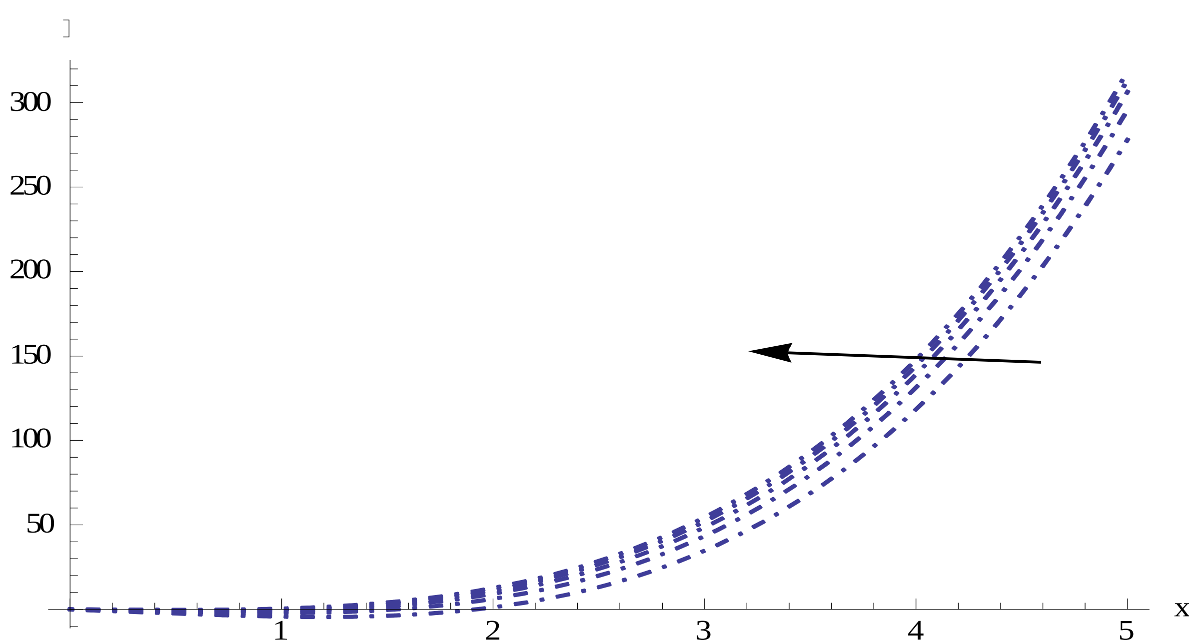

Figure 2. Showing increase of Semenov parameter (ψ) on temperature of Newtonian fluids.

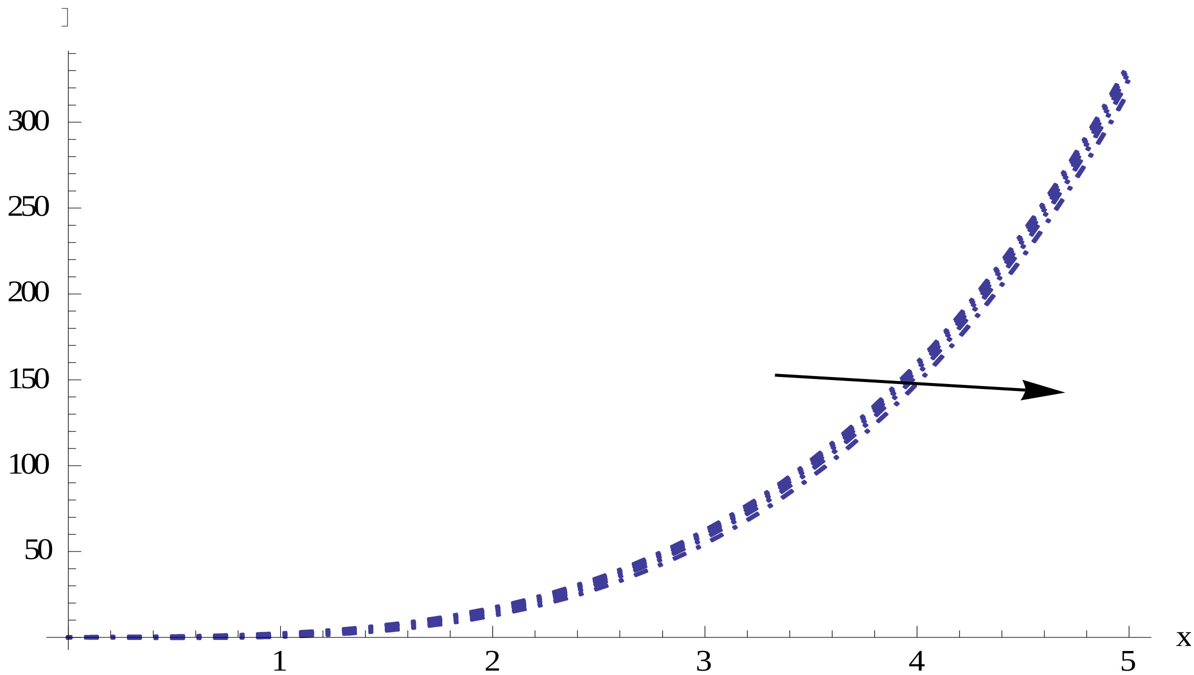

Figure 3. Showing increase of heat term ($\lambda_1 + \lambda$) on temperature of Newtonian fluids.

Case 2: This is non-Newtonian fluids case ($\alpha = 2$) and the procedure for Newtonian fluids followed, which transform equation (9) into

$$x^2 \frac{\partial^2 \theta}{\partial x^2} - \frac{1}{\beta \delta \rho' R'} \frac{\partial \theta}{\partial x} + x^2 \left[-\psi + \left(\lambda_1 + \lambda \right) \right] \theta = 0 \qquad (21)$$

With the additional boundary condition $\frac{d\theta}{dx} = 0,\ x = 1$ (22)

To solve equation (21), we employ the power series solution method of the form

$$\theta = \sum_{i=0}^{\infty} a_i x^i \qquad (23)$$

We put equation (23) into equation (21) and simplify, the resulting expression is

$$a_i i(i-1) - \frac{1}{\beta \delta \rho'^2 R'^2} a_{i+1} (i+1) + [-\psi + (\lambda_1 + \lambda)] a_{i-2} = 0 \qquad i \geq 2 \qquad (24)$$

For $i = 2,3,4$, we obtain the following expressions

$$a_0 = \frac{3a_3 - 2a_2\beta\delta\rho'^2 R'^2}{(\beta\delta\rho'^2 R'^2)[-\psi + (\lambda_1 + \lambda)]} \tag{25}$$

$$a_1 = \frac{4a_4 - 6a_3\beta\delta\rho'^2 R'^2}{(\beta\delta\rho'^2 R'^2)[-\psi + (\lambda_1 + \lambda)]} \tag{26}$$

$$a_2 = \frac{5a_5 - 12a_4\beta\delta\rho'^2 R'^2}{(\beta\delta\rho'^2 R'^2)[-\psi + (\lambda_1 + \lambda)]} \tag{27}$$

If we terminate the series of equation (23) at $i = 2$ and impose the boundary conditions of equations (10) and (22), we obtain, $a_0 = \frac{1}{2}$, $a_1 = -\frac{1}{2}$ and $a_2 = \frac{1}{4}$ with this values we determined a_3, a_4 and a_5 as well as write our solution as

$$\theta(x) = \left[\frac{3[-\psi + (\lambda_1 + \lambda) + 1] + \beta\delta\rho'^2 R'^2}{(\beta\delta\rho'^2 R'^2)[-\psi + (\lambda_1 + \lambda)]}\right] + \frac{[-\psi + (\lambda_1 + \lambda)] + (\beta\delta\rho'^2 R'^2)[[-\psi + (\lambda_1 + \lambda)] + 1]}{[-\psi + (\lambda_1 + \lambda)]}x$$

$$+ \frac{[-\psi + (\lambda_1 + \lambda)](1 + 5\beta\delta\rho'^2 R'^2) + 2\beta\delta\rho'^2 R'^2}{4}x^2 + \ldots\ldots\ldots \tag{28}$$

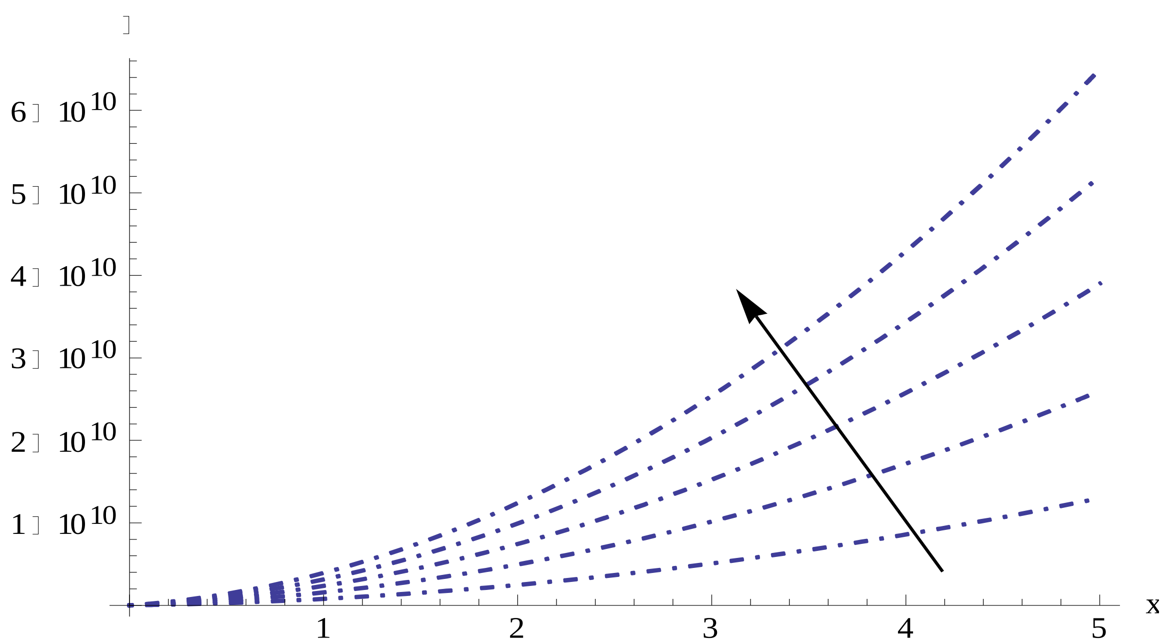

Figure 4. Showing increase of Frank-Kamenestkii parameter (δ) on temperature of non- Newtonian fluid.

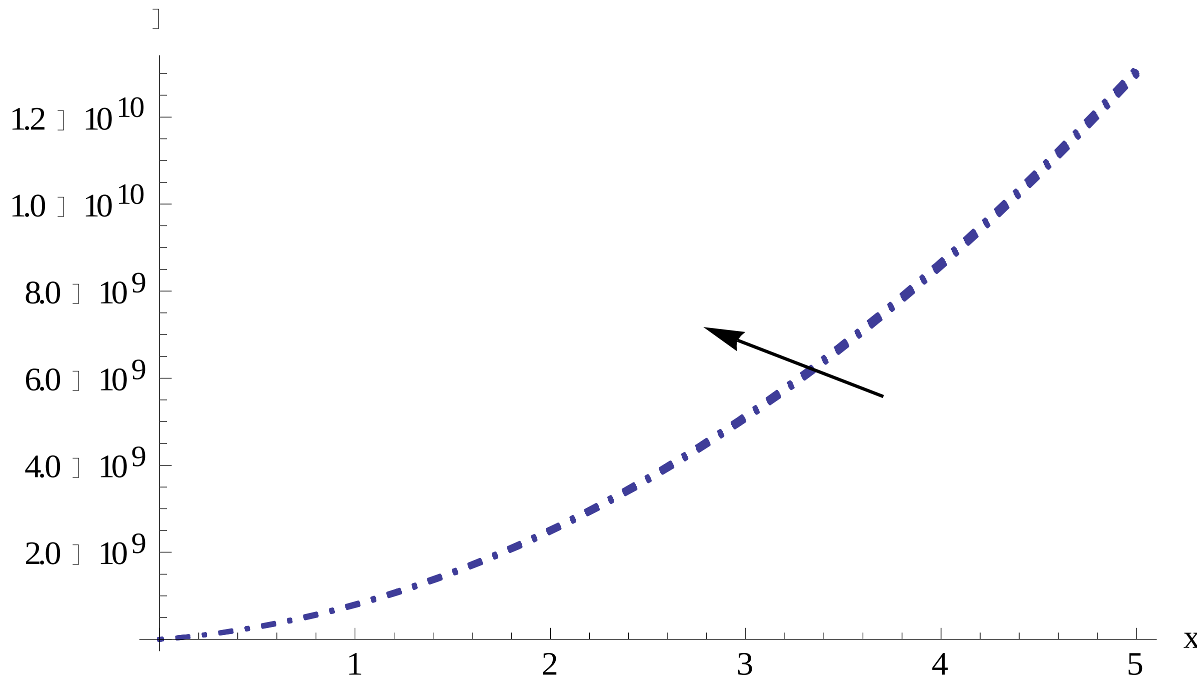

Figure 5. Showing increase of Semenov parameter (ψ) on temperature of Newtonian fluid.

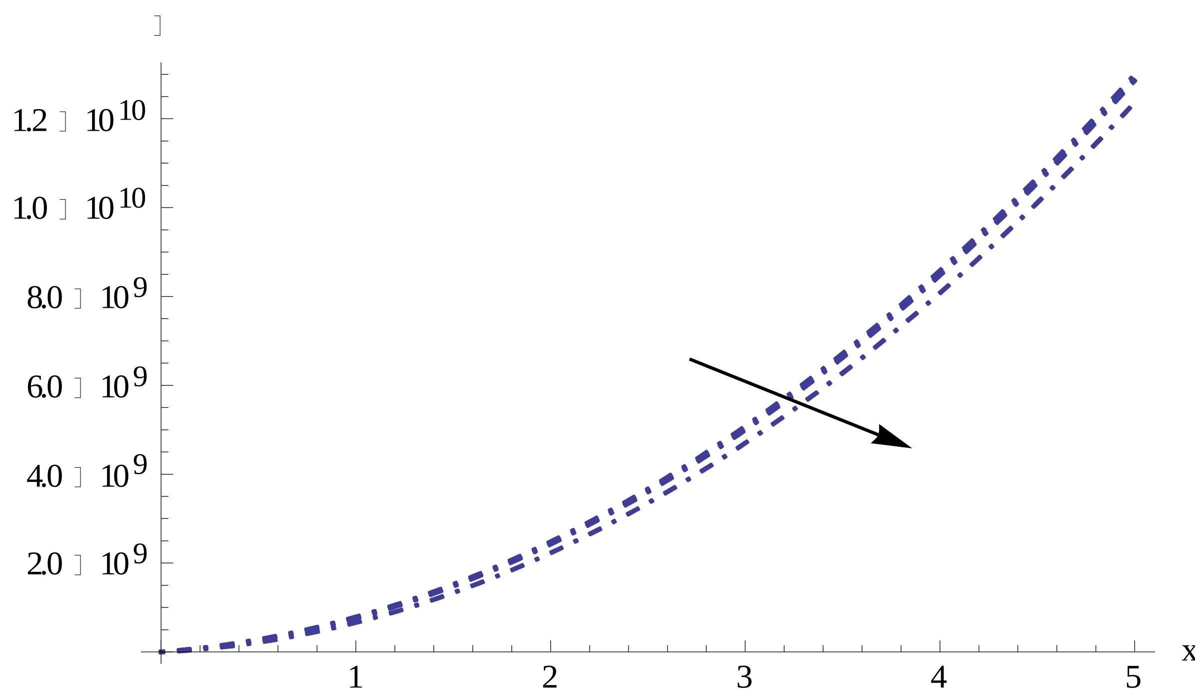

Figure 6. Showing increase of heat term ($\lambda_1 + \lambda$) on temperature of Newtonian fluid.

6. DISCUSSION AND CONCLOSION

In order to get physical insight and numerical validation of the problem, a typical value of universal fluid constant 8.31 and fluid density of water 1000 is as well as 2500 for non-Newtonian fluid density is chosen. The values of other parameters made use of are:

$\beta = 1.5, R' = 1.5, \rho' = 1000$, $\delta = 0.4, 0.8, 1.2, 1.6, 2.0$, $\psi = 0.7, 1.2, 1.7, 2.2, 2.7$
$(\lambda_1 + \lambda) = 4.2, 4.7, 5.2, 5.7, 6.2$

In the analysis, Figure 1 shows increase in Frank-Kamenestkii parameter on the temperature of Newtonian fluid. Increase in Frank-Kamenestkii parameter results in decrease in the temperature of the fluid. This temperature is observed as the minimum temperature of the reaction thereby slowing down initiation of thermal explosion. The same observation is found in figure 4 but the rate at which temperature decreases is more rapid. The observation is consistent with the work of [2], [11] and [16]. Figure 2 displays the effect of Semenov parameter on the temperature of a reacting fluid. It is observed increase in Semenov parameter decreases the temperature in both the Newtonian and non-Newtonian case (Figure 5). The decrease is more prominent in the Newtonian case than the non-Newtonian case and the observation is consistent with the work of [11] and [15]. Increase n heat term as shown in Figure 3 for the Newtonian case, shows corresponding increase in the temperature of the fluid. The additional heat source lead to early initiation of thermal explosion. The same observation is also found in the non-Newtonian case as depicted in Figure 6. The observation is consistent with the studies [11] and confirmed the assertion that an external heat source is necessary for the attainment of thermal equilibrium.

References

[1] Adegbie, K. S (2008). On the delay type behaviour of thermal explosion in a combustible gas mixture cointaning fuel droplets with Arrhenius power-law model. *Journal of Nigerian Association of Mathematical Physics,*13:69-82

[2] Goldfard, I., Sazhin, S. and Zinoviev, A. (2004). Delayed thermal explosion in flammable gas containing fuel droplets: Asymptotic analysis. *Journal of Egineering Mathematics,* 50: 399-414.

[3] Goldfard, I., Goldshtein, V. and Zinoviev, A. (2002). Delayed thermal explosion in porous media: Method of Invariant Manifolds. *IMA J of Applied Math.* 67: 263-280.

[4] Goldfard, I and Zinoviev, A (2003). A study of delayed spontaneous insulation fires. *Physics Letters A,* 311: 491-500.

[5] Goldfard, I., Goldshtein, V., Greenberg, J. B and Kuzmenko, G (2000). Thermal explosion in a droplet gas cloud. *Combustion Theory Modeling.* 4: 289-316.

[6] Goldfard, I., Goldshtein V., Kuzmenko, G and Sazhin, S (1999). Thermal radiation effect on thermal explosion in a gas containing fuel droplets. *Combustion Theory Modeling*. 3: 769-787.

[7] Goldshtein, V. and Sobolev, V. (1992). Integral manifolds in chemical kinetics and combustion in singularity theory and some problems of functional analysis. *AMS Translations series,* 2: 153-164

[8] Goldshtein, V., Goldfard, I., Shreiber, I. and Zinoview, A. (1998). Oscillations in a combustible gas bubble. *Combustion Theory Modeling.* 2: 1-17

[9] Goldfard, I., Goldshtein, V., Karz, D. and Sazhin, S. (2007). Radiation effect on thermal explosion in gas containing evaporated fuel droplets. *International Journal of Thermal Sciences.* 46: 358-370.

[10] Goldshtein, V. and Zinovien, A. (1997). Thermal explosion in multiphase media. Nonlinear Analysis Theory, *Method and Applications.* 30(8): 4771-4780.

[11] Ajadi, S. O and Gol'dshtein V (2010). Thermal explosion characteristics in the presence of an additional heat source. Journal of Mathematical sciences, 1:36-48

[12] Ayeni, R. O., Okedoye, A. M., Popoola, A. O. and Ayodele, T. O (2005). Effect of radiation on the critical Frank-kamenestkii parameter of thermal ignition in a combustible gas containing fuel droplets. *Journal of Nigerian Association of Mathematical Physics.* 9: 17-23.

[13] Lamidi, O. T. and Ayeni, R.O. (2007). Influence of power law index in an unsteady exothermic reaction. *Journal of Nigerian Association of Mathematical Physics*. 11: 545-548.

[14] Lamidi, O. T., Ajala, A. O., Okedoye, A. M. and Ayeni, R.O. (2008). Effect of power law exponent in endothermic reactions. *Journal of Nigerian Association of Mathematical Physics.* 13: 231-234.

[15] Ngiangia, A. T., Amadi, O. and Harry, S. T. (2013). Approximation of power law exponent to Newtonian fluids in reactions pathway. *International Journal of Dynamics of Fluids.* 9(1), 29-33.

[16] Ngiangia, A. T. (2015). The effect of Frank-Kamenstkii parameter on Newtonian fluids and non-Newtonian fluids. *Journal of Advances in Mathematics.* 10(8): 3705-3710

[17] Hughes, W. F and Brighton, J. A. (1999). Fluid Dynamics (Third edition). Schaum's Outlines, McGraw-Hill, New York.

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