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## Quark star model with Tolman VII Type Potential

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### ABSTRACT

In this paper, we studied the behavior of relativistic objects with charged anisotropic matter distribution within the framework of MIT-Bag Model considering Tolman VII form for the gravitational potential  $Z$ . A physical analysis of electromagnetic field indicates that is regular in the origin and well behaved. In the obtained solution there is a singularity in the charge density. We show as the presence of an electrical field causes a modification in the radial pressure, the tangential pressure, the anisotropic factor and the mass of the stellar object.

**Keywords:** Relativistic objects; anisotropic factor; MIT-Bag model; Tolman VII type potential; Charge density

### 1. INTRODUCTION

From the development of Einstein's theory of general relativity, the modelling of superdense matter configurations is an interesting research area [1,2]. In the last decades, such models allow explain the behavior of massive objects as neutron stars, quasars, pulsars, black holes and white dwarfs [3-5]. Malaver [3] studied the behavior of the thermal capacity  $C_v$  for Schwarzschild's black hole when  $T \gg T_C$  and  $T \ll T_C$  where  $T_C$  is the characteristic temperature of the Schwarzschild black hole and found that the value for  $C_v$  if  $T \gg T_C$  is the same that would be obtained in an ideal diatomic gas if only are considered the degrees of freedom rotational. Komathiraj and Maharaj [4] find new classes exact solutions to the

Einstein-Maxwell system of equations for a charged sphere with a particular choice of the electric field intensity and one of the gravitational potentials. Sharma et al. [5] have obtained a class of solutions to the Einstein-Maxwell system assuming a particular form for the hypersurface ( $t = \text{constant}$ ) containing a parameter  $\lambda$ .

In theoretical works of realistic stellar models, it is important to include the pressure anisotropy [6-8]. Bowers and Liang [6] extensively discuss the effect of pressure anisotropy in general relativity. The existence of anisotropy within a star can be explained by the presence of a solid core, phase transitions, a type III super fluid, a pion condensation [9] or another physical phenomena as the presence of an electrical field [10]. The physics of ultrahigh densities is not well understood and many of the strange stars studies have been performed within the framework of the MIT-Bag model [11]. In this model, the strange matter equation of state has a simple linear form given by  $p = \frac{1}{3}(\rho - 4B)$  where  $\rho$  is the energy density,  $p$  is the isotropic pressure and  $B$  is the bag constant. Many researchers have used a great variety of mathematical techniques to try to obtain exact solutions for quark stars within the framework of MIT-Bag model: Komathiraj and Maharaj [11] found two new classes of exact solutions to the Einstein-Maxwell system of equations with a particular form of the gravitational potential and isotropic pressure. Malaver [12,13] also has obtained some models for quark stars considering a potential gravitational that depends on an adjustable parameter. Thirukkanesh and Maharaj [14] studied the behavior of compact relativistic objects with anisotropic pressure in the presence of the electromagnetic field. Maharaj et al. [15] generated new models for quark stars with charged anisotropic matter considering a linear equation of state. Thirukkanesh and Ragel [16] obtained new models for compact stars with quark matter. Sunzu et al. found new classes of solutions with specific forms for the measure of anisotropy [17].

With the use of Einstein's field equations, important advances have been made to model the interior of a star. In particular, Feroze and Siddiqui [18,19] and Malaver [20,21] consider a quadratic equation of state for the matter distribution and specify particular forms for the gravitational potential and electric field intensity. Mafa Takisa and Maharaj [22] obtained new exact solutions to the Einstein-Maxwell system of equations with a polytropic equation of state. Thirukkanesh and Ragel [23] have obtained particular models of anisotropic fluids with polytropic equation of state which are consistent with the reported experimental observations. Malaver [24,25] generated new exact solutions to the Einstein-Maxwell system considering Van der Waals modified equation of state with and without polytropical exponent and Thirukkanesh and Ragel [26] presented an anisotropic strange quark matter model by imposing a linear barotropic equation of state with Tolman IV form for the gravitational potential. Bhar et al. [27] studied the behavior of uncharged relativistic objects with anisotropic matter distribution considering the Tolman VII form for the gravitational potential. Mak and Harko [28] found a relativistic model of strange quark star with the suppositions of spherical symmetry and conformal Killing vector.

The objective of this paper is to obtain new exact solutions to the Maxwell-Einstein system for charged anisotropic matter with the barotropic equation of state that presents a linear relation between the energy density and the radial pressure in static spherically symmetric spacetime using Tolman VII form for the gravitational potential  $Z$ . We have obtained a new class of static spherically symmetrical models where the presence of charge modifies the radial pressure, the tangential pressure and the mass of the compact objects. This

article is organized as follows, in Section 2, we present Einstein's field equations of anisotropic fluid distribution. In Section 3, we make a particular choice for gravitational potential  $Z(x)$  that allows solving the field equations and we have obtained a new model for charged anisotropic matter. In Section 4, a physical analysis of the new solutions is performed. Finally in Section 5, we conclude.

## 2. EINSTEIN FIELD EQUATIONS

We consider a spherically symmetric, static and homogeneous and anisotropic spacetime in Schwarzschild coordinates given by

$$ds^2 = -e^{2\nu(r)} dt^2 + e^{2\lambda(r)} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (1)$$

where  $\nu(r)$  and  $\lambda(r)$  are two arbitrary functions.

The Einstein field equations for the charged anisotropic matter are given by

$$\frac{1}{r^2}(1 - e^{-2\lambda}) + \frac{2\lambda'}{r} e^{-2\lambda} = \rho \quad (2)$$

$$-\frac{1}{r^2}(1 - e^{-2\lambda}) + \frac{2\nu'}{r} e^{-2\lambda} = p_r \quad (3)$$

$$e^{-2\lambda} \left( \nu'' + \nu'^2 + \frac{\nu'}{r} - \nu'\lambda' - \frac{\lambda'}{r} \right) = p_t \quad (4)$$

$$\sigma = \frac{1}{r^2} e^{-\lambda} (r^2 E)' \quad (5)$$

where  $\rho$  is the energy density,  $p_r$  is the radial pressure,  $E$  is electric field intensity,

$p_t$  is the tangential pressure and primes denote differentiations with respect to  $r$ . Using the transformations,  $x = cr^2$ ,  $Z(x) = e^{-2\lambda(r)}$  and  $A^2 y^2(x) = e^{2\nu(r)}$  with arbitrary constants  $A$  and  $c > 0$ , suggested by Durgapal and Bannerji [29], the metric (1) take the form

$$ds^2 = -A^2 y^2(x) dt^2 + \frac{1}{4cxz} dx^2 + \frac{x}{c} (d\theta^2 + \sin^2\theta d\phi^2) \quad (6)$$

and the Einstein field equations can be written as

$$\frac{1-Z}{x} - 2\dot{Z} = \frac{\rho}{c} + \frac{E^2}{2c} \quad (7)$$

$$4Z \frac{\dot{y}}{y} - \frac{1-Z}{x} = \frac{p_r}{c} - \frac{E^2}{2c} \quad (8)$$

$$4xZ \frac{\ddot{y}}{y} + (4Z + 2x\dot{Z}) \frac{\dot{y}}{y} + \dot{Z} = \frac{p_t}{c} + \frac{E^2}{2c} \quad (9)$$

$$p_t = p_r + \Delta \quad (10)$$

$$\frac{\Delta}{c} = 4xZ \frac{\ddot{y}}{y} + \dot{Z} \left( 1 + 2x \frac{\dot{y}}{y} \right) + \frac{1-Z}{x} - \frac{E^2}{c} \quad (11)$$

$$\sigma^2 = \frac{4cZ}{x} (x\dot{E} + E)^2 \quad (12)$$

$\sigma$  is the charge density,  $\Delta = p_t - p_r$  is the anisotropic factor and dots denote differentiation with respect to  $x$ . With the transformations of [28], the mass within a radius  $r$  of the sphere take the form

$$M(x) = \frac{1}{4c^{3/2}} \int_0^x \sqrt{x} \rho(x) dx \quad (13)$$

In this paper, we assume the following lineal equation of state within the framework of MIT-Bag model

$$p_r = \frac{1}{3} \rho \quad (14)$$

### 3. THE MODEL

Following Tolman [30] and Bhar et al. [27], we take the form of the gravitational potential,  $Z(x)$  as

$$Z(x) = 1 - ax + bx^2 \quad (15)$$

where a and b are real constants. We have considered the form of the electrical field proposed for Feroze and Siddiqui [19]

$$E^2 = \frac{2c(1-Z)}{x} \tag{16}$$

Using  $Z(x)$  and eq. (16) in eq.(7) we obtain

$$\rho = 2c(a - 2bx) \tag{17}$$

Substituting (17) in eq.(14), the radial pressure can be written in the form

$$P_r = \frac{2c}{3}(a - 2bx) \tag{18}$$

Using (17) in (13), the expression of the mass function is

$$M(x) = \frac{x^{3/2}(5a - 6bx)}{15\sqrt{c}} \tag{19}$$

for the electric field intensity  $E^2$

$$E^2 = 2c(a - bx) \tag{20}$$

and for charge density

$$\sigma^2 = \frac{2c^2(1 - ax + bx^2)(-3bx + 2a)^2}{x(a - bx)} \tag{21}$$

The tangential pressure is given for

$$P_t = c \frac{[19b^2x^3 - 31abx^2 + (16a^2 + 3b)x - 12a]}{9(1 - ax + bx^2)} \tag{22}$$

The measure of anisotropy  $\Delta$  can be written as

$$\Delta = c \frac{[31b^2x^3 - 49abx^2 + (22a^2 + 15b)x - 18a]}{9(1 - ax + bx^2)} \tag{23}$$

Substituting (16) and (18) in (8), we have

$$\frac{\dot{y}}{y} = \frac{(a-2bx)}{6(1-ax+bx^2)} \quad (24)$$

Integrating (24), we obtain

$$y(x) = c_1(1-ax+bx^2)^{-1/6} \quad (25)$$

The metric functions  $e^{2\lambda}$  and  $e^{2\nu}$  can be written as

$$e^{2\lambda} = \frac{1}{1-ax+bx^2} \quad (26)$$

$$e^{2\nu} = A^2 c_1^2 (1-ax+bx^2)^{-1/3} \quad (27)$$

The metric for this model is

$$ds^2 = -A^2 c_1^2 (1-ax+bx^2)^{-1/3} dt^2 + \frac{dx^2}{4xc(1-ax+bx^2)} + \frac{x}{c} (d\theta^2 + \sin^2 \theta d\phi^2) \quad (28)$$

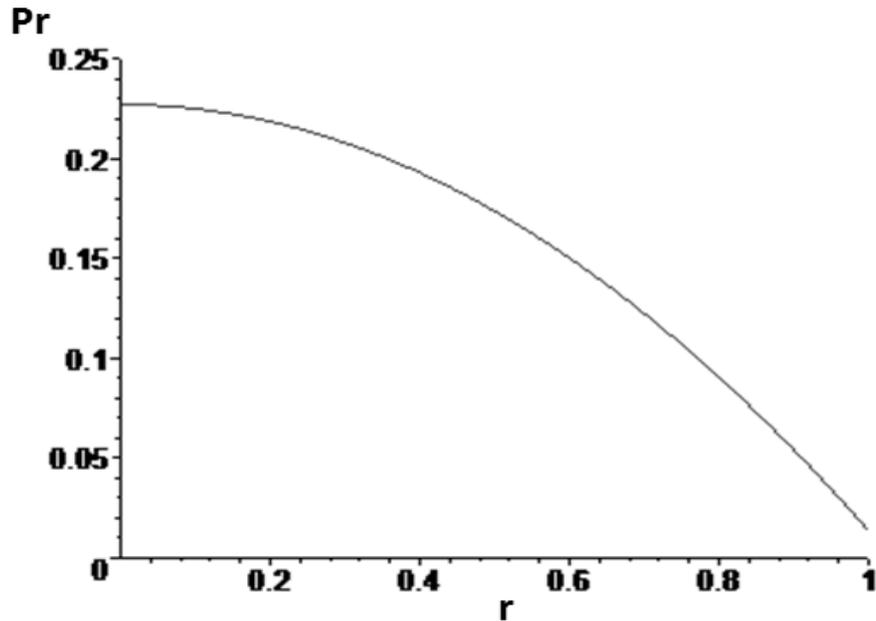


Fig. 1. Radial Pressure.

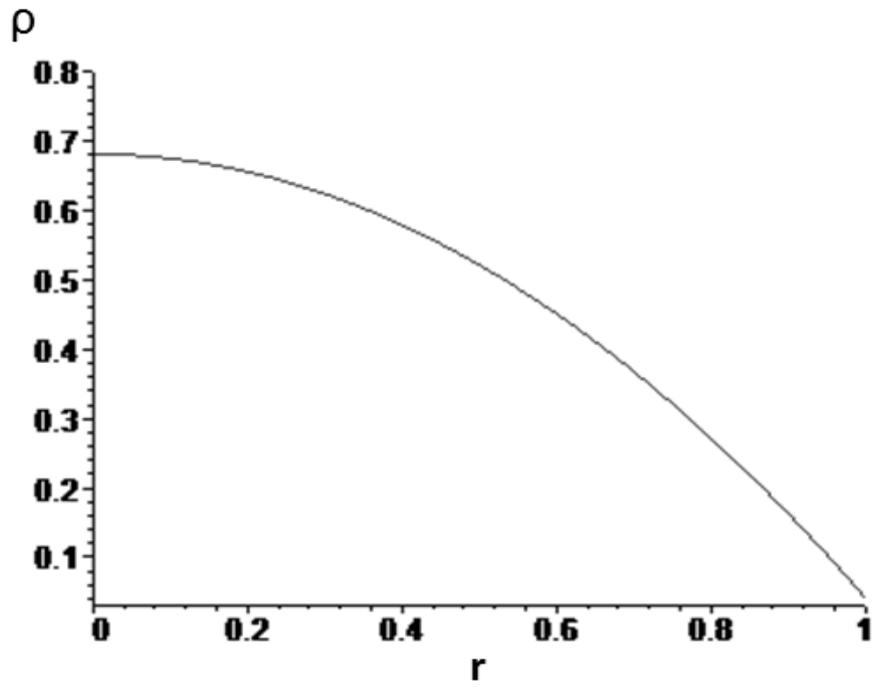


Fig. 2. Energy density.

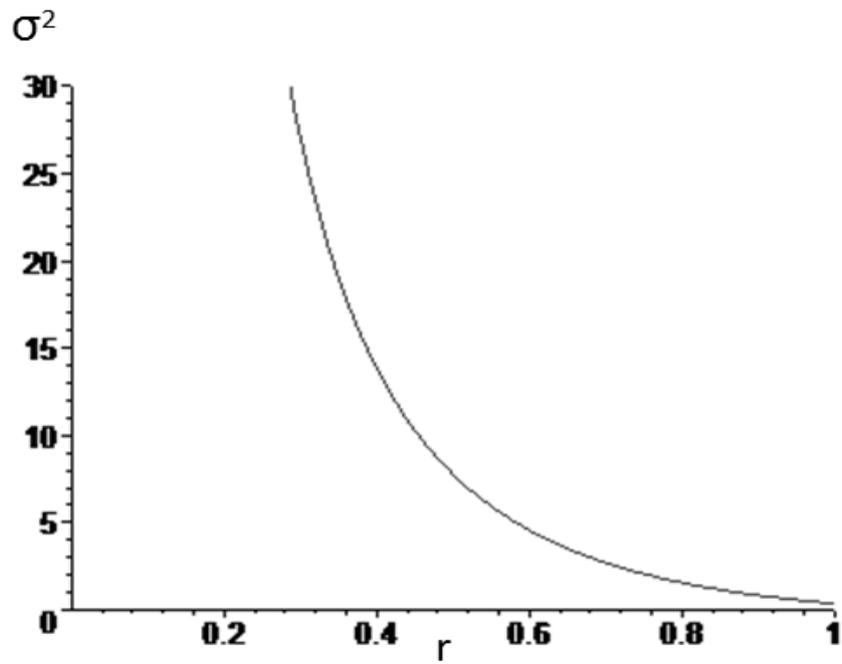


Fig. 3. Charge density.

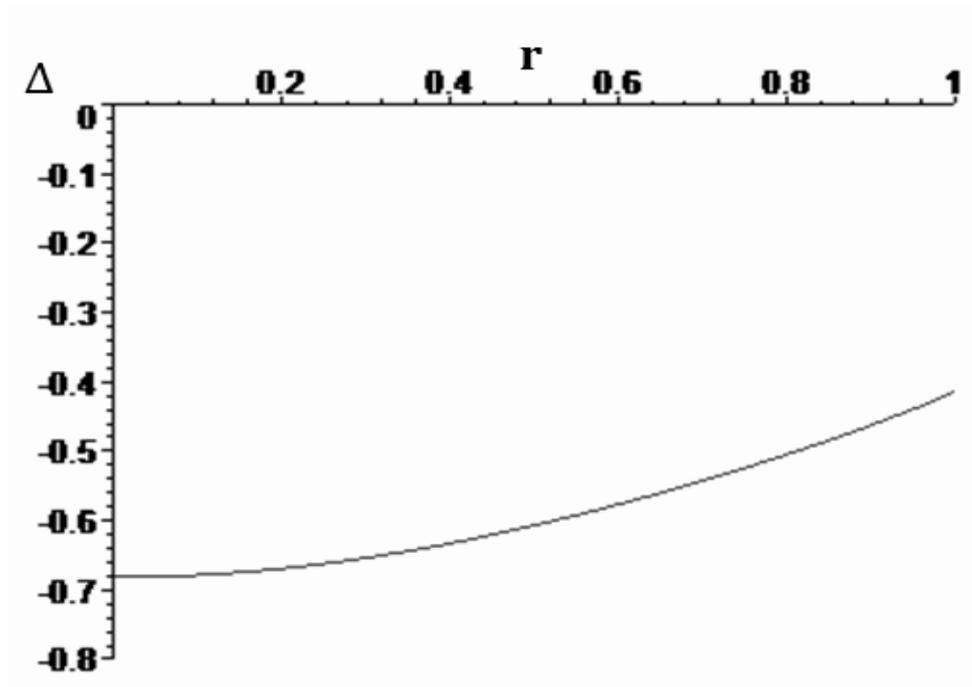


Fig. 4. Anisotropy.

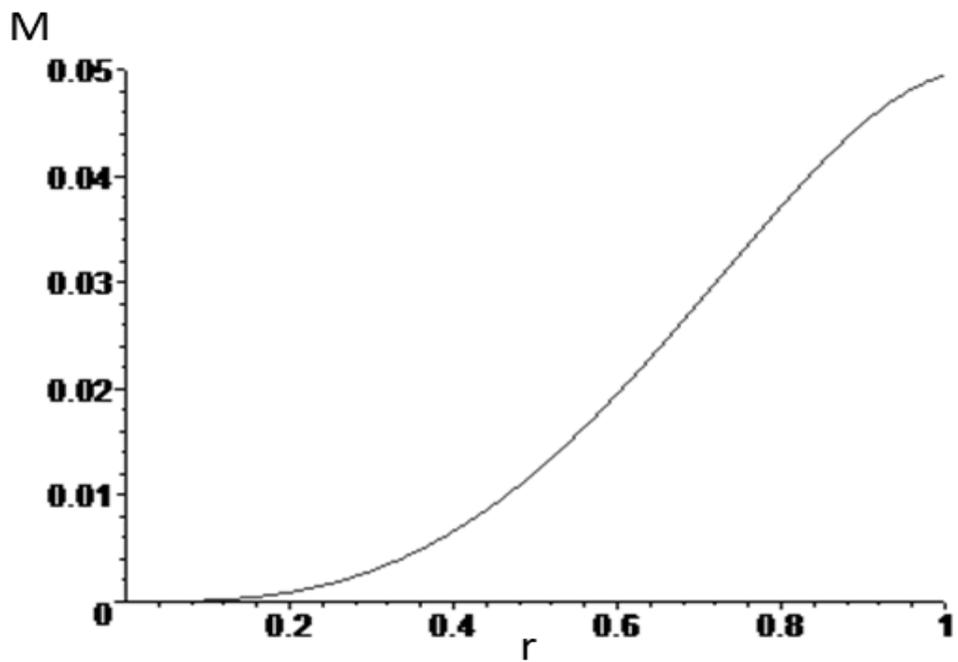


Fig. 5. Mass.

Figures 1, 2, 3, 4 and 5 represent the graphs of  $P_r$ ,  $\rho$ ,  $\sigma^2$ ,  $\Delta$  and  $M(x)$ , respectively. The graphs has been plotted for a particular choice of parameters  $a = 0.341$ ,  $b = 0.1603$  with a stellar radius of  $r = 1$  km.

#### 4. PHYSICAL ANALYSIS

Any physically acceptable solutions must satisfy the following conditions:

- (i) Regularity of the gravitational potentials in the origin.
- (ii) Radial pressure must be finite at the centre.
- (iii)  $P_r > 0$  and  $\rho > 0$  in the origin.

The new models satisfy the system of equations (7) - (12) and constitute another new solution for a charged quark star with anisotropy. The metric functions  $e^{2\lambda}$  and  $e^{2\nu}$  can be written in terms of elementary functions and the variables energy density, pressure and charge density also are represented analytical. In  $r = 0$  we have  $e^{2\lambda(0)} = 1$ ,  $e^{2\nu(0)} = A^2 c_1^2$  and  $(e^{2\lambda(r)})'_{r=0} = (e^{2\nu(r)})'_{r=0} = 0$ . This shows that the potential gravitational is regular in the origin. In the centre  $\rho(0) = 2ca$ ,  $P_r = \frac{2}{3}ca$ . In this new solution, the mass function is continuous and behaves well inside of the star and the charge density  $\sigma$  has a singularity at the center.

In Figure 1, the radial pressure is finite and decreasing with the radial coordinate. In Figure 2, that represent energy density we observe that is continuous, finite and monotonically decreasing function. In Figure 3, the charge density  $\sigma$  is singular at the origin, non-negative and decreases. The anisotropic factor  $\Delta$  is shown in Figure 4. The figure indicates that  $\Delta < 0$  in this model and is finite and continuous increasing function. The mass is an increasing function with the radial distance, continuous and  $M(x) = 0$  at  $r = 0$  as indicated in Figure 5.

#### 5. CONCLUSION

In this paper, we have generated new exact solutions to the Einstein-Maxwell system considering Tolman VII form for the gravitational potential  $Z$  with a linear equation of state which is relevant in the description of charged anisotropic matter. The new obtained model may be used to model relativistic stars in different astrophysical scenes. The relativistic solutions to the Einstein-Maxwell systems presented are physically reasonable. The charge density  $\sigma$  admits a singularity at the centre of the stellar object and the mass function is an increasing function, continuous and finite. The gravitational potentials are regular at the centre and well behaved.

We show as a presence of charge affects the radial pressure, tangential pressure, anisotropic factor and the mass of the stellar object when considering a Tolman VII type

potential. The model presented in this article may be useful in the description of relativistic compact objects with charge, strange quark stars and configurations with anisotropic matter.

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